Muon Colliders

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Contents

1 INTRODUCTION

WHY CONSIDER A MUON COLLIDER

Why are leptons (e.g. e or μ) 'better' than protons

- Protons are made of many pieces (quarks and gluons)
- Each carries only a fraction of th proton energy
- Fundamental interactions occur only between these individual pieces
- And the interaction energies are only fractions ($\approx 1/10$) of the proton energies

- Leptons (e's and μ's) are point like
- Their interaction energies are their whole energies

$$\text{E(3 TeV } e^+e^- \text{ CLIC or } \mu^+\mu^-) \equiv 2 \times \text{E(14 TeV } p\bar{p} \text{ LHC)}$$

- In addition the energy and quantum state is known for e^+e^-or $\mu^+\mu^-$ but unknown for the parton-parton interaction with protons

S-Channel advantage of muons over electrons

- When all the collision energy $\longrightarrow$ a single state, it is called the "S-Channel"
- A particularly interesting S-Channel interaction would be

$$e^+e^- \longrightarrow Higgs \qquad or \qquad \mu^+\mu^- \longrightarrow Higgs$$

The cross sections σ for these interactions

$$\sigma \propto m^2$$

so

$$\sigma(e^+e^- \longrightarrow H) \approx 40,000 \times \sigma(\mu^+\mu^- \longrightarrow H)$$

Muons generate less 'Beamstrahlung'

- When high energy electrons in one bunch pass through the other bunch they see the EM fields of the other moving bunch
- These fields are enough to generate synchrotron radiation (called beamstahlung)
- So the energy of the collision is not so well known

$$\sigma_E \approx 30\% \text{ (at 3 TeV } e^+e^- \text{ CLIC)}$$

- And the luminosity at the requires energey is less

$$\mathcal{L} \approx 1/3 \text{ (for } E \pm 1\% \text{ at 3 TeV CLIC)}$$

- But for muons the synchrotron radiation ($\propto 1/m^3$) is negligible
- This could be a particular advantage for $\mu^+\mu^- \rightarrow H$ because with a narrow enough σ_E one could measure the width of a narrow Higgs

Why are Linear colliders linear?

- Earlier electon positron colliders (LEP), like proton colliders, were rings
- But proposed high energy electron colliders are linear

WHY

- Synchrotron radiation of particles bent in the ring magnetic field

$$\Delta E(\text{per turn}) = \left(\frac{4\pi\ mc^2}{3}\right)\ \left(\frac{r_o}{\rho}\right)\ \beta_v^3\gamma^4 \tag{1}$$

$$\rho \propto \frac{\beta\gamma}{B} \tag{2}$$

$$\Delta E(\text{per turn}) \approx\propto \ B\ \gamma^3 \tag{3}$$

- For electrons (m≈ 0.5 MeV) this becomes untenable for $E >> 0.1$ TeV
- Above this (LEP's) energy, electron colliders must be linear

- But for muons (m≈ 100 MeV) rings are ok up to around 20 TeV equivalent to a proton collider of 200 TeV

The advantages of rings

- Muon go round a ring many times
  - Muons live 2 μ seconds   at the speed of light that is only 150 m   But

$$\tau_{\text{lab frame}} = \tau_{\text{rest frame}} \times \gamma$$

  - For a 1 TeV muon: $\gamma \approx$ 10,000 $\tau \approx 20$ msec   they go 1500 km
  - For $< B >$=10 T, a 1 TeV ring will have a circumference of

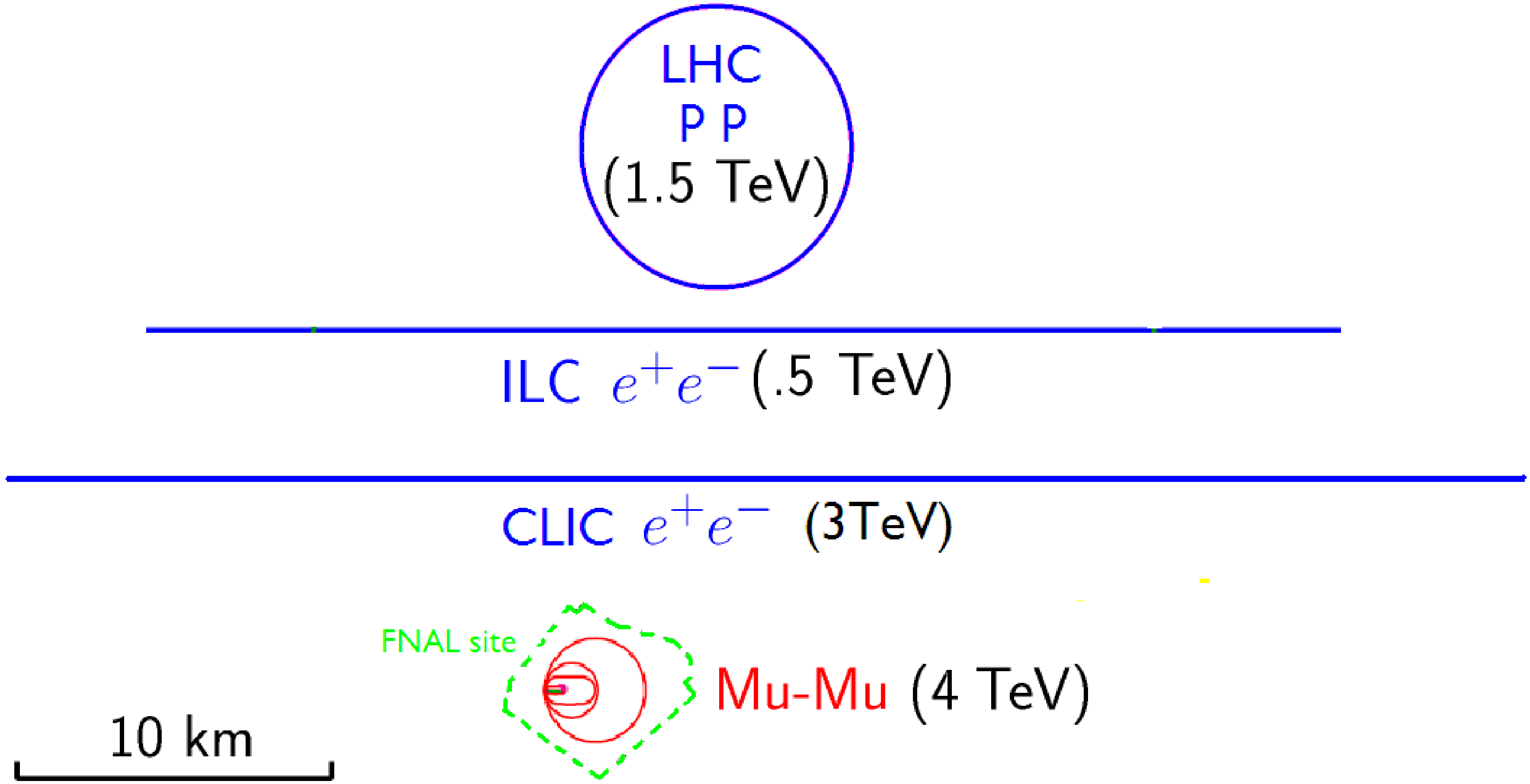
$$C = \frac{2\pi\ [pc/e]}{c\ B} = \frac{2\pi\ 10^{12}}{3\ 10^{8}\ 10} = 2 \quad \text{km}$$

    so they will go round , on average, 1500/2=700 times
  - For the same luminosity, the spot is 700 times larger than in a linear collider $\rightarrow$ easier tolerances

- There can be 2 or more Detectors
  giving an even larger total luminosity gain

- Acceleration must also be fast, in a number of turns $<<$ 700
  still much easier than in the single pass required for e^+e^-

So they are much smaller



And hopefully cheaper

Luminosity Dependence

$$\mathcal{L} \;=\; n_{\rm turns}\; f_{\rm bunch}\; \frac{N_\mu^2}{4\pi\sigma_\perp^2} \tag{4}$$

$$\Delta\nu \;=\; \frac{N r_o\, \beta^*}{4\pi\gamma\sigma_\perp} \;=\; \frac{r_o}{4\pi}\frac{N_\mu}{\epsilon_\perp} \tag{5}$$

$\epsilon_\perp$ is the normalized rms emittance

$$\mathcal{L} \;\propto\; B_{\rm ring}\; P_{\rm beam}\;\; \Delta\nu\; \frac{1}{\beta^*}$$

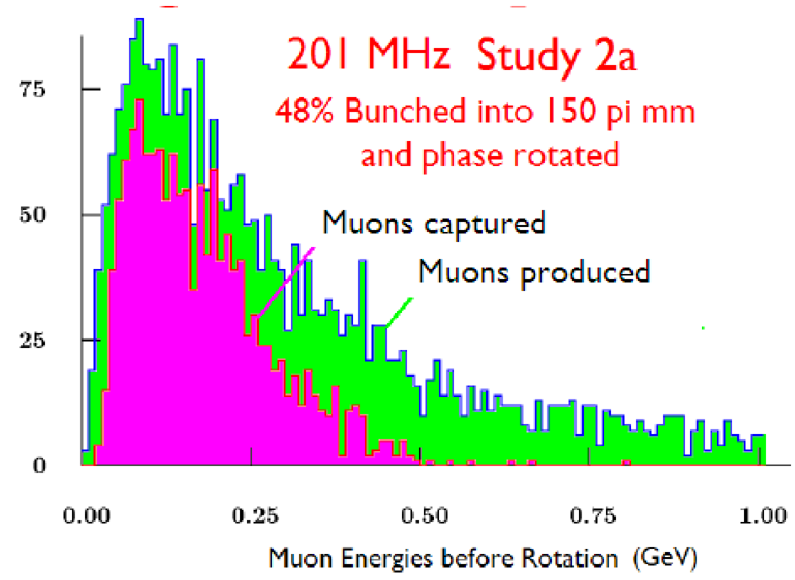
- Higher $\mathcal{L}/P_{\rm beam}$ requires lower β^* or correction of $\Delta\nu$
- Lower emittances do not directly improve Luminosity/Power
- But for fixed $\Delta\nu$, $\epsilon_\perp$ must be pretty small to avoid N_μ becoming unreasonable

The same luminosity easy with $\mu - p$

- Probably with another ring
- The event rate per bunch crossing is now significant but $\ll$ LHC
- Needs study

Why NOT a $\mu^+\mu^-$ collider

- Make muons from the decay of pions
- With pions made from protons on a target
- To avoid excessive proton power, we must capture a large fraction of pions made
  - Use a high field solenoid
    Captures most transverse momenta
  - Use Phase rotation
    Captures most longitudinal moments

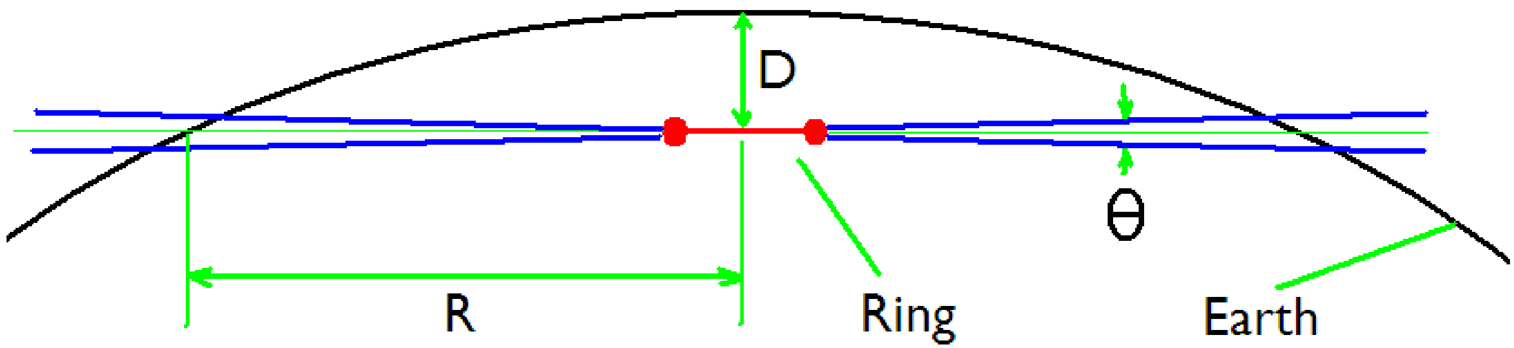


- We capture both forward and backward decays and lose polarization
- The phase space of the pions is now very large:
  - a transverse emittance of 20 pi mm and
  - a longitudinal emittance of 2 pi m
- These emittances must be somehow be cooled by a factor of order 10^7 !
  - $\approx$ 1000 in each transverse direction and
  - 40 in longitudinal direction

Cooling Methods

- Electrons are typically cooled (damped) by synchrotron radiation
  but muons radiate too little ($\Delta E \ \propto \ 1/m^3$)
- Protons are typically cooled by:
  - a co-moving cold electron beam
    too slow
  - Or by stochastic methods
    too slow and only works for low intensities ($\tau \ \propto \ 1/\sqrt{N}$)
- Ionization cooling is probably the only hope
- Although optical stochastic cooling after ionization cooling might be useful for very high energies

Neutrino Radiation Constraint



$$\text{Radiation} \;\propto\; \frac{E_\mu\ I_\mu\ \sigma_\nu}{\theta\ R^2} \;\propto\; \frac{I_\mu\ \gamma^3}{D}$$

$$\text{Radiation} \;\propto\; \frac{\mathcal{L}\ \beta_\perp}{\Delta\nu\ \ <\mathrm{B}>}\ \frac{\gamma^2}{D} \tag{6}$$

For fixed $\Delta\nu$, $\beta_\perp$ and $< B >$; and $\mathcal{L} \propto \gamma^2$:

$$\text{Radiation} \;\propto\; \frac{\beta_\perp}{\Delta\nu\ \ < B >\ \ D}\ \gamma^4 \tag{7}$$

For 3 TeV: D=135 m R=40 Km $\beta_\perp$=5 mm
For 6 TeV: D=540 m R=80 Km $\beta_\perp$=2.5 mm & higher $< B >$

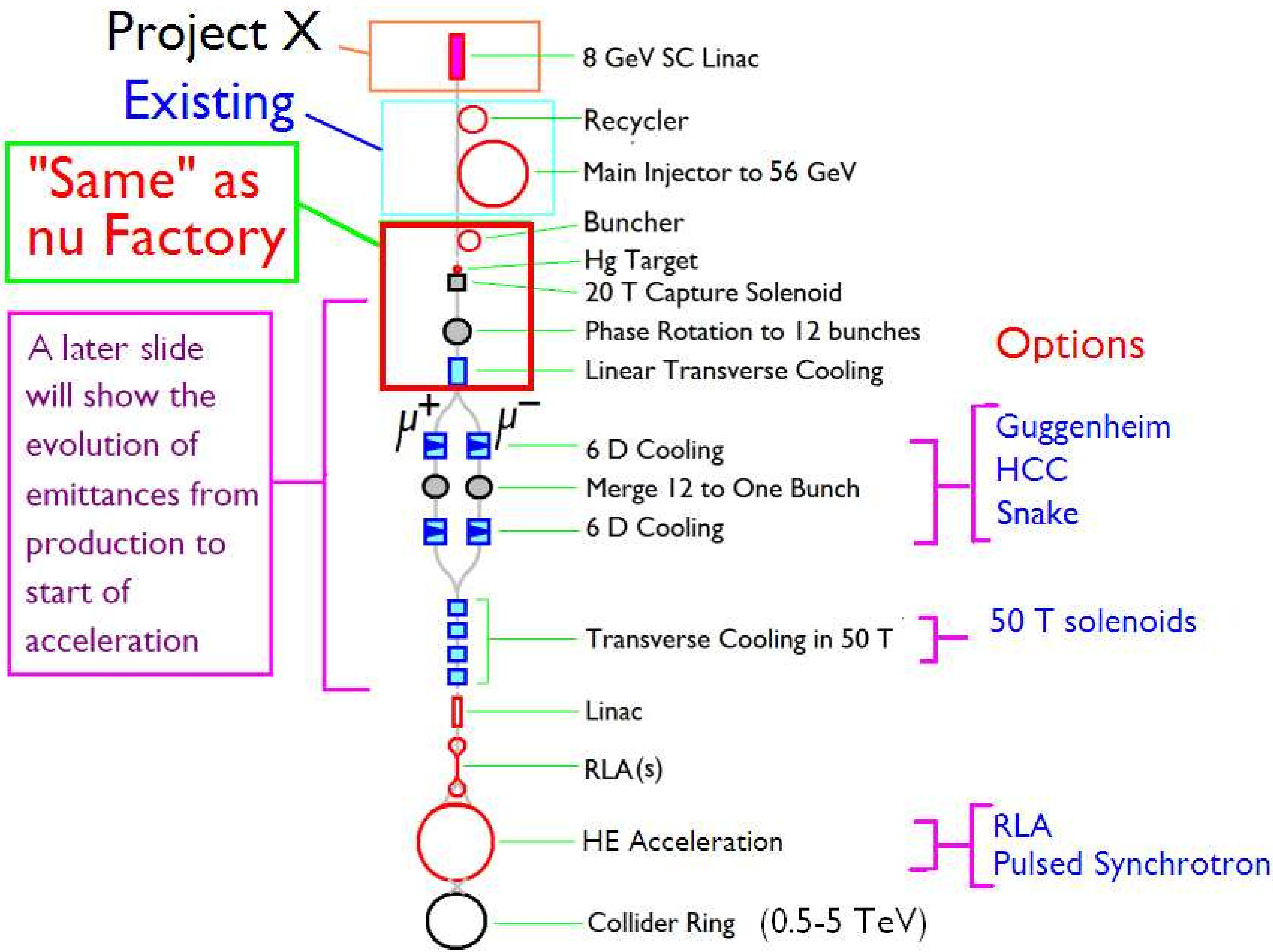
Conclusions on 'Why a muon collider"

- Point like interactions as in linear e^+e^-
  effective energy 10 times hadron machines
- Negligible synchrotron radiation:
  - Acceleration in rings
  - Small footprint
  - Less rf
  - Hopefully cheaper
- Collider is a Ring $\approx$ 1000 crossings per bunch
  - Larger spot
  - Easier tolerances
  - 2 or more Detectors
- Negligible Beamstrahlung Narrow energy spread
- 40,000 greater S channel Higgs Enabling study of widths
- But serious challenge to cool the muons by $\gg 10^7$ times
- Neutrino radiation a significant problem at very high energies
- CLIC better understood, but is it affordable?

CURRENT BASELINE DESIGNS
Parameters

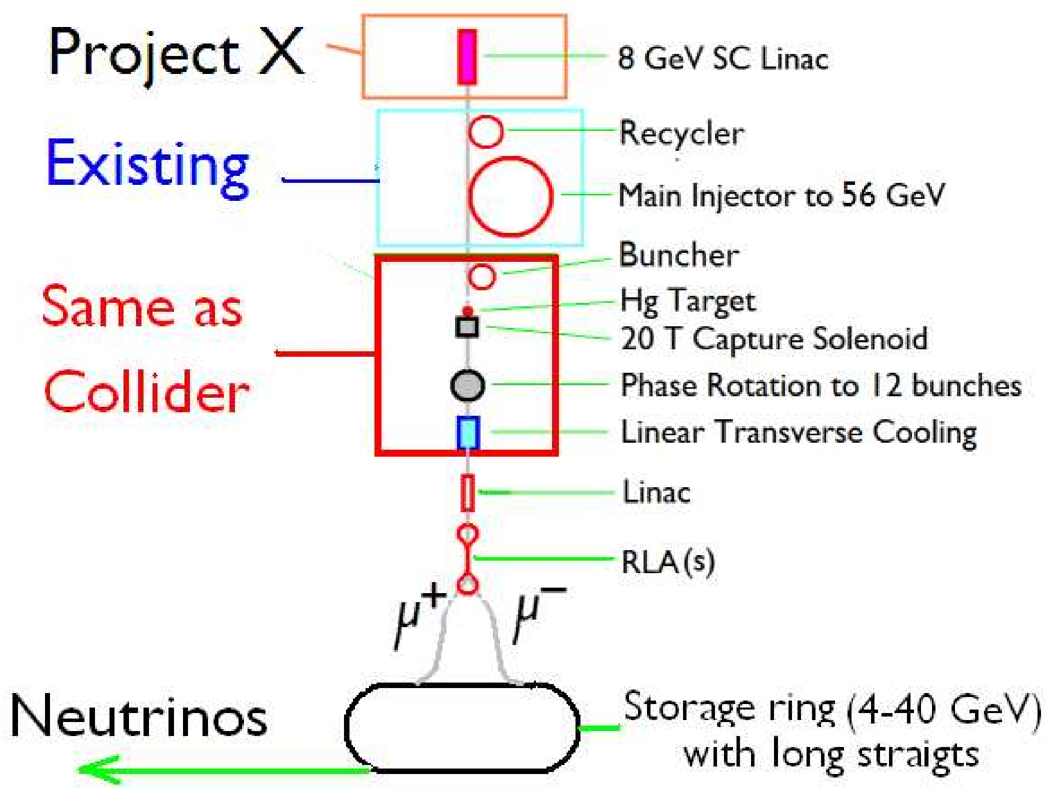
C of m Energy	1.5	3	TeV
Luminosity	0.77	3.4	10^{34} cm^{2}sec^{-1}
Beam-beam Tune Shift	0.087	0.087	
Muons/bunch	2	2	10^{12}
Total muon Power	9	15	MW
Ring $<$bending field$>$	6	8.4	T
Ring circumference	3.1	4.5	km
β^* at IP $= \sigma_z$	10	5	mm
rms momentum spread	0.1	0.1	%
Muon per 8 GeV p	0.008	0.007	
Repetition Rate	15	12	Hz
Proton Driver power	4.8	4.3	MW
Muon Trans Emittance	25	25	pi mm mrad
Muon Long Emittance	72,000	72,000	pi mm mrad

- Based on real Collider Ring designs, though both have problems
- Emittance and bunch intensity requirement same for both examples
- 3 TeV luminosity comparable to CLIC's (for dE/E $< 1\%$)

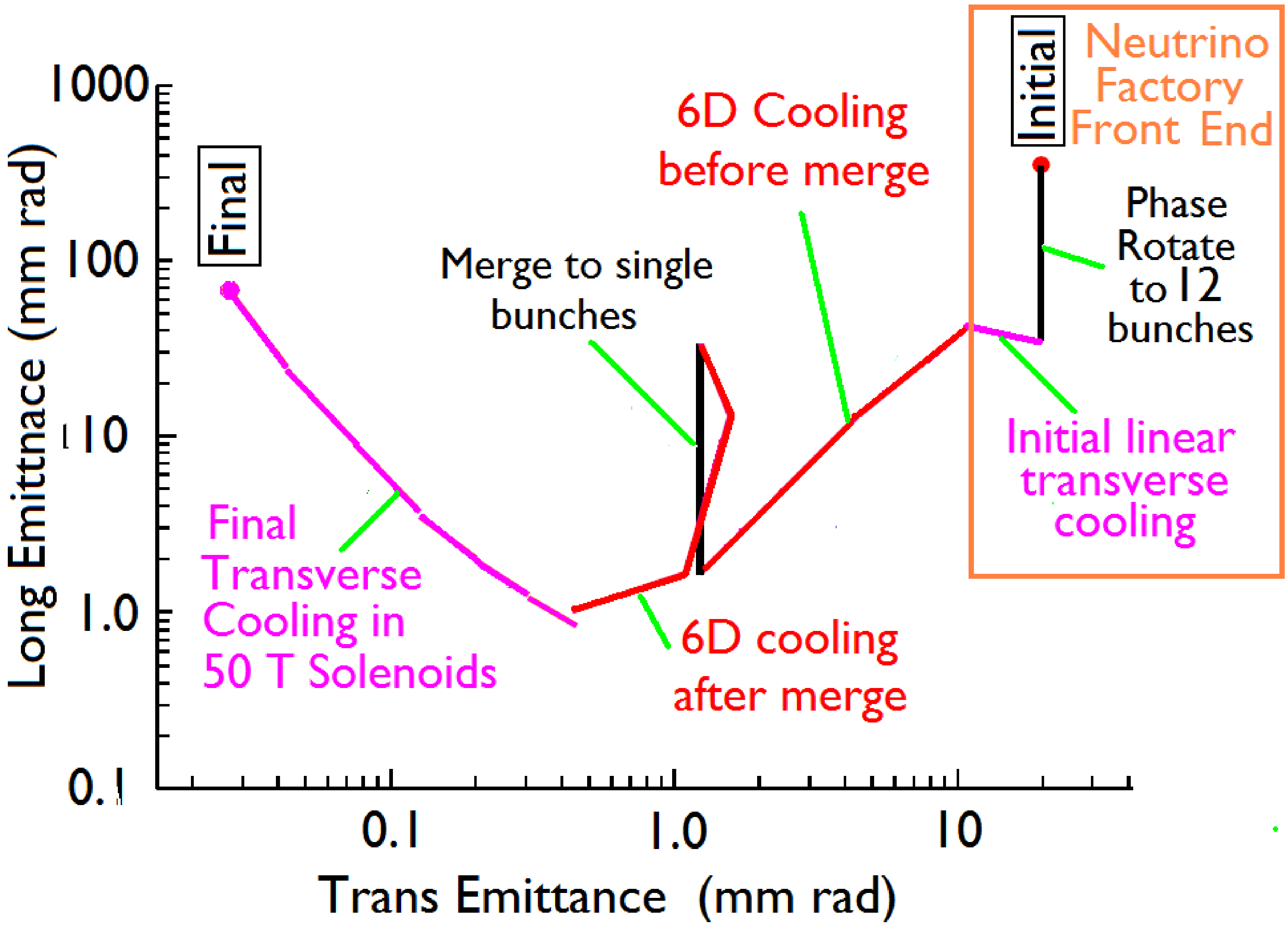

Project X
Existing
"Same" as nu Factory
A later slide will show the evolution of emittances from production to start of acceleration
8 GeV SC Linac
Recycler
Main Injector to 56 GeV
Buncher
Hg Target
20 T Capture Solenoid
Phase Rotation to 12 bunches
Linear Transverse Cooling
μ^+ μ^-
6 D Cooling
Merge 12 to One Bunch
6 D Cooling
Transverse Cooling in 50 T
Linac
RLA(s)
HE Acceleration
Collider Ring (0.5-5 TeV)
Options
Guggenheim
HCC
Snake
50 T solenoids
RLA
Pulsed Synchrotron

What is a Neutrino Factory ?

- The Muon Collider came first, then the neutrino radiation problem, then the idea of using that radiation
- Its muon energy would be 4-40 GeV instead of 0.5 to 5 TeV
- A lot of cooling is not needed. Only enough to fit in the acceleration
- Flux of muons is what counts. More cooling means more decay and less flux, so a lot of cooling is actually bad

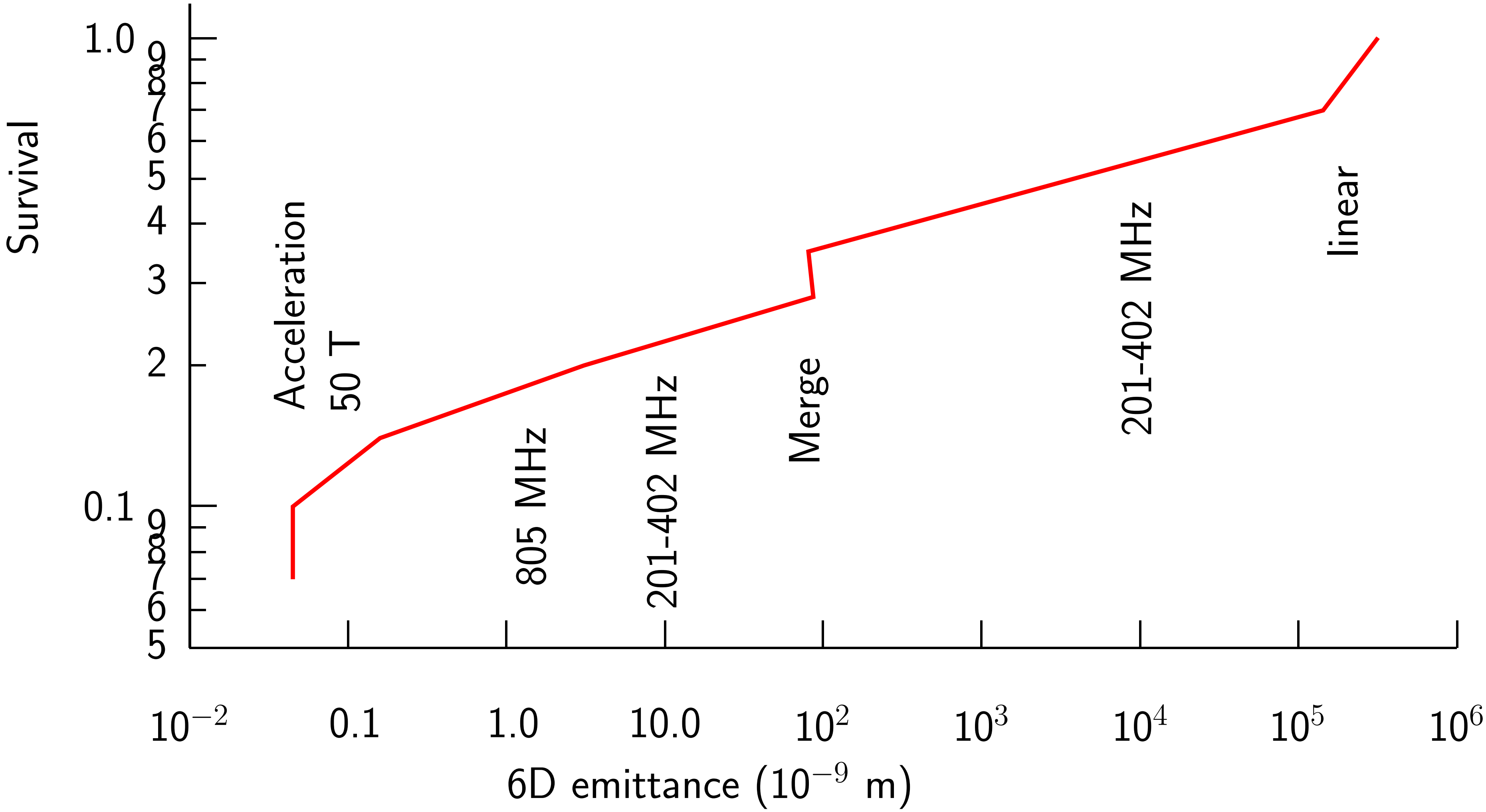


Emittances vs. Stage



- Every stage simulated at some level,
- But with many caveats

Estimated losses vs 6D emittance



- Only 7% survive
- This means that the initial pion, and thus proton, bunches must be intense
- Much more intense than IDS specification for a Neutrino Factory

Proton driver

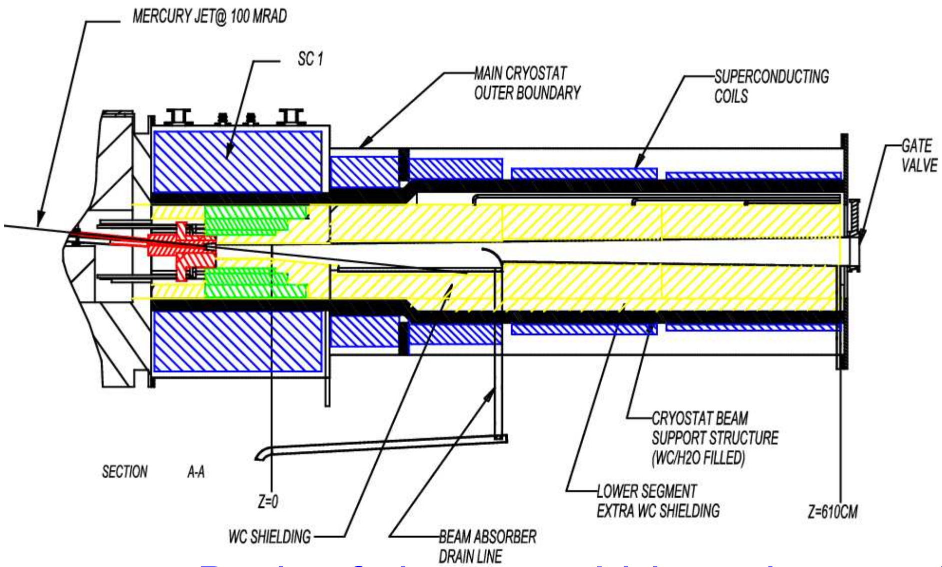
- Project X (8 GeV H^- linac),
- Together with accumulation in the Re-cycler
- And acceleration to 56 GeV in the Main Injector
- 35 Tp per 3 ns bunch requires separate buncher
- Could provide the required 15 Hz protons with power = 4.8 MW

- But pion production per MW greater at 8 GeV
- Then 250 Tp per 3 ns bunch
  severe space charge requires very large acceptance

- This driver could meet Factory requirement, but the reverse need not be true

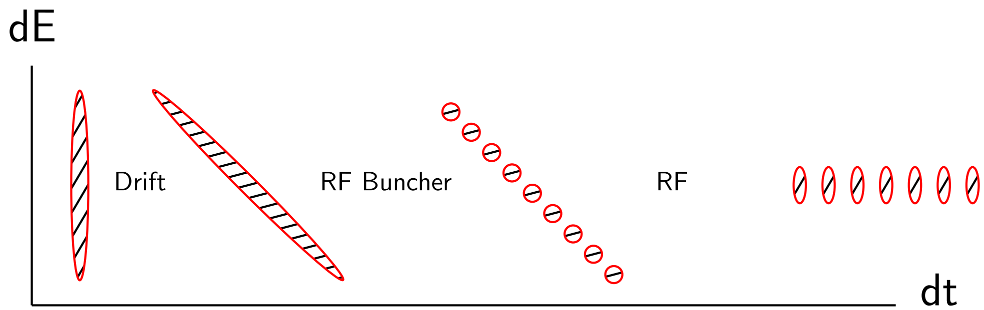
Target and Capture

Mercury Jet Target, 20 T capture
Adiabatic taper to 2 T



Phase Rotation

Drifts & Multiple frequency rf
to Bunch, then Rotate



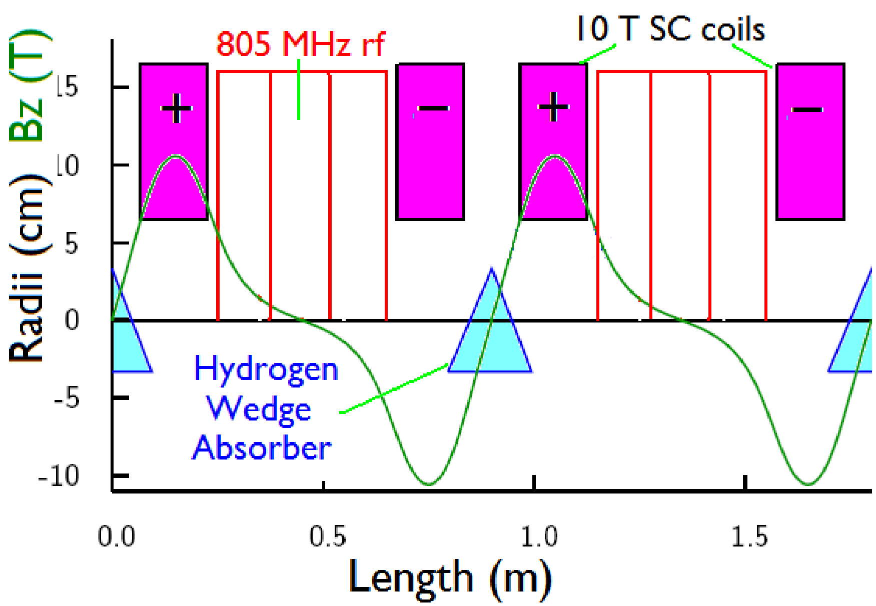
Both of these would be substantially the same as for a Neutrino Factory

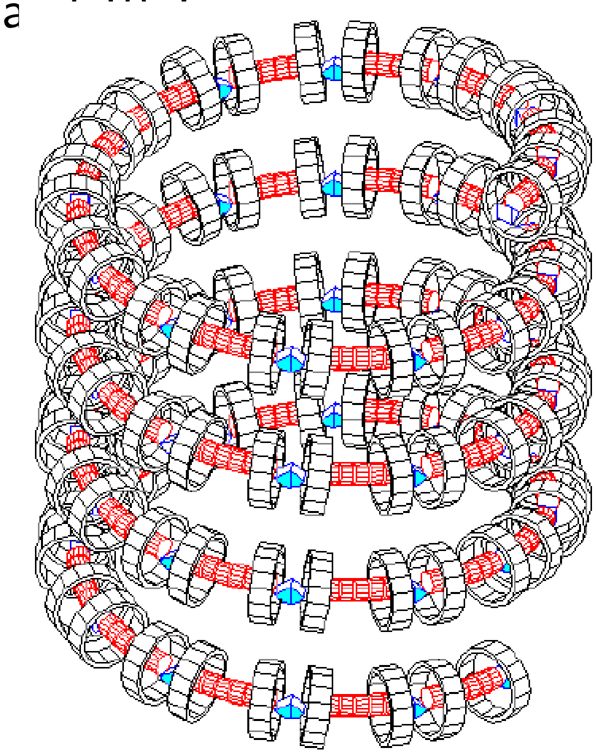
6D Cooling   Several methods under study

a) "Guggenheim" Lattice

- Lattice arranged as 'Guggenheim' upward helix
- Bending gives dispersion
- Higher momenta pass through longer paths in wedge absorbers giving momentum cooling (emittance exchange)
- Starting at 201 MHz and 3 T, ending at 805 MHz a

e.g. 805 MHz 10 T cooling to 400 mm mrad

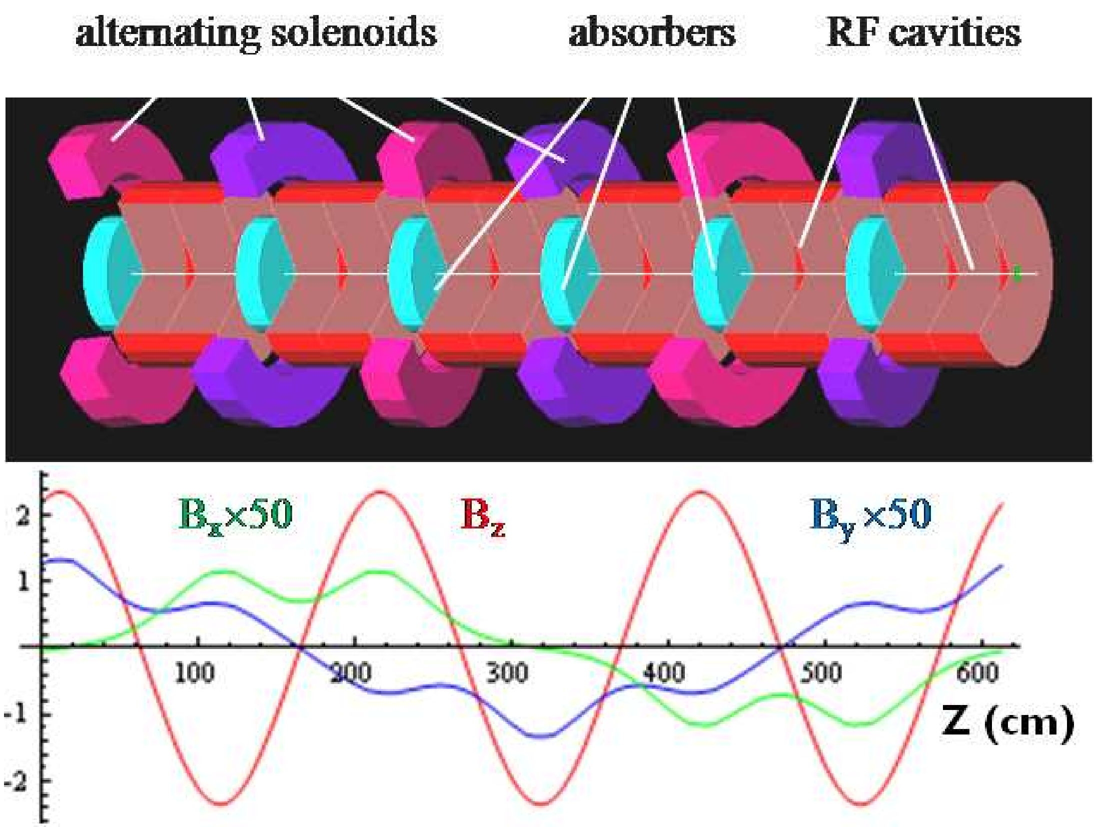




6D Cooling   Several methods under study

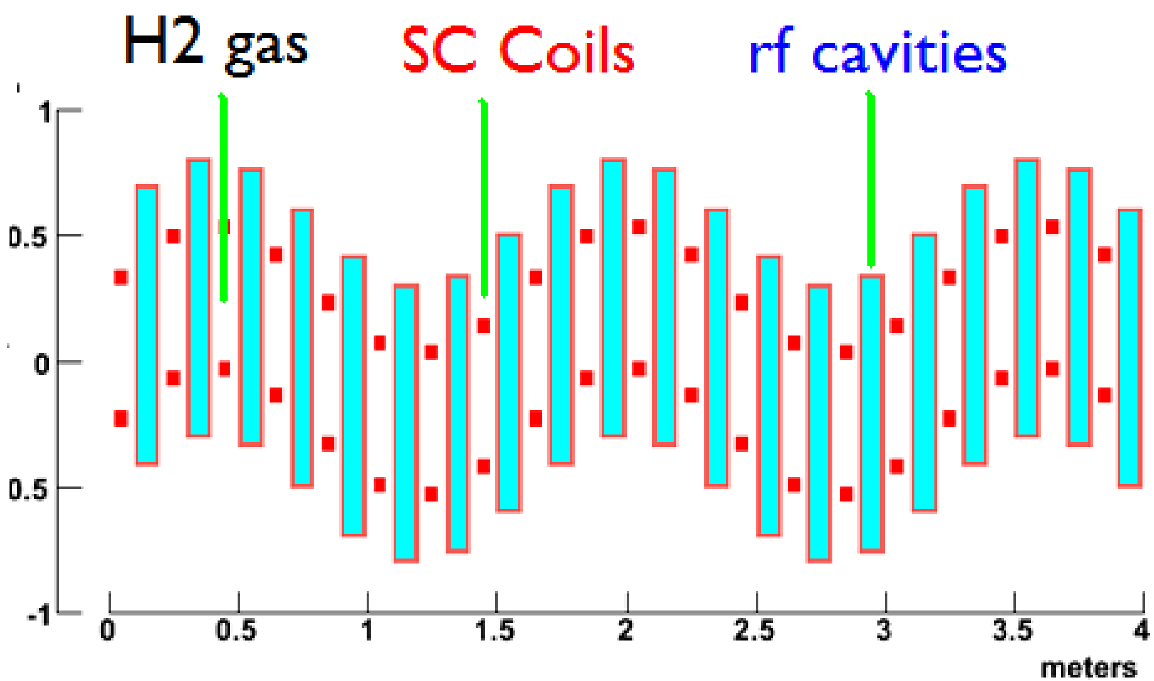
b) Snake

- Tilted alternating solenoids generate alternating dispersion
- Higher momenta pass through absorbers at steeper angles giving momentum cooling (emittance exchange)
- Lattice accepts both signs
- Starting at 201 MHz and 2.5 T, ending at 805 MHz and 10 T



c) Helical Cooling Channel (HCC)

- Muons move in helical paths in high pressure hydrogen gas
- Higher momentum tracks have longer trajectories giving momentum cooling (emittance exchange)



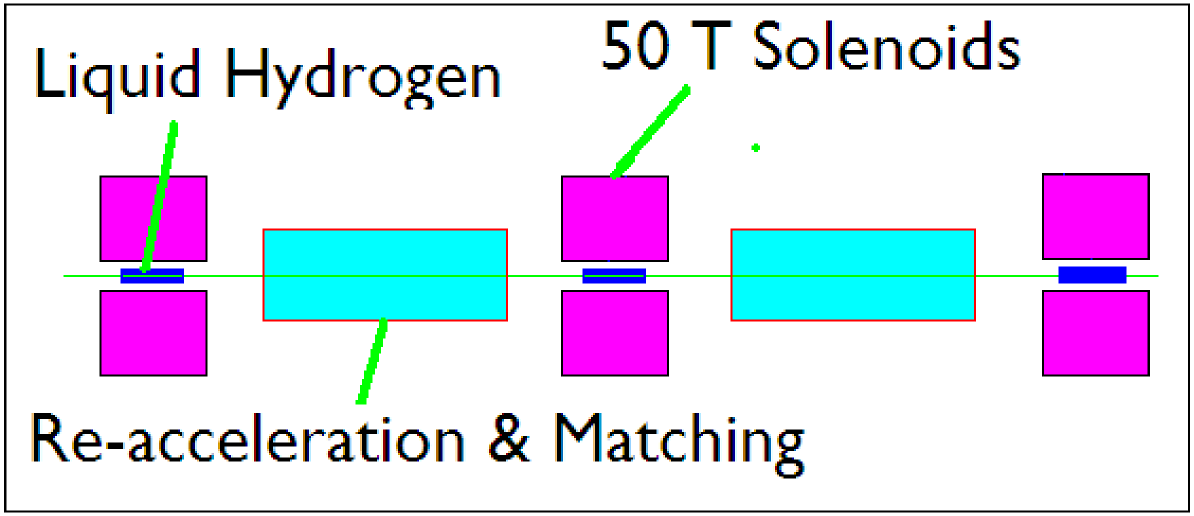
- Initial Bz=4.3 T
- Final Bz=17.2 T !
-

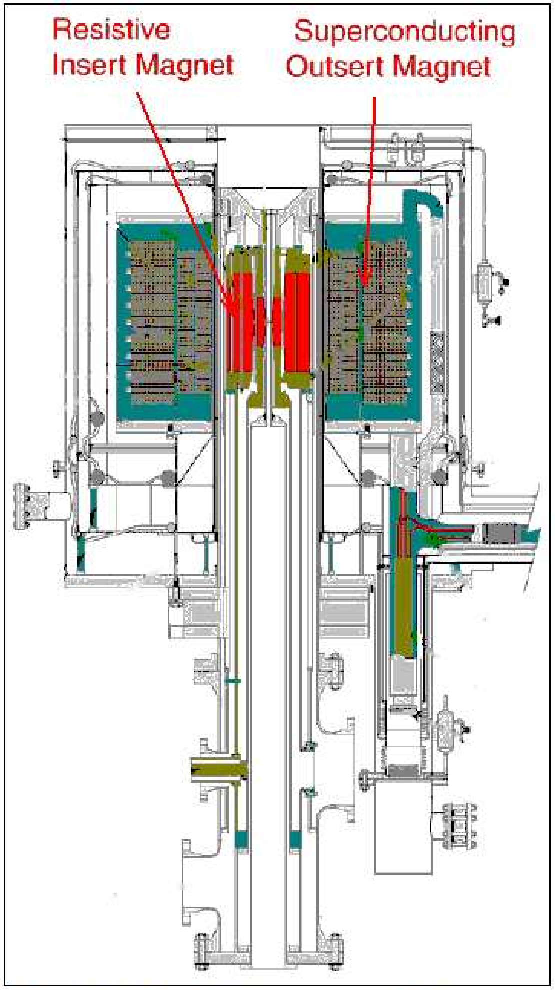
.bmp y=4.

- Engineering integration of rf not well defined
- Possible problem of rf breakdown with intense muon beam transit

Final Transverse Cooling in High Field Solenoids

- Lower momenta allow transverse cooling to required low transverse emittance, but long emittance rises: Effectively reverse emittance exchange



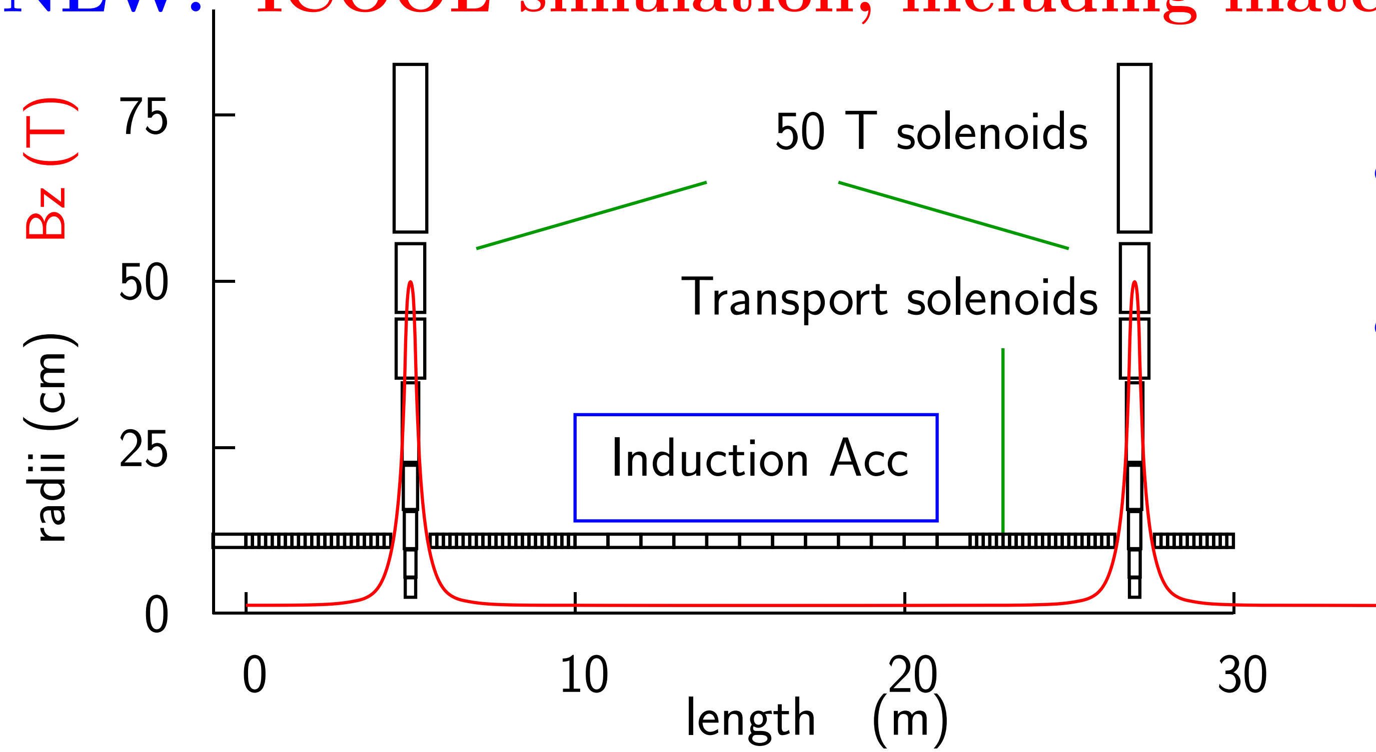


NHMFL 45 T Hybrid Magnet

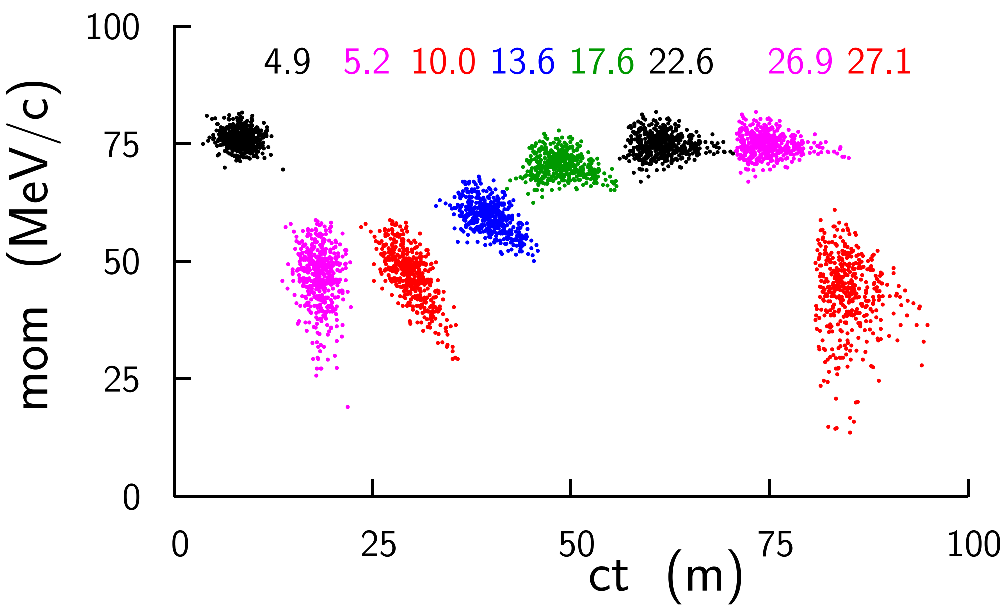
- Need five 50 T solenoids
- ICOOL Simulation of cooling in solenoids
- Simulation of re-acceleration/matching started

- 45/50 T Solenoids ?
  - 45 T hybrid at NHMFL, but uses 25W
  - Could achieve 50T with 37 MW
  - 30 T all HTS designed at NHMFL

NEW: ICOOL simulation, including matching



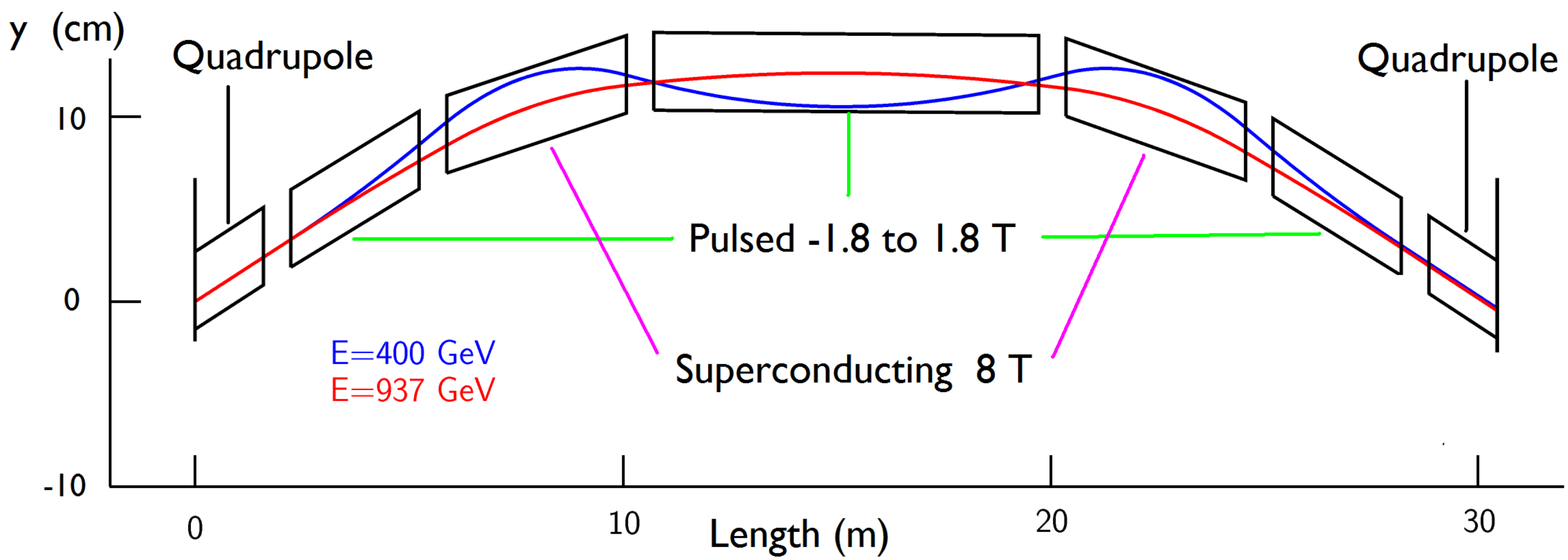
- 50 T magnet design from PBL SBIR phase 1
- 1.4 MV/m Induction Linac



Acceleration

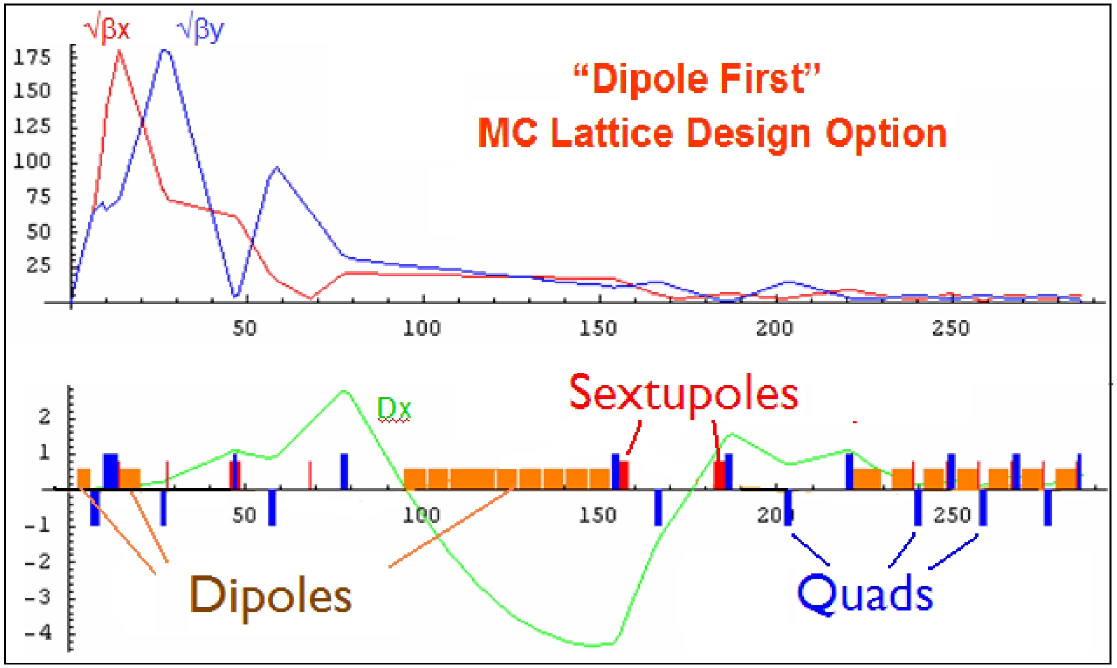
- Sufficiently rapid acceleration is straightforward in Linacs and Recirculating linear accelerators (RLAs)
  Using ILC-like 1.3 GHz rf
- Lower cost solution would use Pulsed Synchrotrons

- Pulsed synchrotron 30 to 400 GeV
  (in Tevatron tunnel)
- Hybrid SC & pulsed magnet synchrotron 400-900 GeV
  (in Tevatron tunnel)



Collider Ring

- 1.5 TeV (c of m) Design

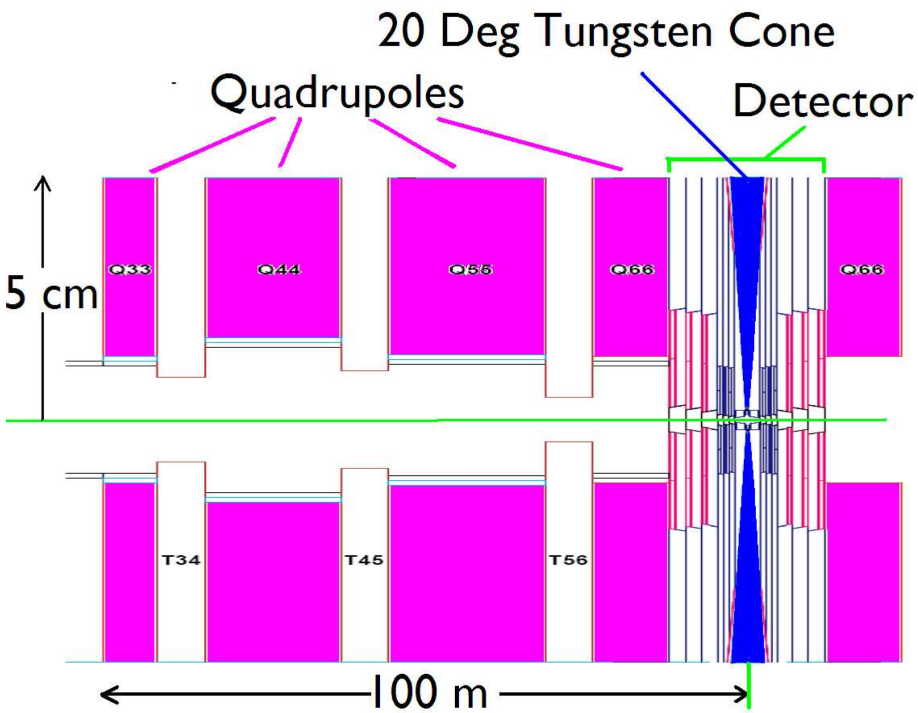


NEW: – Meets requirements at 1.5 TeV

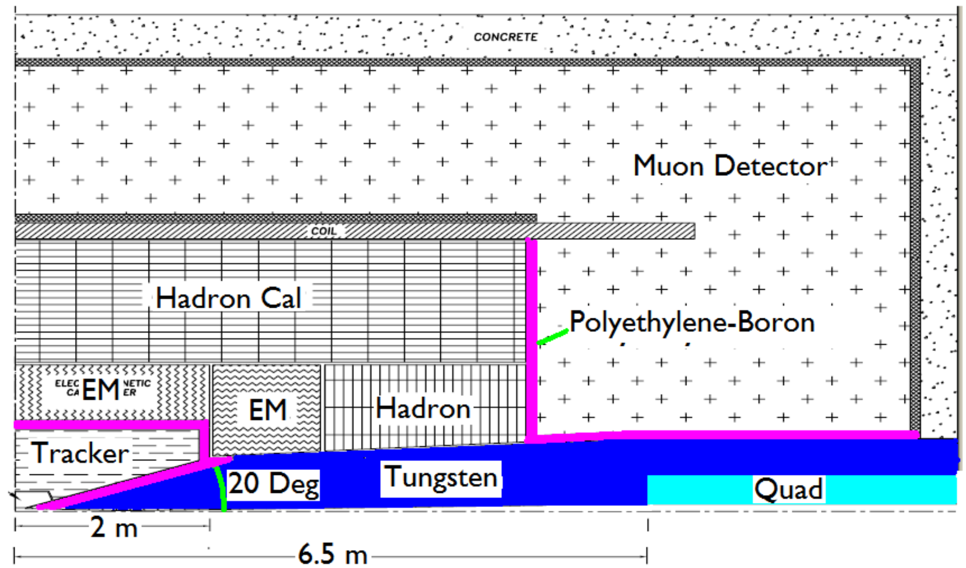
– But early dipole may deflect unacceptable background into detector

- 4 TeV (c of m) 1996 design by Oide
  - Meets requirements in ideal simulation
  - But is too sensitive to errors to be realistic
- The experts believe that the required rings should be possible

Detector From 1996 Study of 4 TeV Collider



Shielding



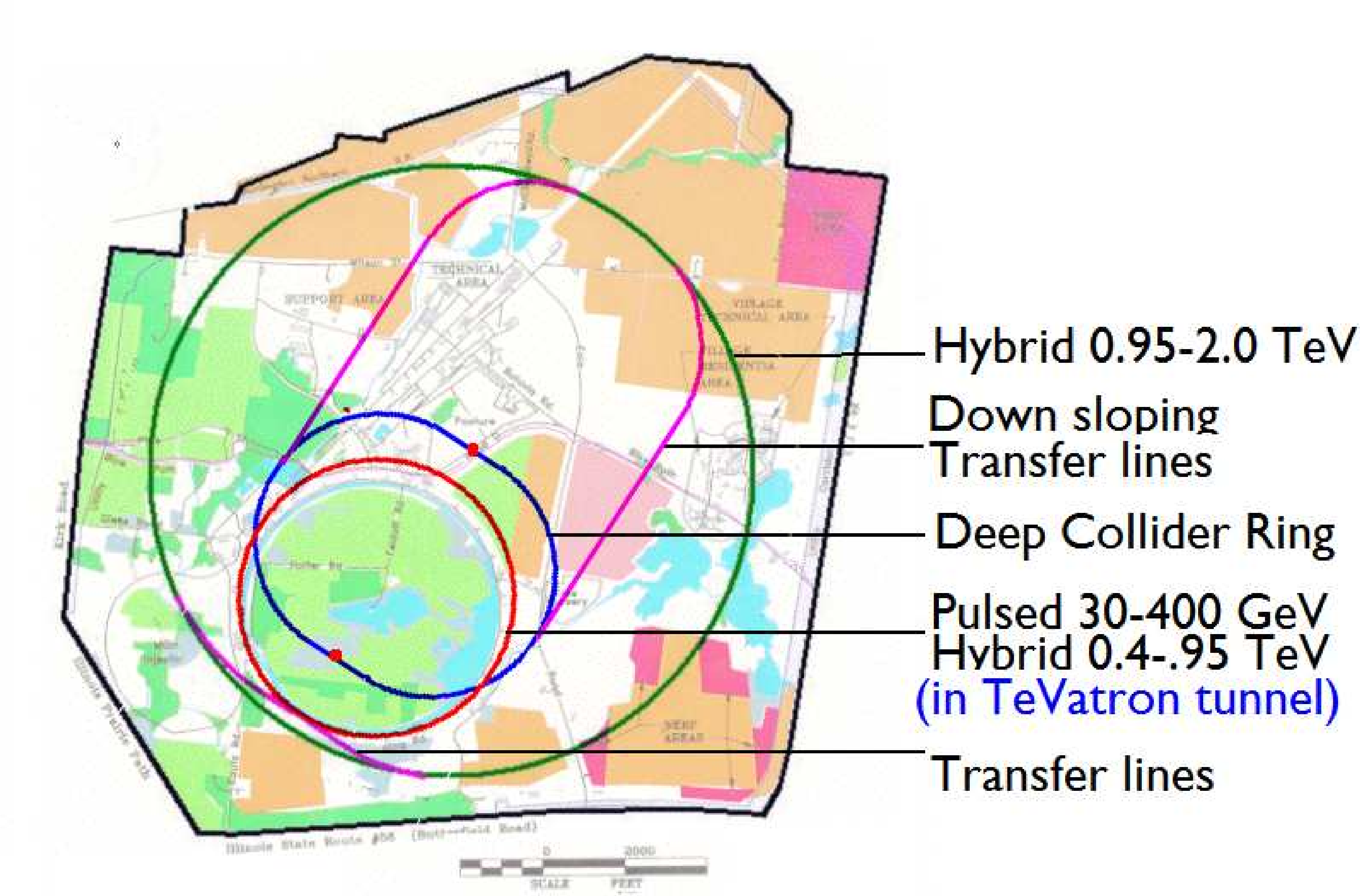
Detector

- Sophisticated shielding designed in 1996 4 TeV Study
- GEANT simulations then indicated acceptable backgrounds
- Would be less of a problem now with finer pixel detectors

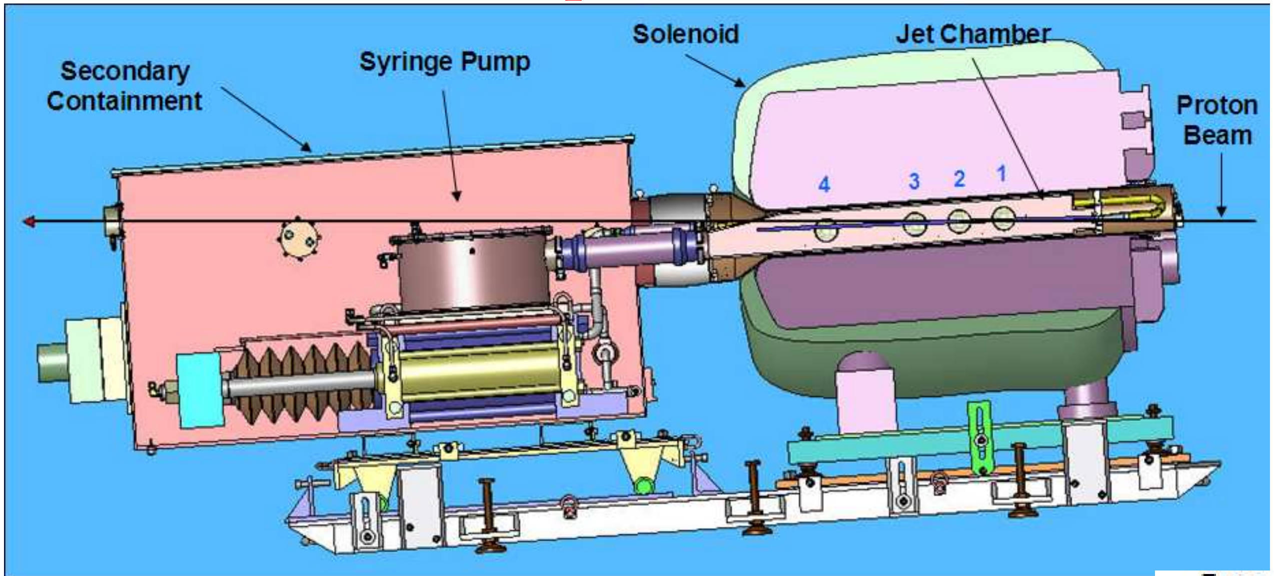
  BUT

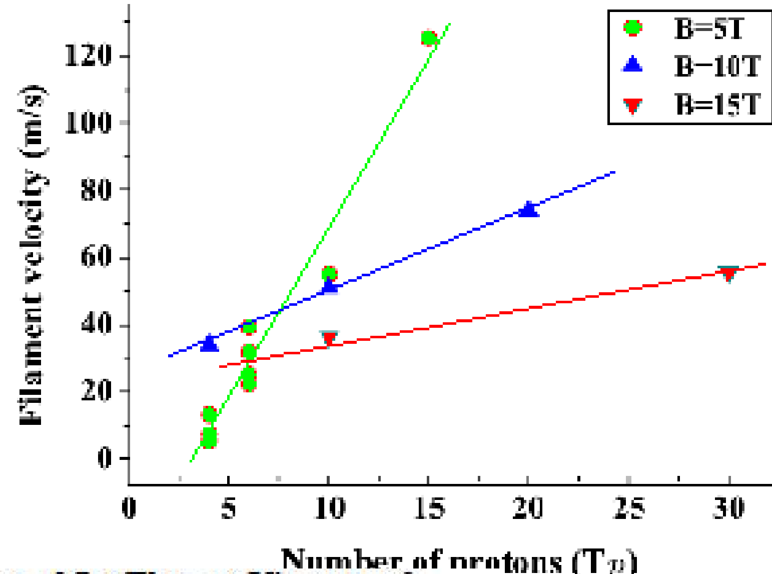
- Tungsten shielding takes up 20 degree cone

Layout of 3 TeV Collider using pulsed synchrotrons

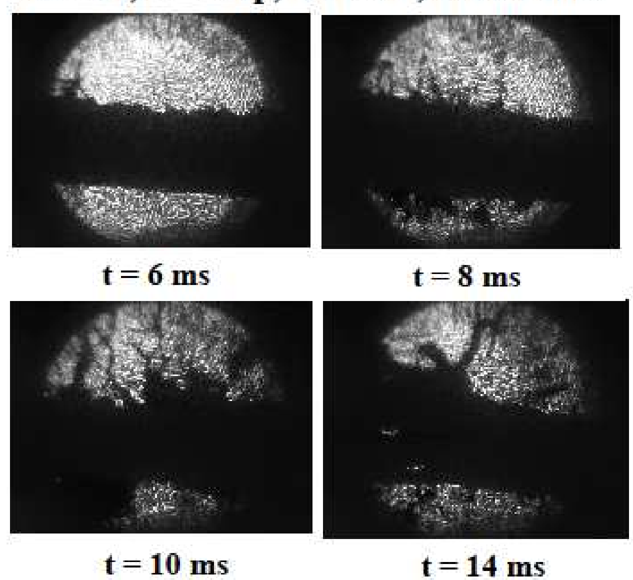


R&D AND EXPERIMENTS
MERIT Experiment at CERN





Images of Jet Flow at Viewport 3,
B=10T, N=10Tp, L=17cm, 2ms/frame

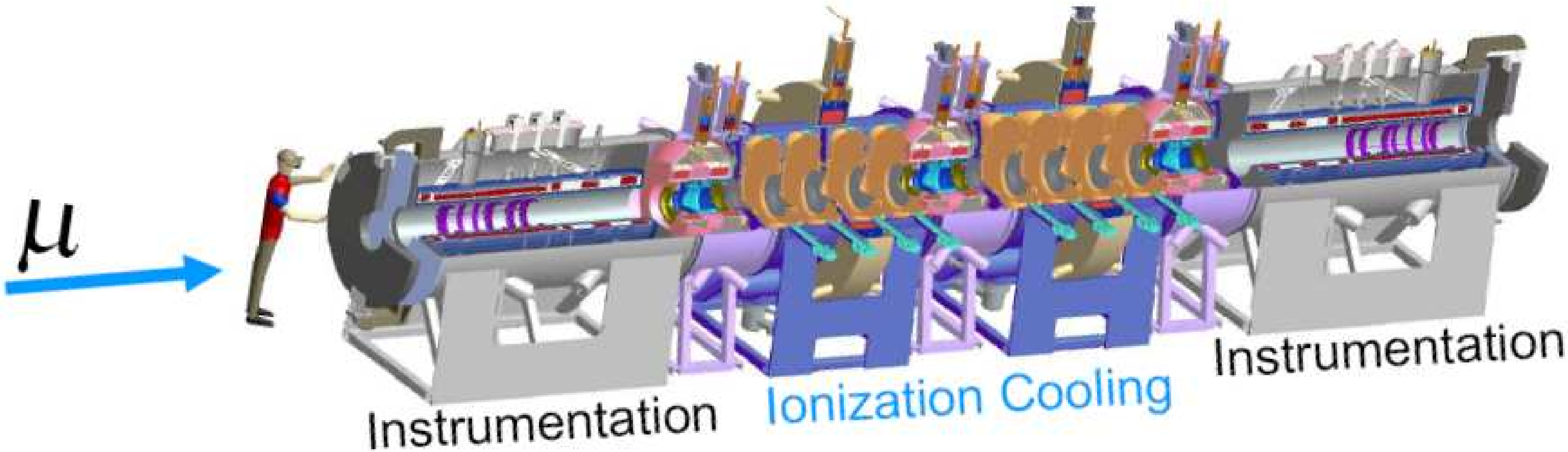


- 15 T pulsed magnet
- 1 cm rad mercury jet
- Up to 30 Tp   cf 40 Tp at 56 GeV
- Magnet lowers splash velocities
- Density persists for 100 micro sec
- No problems found

Muon Ionization Cooling Experiment (MICE)

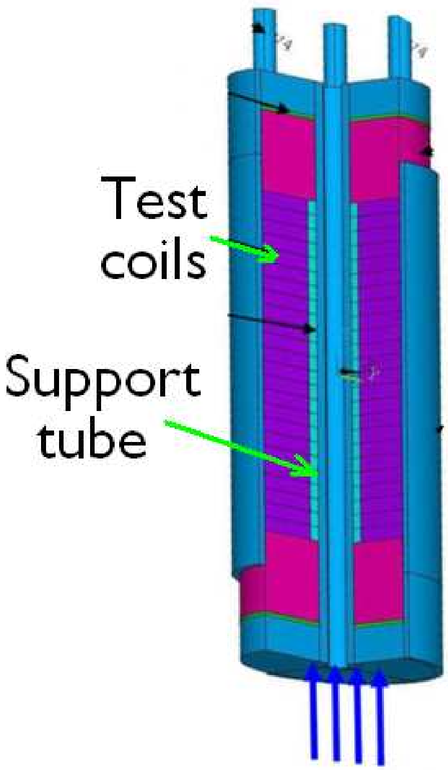
International collaboration at RAL, US, UK, Japan (Blondel)

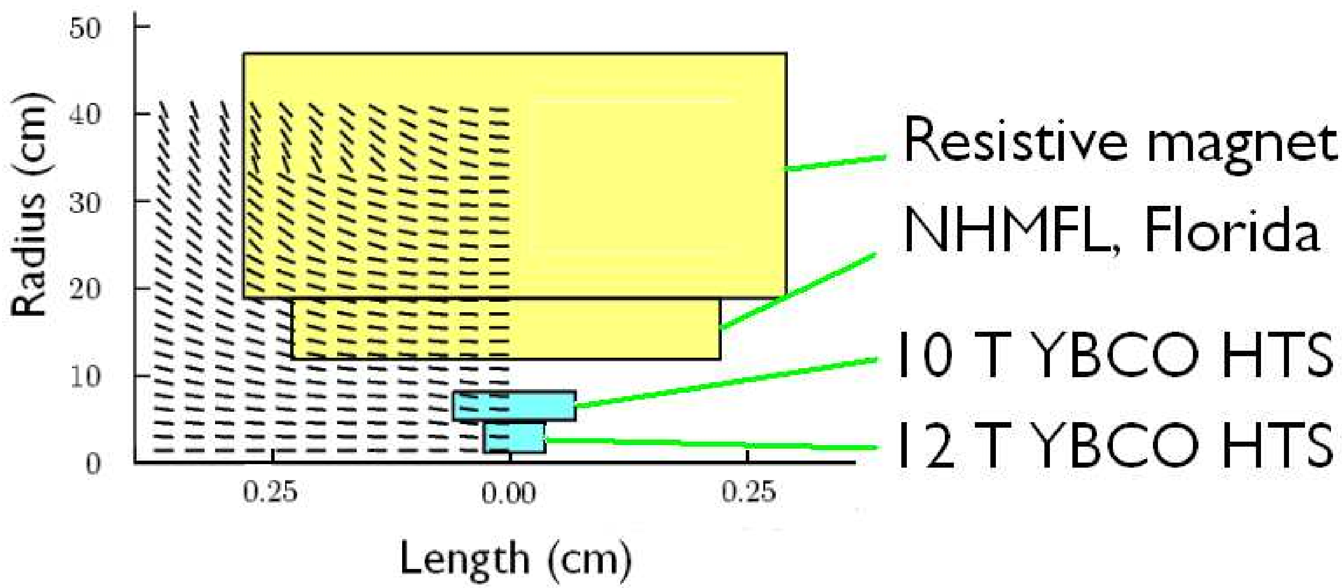
- Will demonstrate transverse cooling in liquid hydrogen, including rf re-acceleration
- Uses a somewhat different version of 'Super FOFO'
  But, as now configured, has now bending or emittance exchange



- Allows early test of emittance exchange without re-acceleration
- Later phase might test emittance exchange with re-acceleration

HTS R&D towards a 50 T solenoid





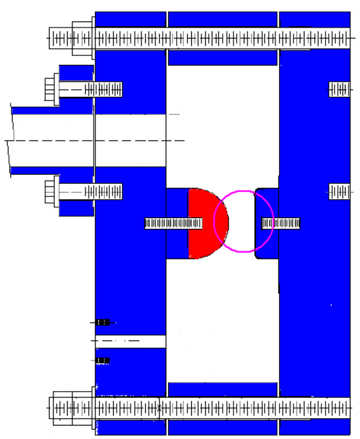
- BNL/PBL Program (SBIR)
- Test HTS coils under construction
- 12 + 10 T = 22 T stand alone
- Approx 40 T in 19 T NHMFL magnet
- Design for 19 T NbTi + Nb_3Sn design is straightforward

- FNAL program
- Testing multiple small coils
- In existing 12 T facility
- Fields up to 25 T

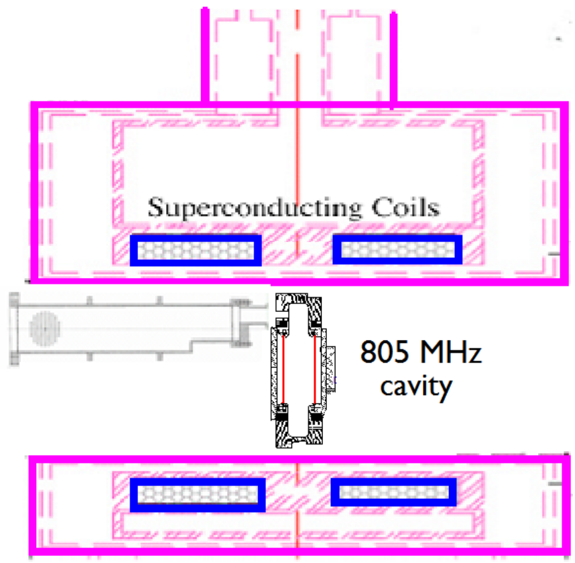
MuCool, and MuCool Test Area (MTA) at FNAL

International collaboration US, UK, Japan (Bross)

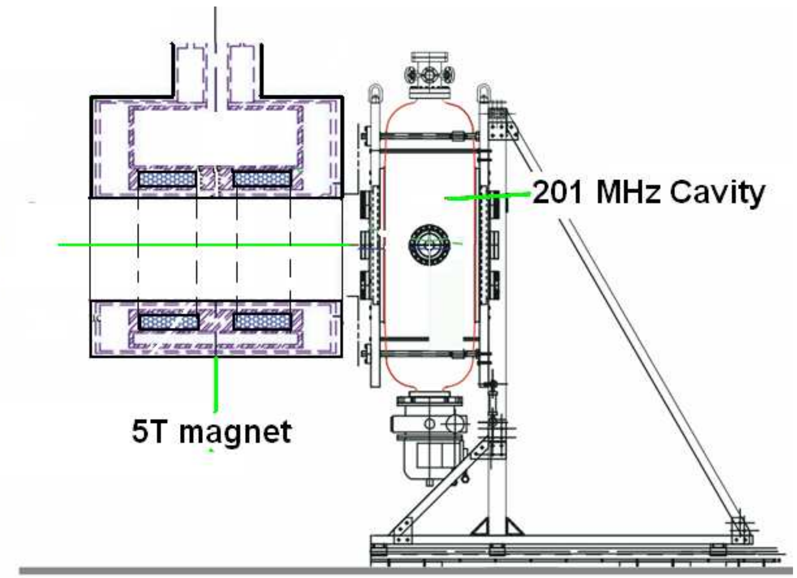
- Liquid hydrogen absorber tested
- Open & pillbox 805 MHz cavities in magnetic fields to 4 T
- 201 MHz cavity tested to magnetic field of 0.7 T
  Later to 2T
- High pressure H2 gas 805 MHz pillbox cavity tested
- Soon: 805 MHz gas Cavity with proton beam



HP Gas cavity

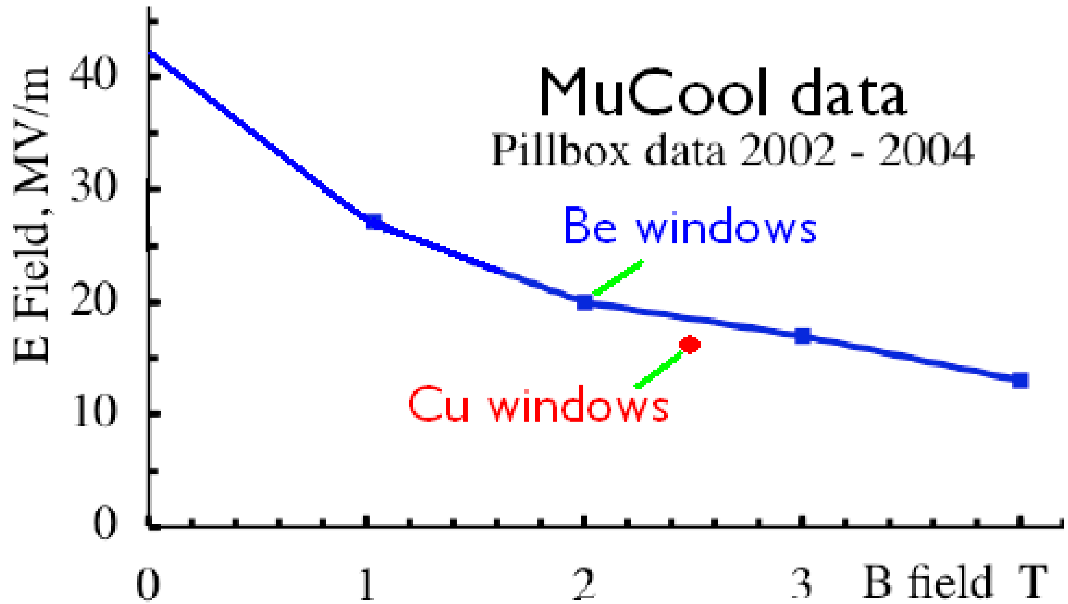


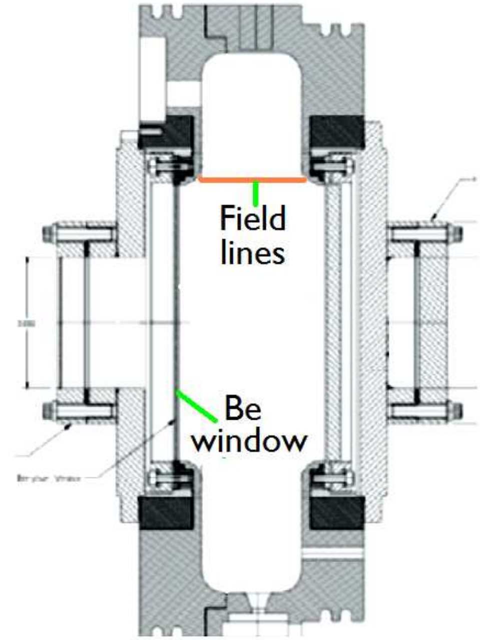
805 MHz in 4 T magnet



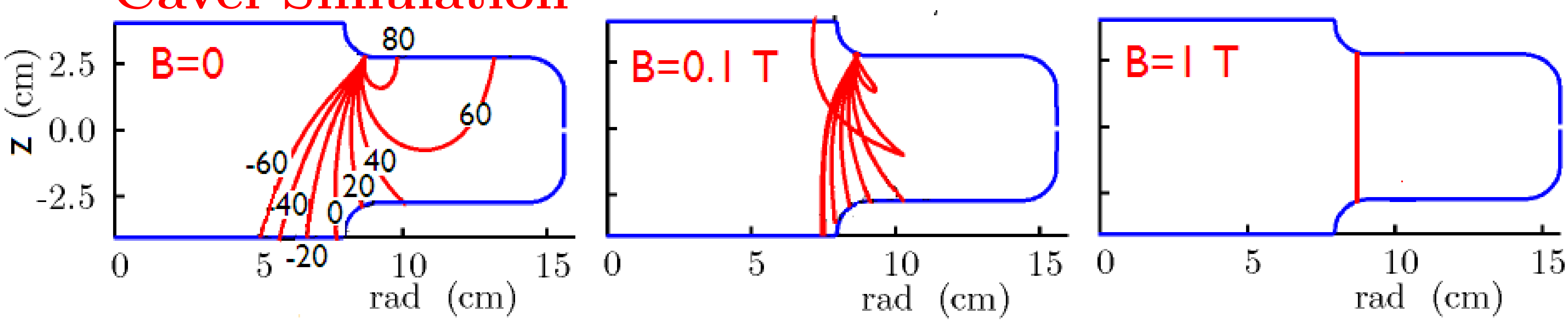
201 MHz next to magnet

Technical challenge: rf breakdown in magnetic fields



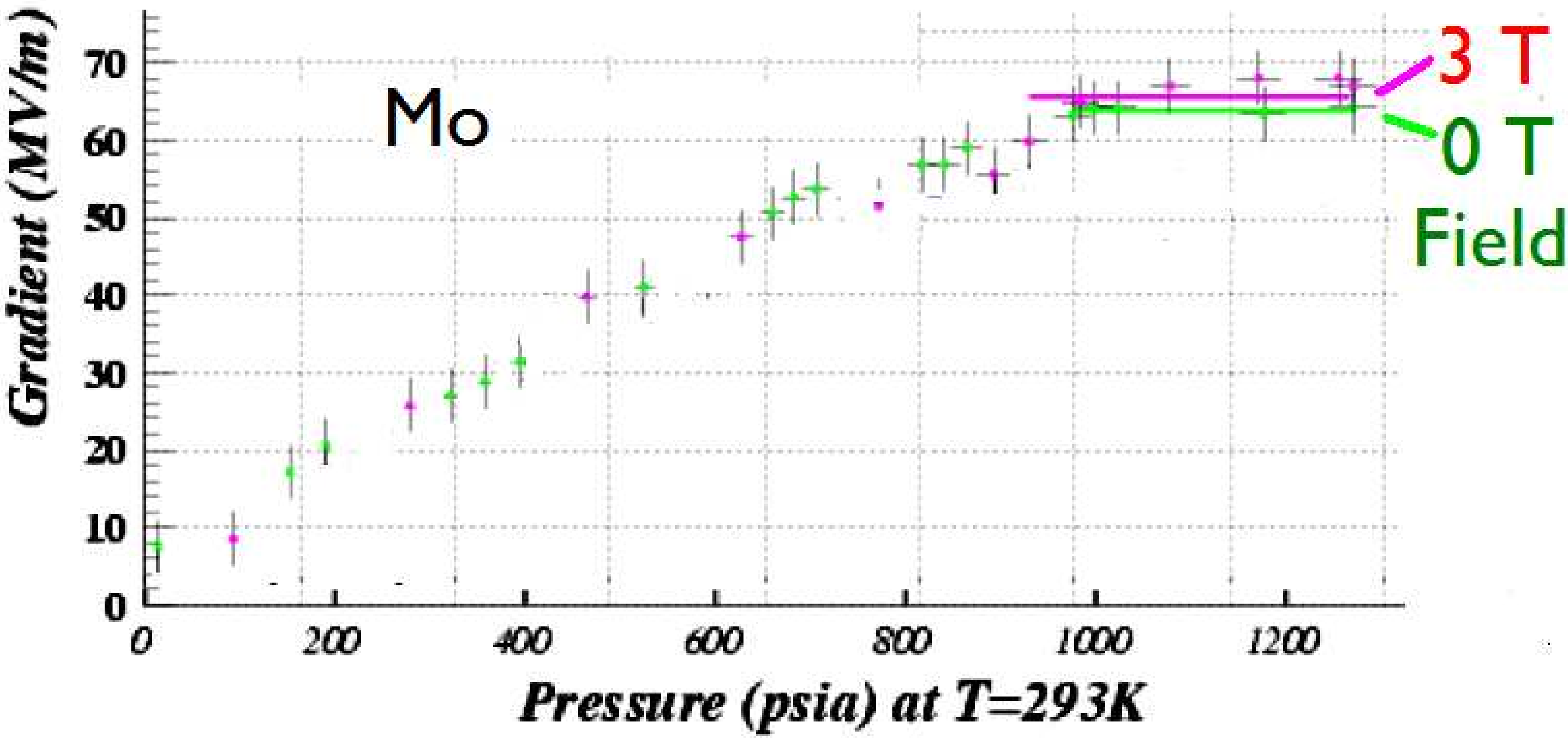


Cavel Simulation



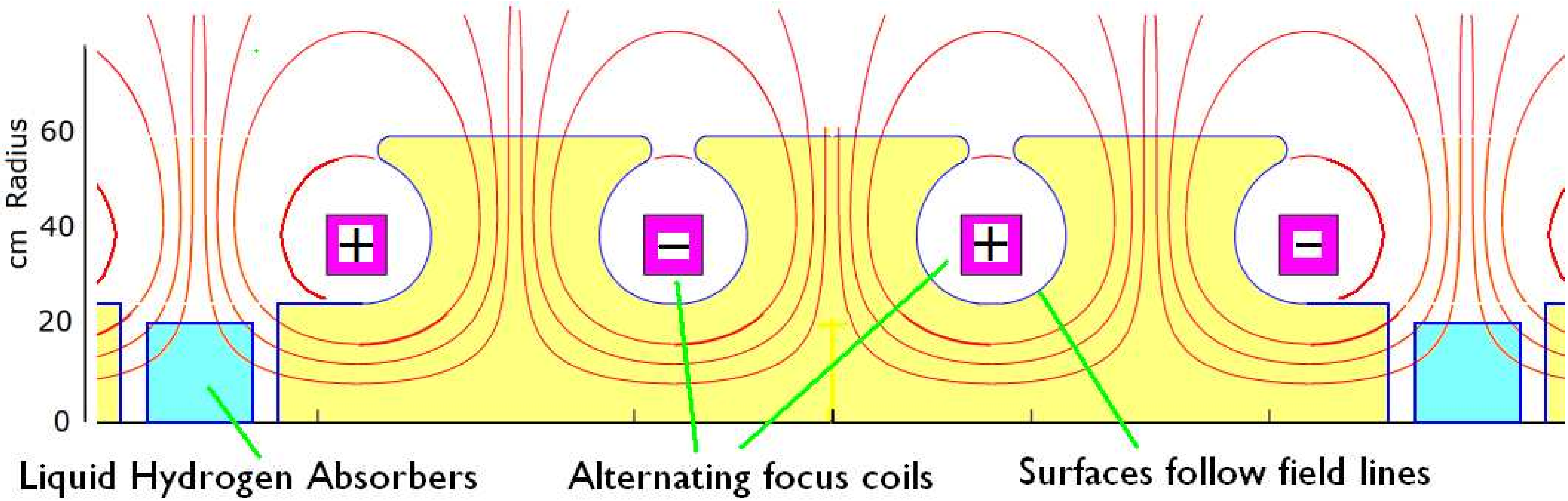
1. "Dark Current" electrons accelerated and focused by magnetic field
2. Damage spots by thermal fatigue causing breakdown

Solution 1) Gas filled cavities show no such effect



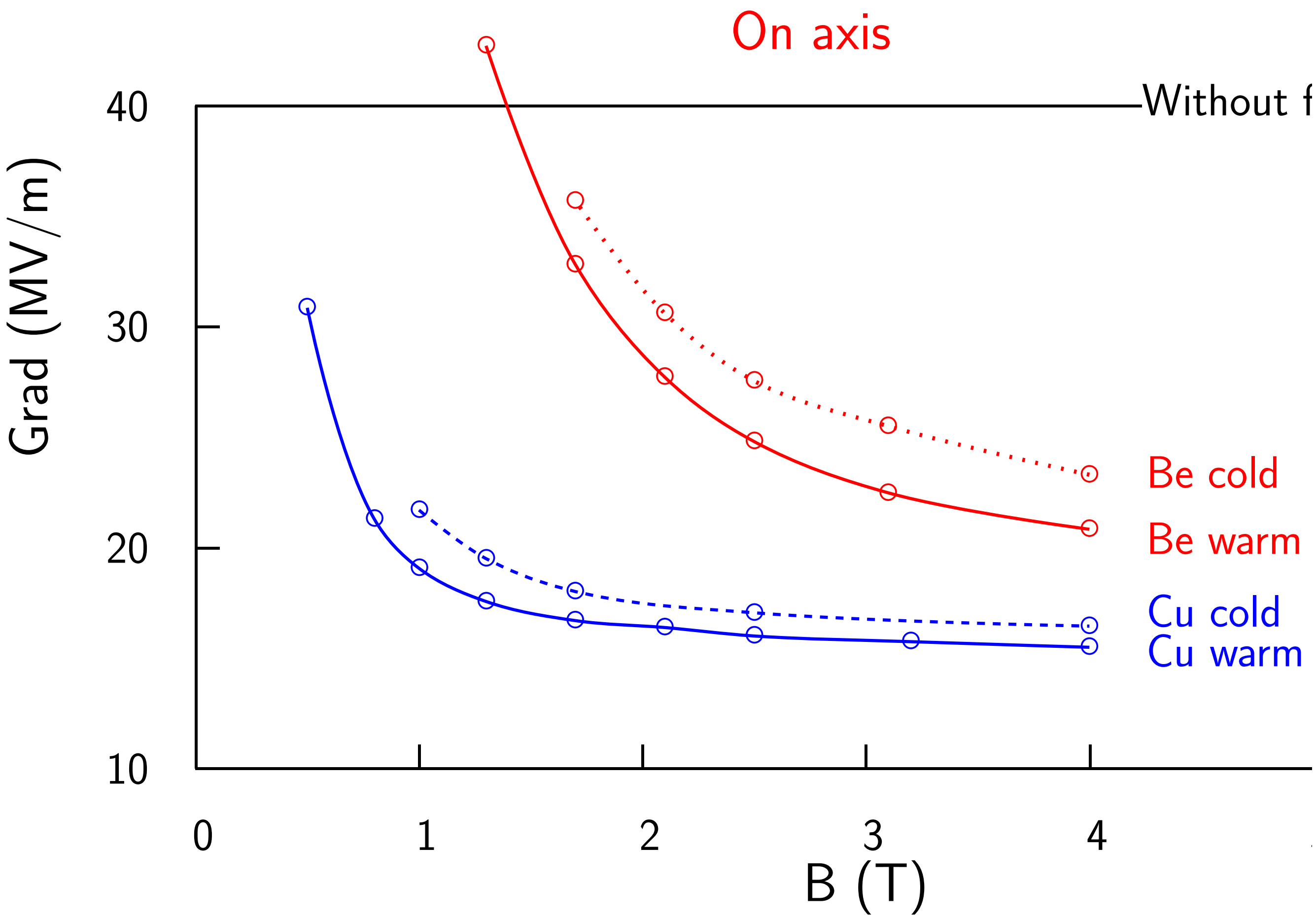
- But a beam passing through may cause breakdown or rapid loss of rf
- Experiment to be performed in proton beam at Fermilab
- Not so suitable for lower emittances since hydrogen everywhere does not allow focus to get low β

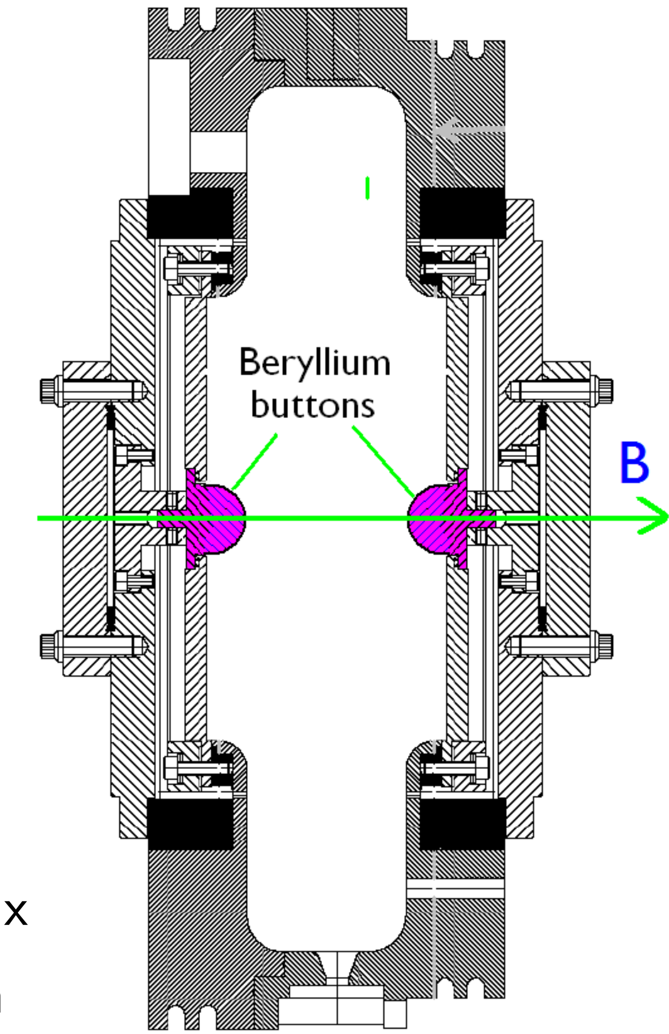
Solution 2) Magnetic Insulation



- All tracks return to the surface & Energies very low
- No dark current, No X-Rays, no danger of melting surfaces
- Rather certain to work but less efficient:
  - Surface/acceleration fields worse for 'open' cavities
  - Shunt impedance worse needing more rf power

NEW: Solution 3) Cavities made of Beryllium





Planned test

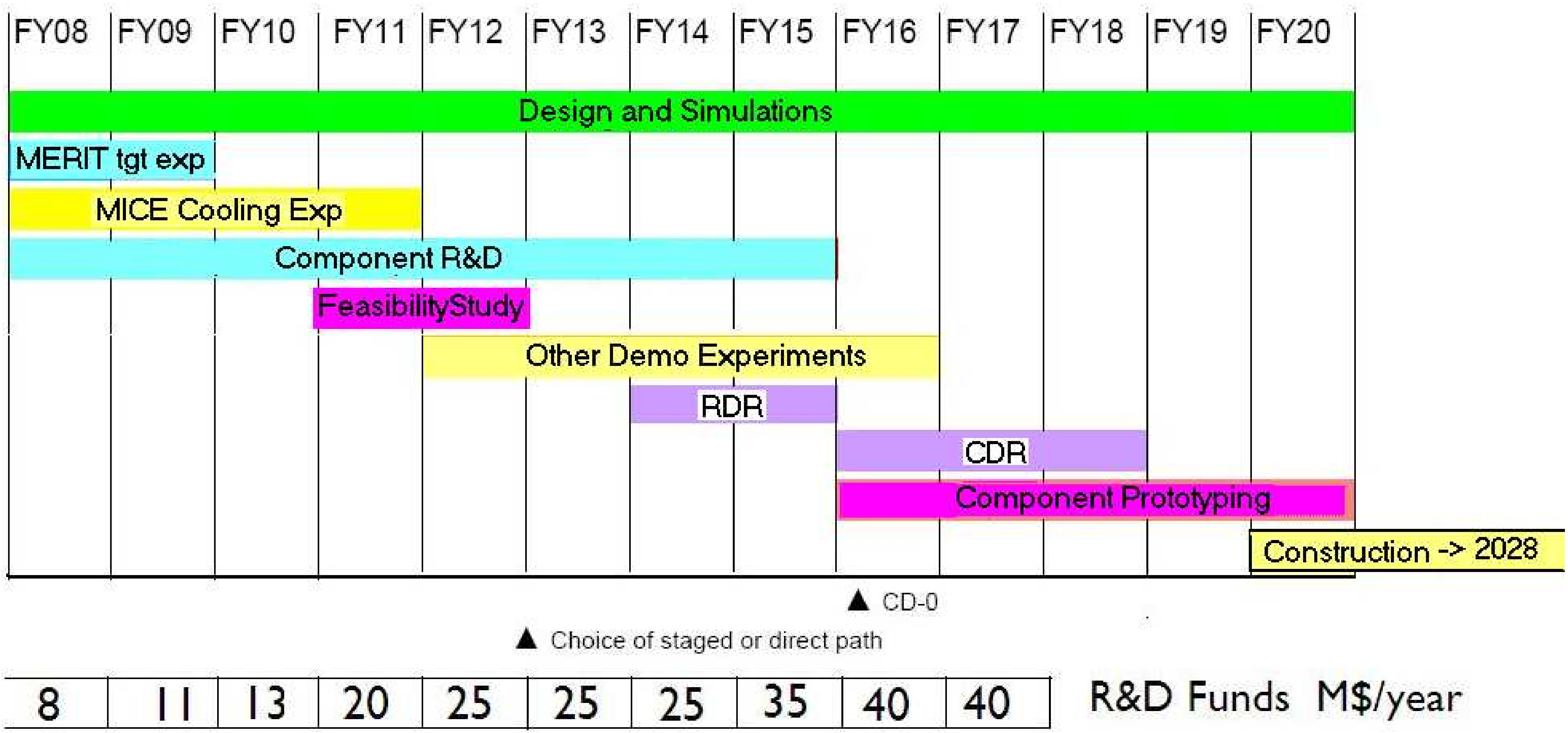
- Be has lower density so electrons go deep leaving less dE/dx
- Be also has low coefficient of expansion α giving less strain
- When cold (77 deg. K) conductivity is improved and α reduced
- Fields & gradients must still be kept low, but probably best choice

Conclusion on Baseline design

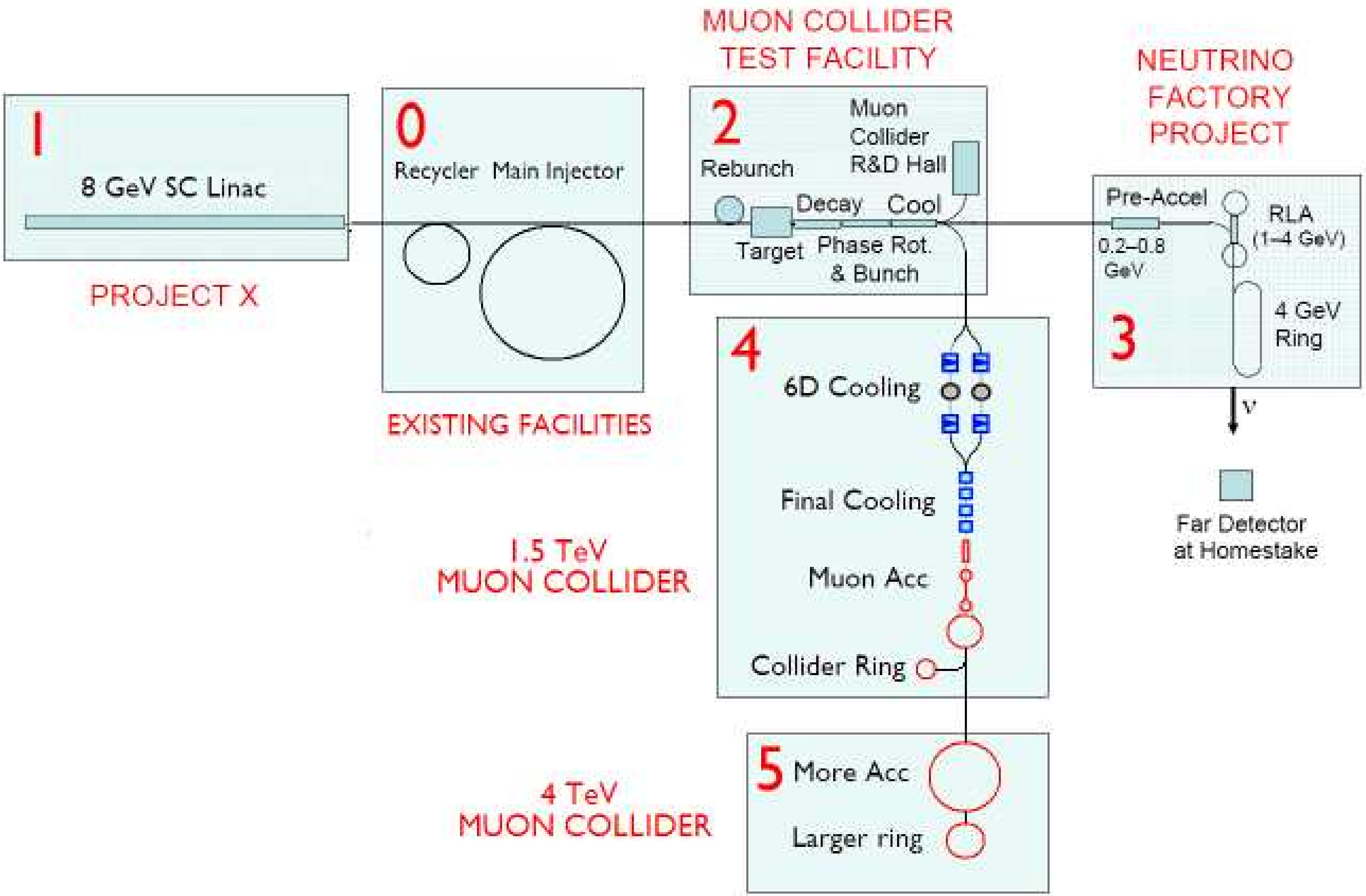
- All stages for a "baseline" design have been simulated at some level
- Matching and tapering of 6D cooling remains to be designed
- Good collider ring design exist for both 1.5 TeV
  although backgrounds form 'dipole first' needs study
- 3 TeV design following Oide's 4 TeV design under study
- Detector design and shielding has been studied in 1996 and looks OK

- The biggest technical problem is rf breakdown in magnetic fields
  but multiple solutions are under study

R&D plan submitted to DoE



A Phased Approach (as presented to P5)



2 DEFINITIONS AND UNIT CONVENTIONS

Units

When discussing the motion of particles in magnetic fields, I will use MKS units, but this means that momentum, energy, and mass are in Joules and kilograms, rather than in the familiar 'electron Volts'. To make the conversion easy, I will introduce these quantities in the forms: $[pc/e]$, $[E/e]$, and $[mc^2/e]$, respectively. Each of these expressions are then in units of straight Volts corresponding to the values of p, E and m expressed in electron Volts. For instance, I will write, for the bending radius in a field B:

$$\rho \ = \ \frac{[pc/e]}{B\ c} \tag{8}$$

meaning that the radius for a 3 GeV/c particle in 5 Tesla is

$$\rho \ = \ \frac{3\ 10^9}{5\ \times\ 3\ 10^8} \ = \ 2m$$

Emittance

Emittances will always be assumed to be normalized rms values

$$\epsilon \;=\; \text{normalized emittance} \;=\; \frac{[\text{Phase Space Area c/e}]}{\pi\ [\text{mc}^2/\text{e}]} \tag{9}$$

The phase space can be transverse: p_x vs x, p_y vs y, or longitudinal Δp_z vs z, where Δp_z and z are with respect to the moving bunch center.

If x and p_x are both Gaussian and uncorrelated, then the area is that of an upright ellipse, and:

$$\epsilon_\perp = \frac{\pi\ \sigma_{[pc/e]_\perp}\sigma_x}{\pi\ [mc^2/e]} \;=\; (\gamma\beta_v)\sigma_\theta\sigma_x \qquad (\pi\ m\ rad) \tag{10}$$

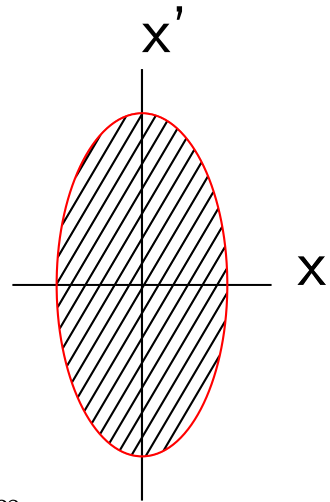
$$\epsilon_\parallel = \frac{\pi\ \sigma_{[pc/e]_\parallel}\sigma_z}{\pi\ [mc^2/e]} \;=\; (\gamma\beta_v)\frac{\sigma_p}{p}\ \sigma_z \qquad (\pi\ m\ rad) \tag{11}$$

$$\epsilon_6 = \epsilon_\perp^2\ \ \epsilon_\parallel \qquad (\pi\ m)^3 \tag{12}$$

The subscript v on β_v indicates that $\beta_v = v/c$. The π, added to the dimension, is a reminder that the emittance is phase space$/\pi$.

Un-normalize emittances $\epsilon_o = \sigma_\theta\sigma_x$ (without the $\beta_v\gamma$), are often used, but not by me. Emittances are also sometimes quoted defining ellipses with 95% of Gaussian beams: $\epsilon_{95\%} \approx 6 \times \epsilon_{rms}$

$\beta_\perp$ of Beam



For an upright phase ellipse in x' vs x,

$$\beta_\perp = \left(\frac{\text{width}}{\text{height}} \quad \text{of phase ellipse}\right) = \frac{\sigma_x}{\sigma_\theta} \tag{13}$$

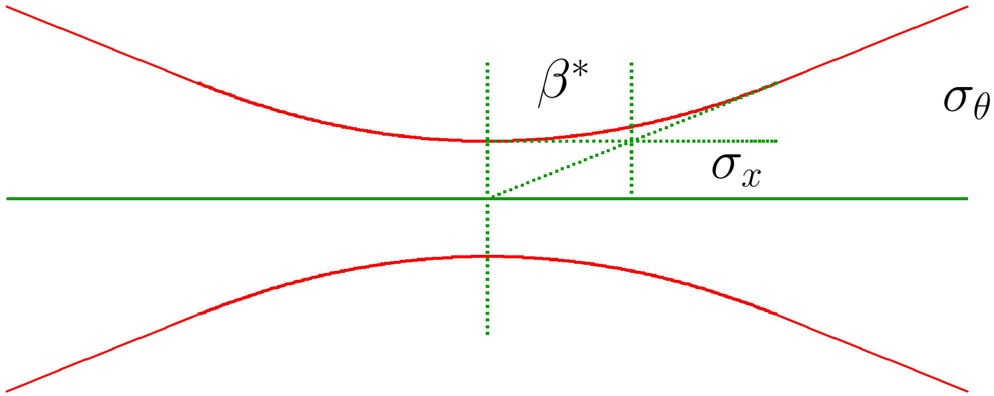
Then, using the emittance definition:

$$\sigma_x = \sqrt{\epsilon_\perp \, \beta_\perp \, \frac{1}{\beta_v \gamma}} \tag{14}$$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp} \, \frac{1}{\beta_v \gamma}} \tag{15}$$

$\beta_{lattice}$ can also be defined for a repeating lattice, where it is that β_{beam} that is matched to the lattice. Equation 14, but not eq. 15 are valid even when the ellipse is tilted.

$\beta_\perp$ (or β^*) beam at focus



$$\sigma_x = \sigma_o \sqrt{1 + \left(\frac{z}{\beta^*}\right)^2}$$

From eqiation 14

$$\beta_x = \beta^* \left(1 + \left(\frac{z}{\beta^*}\right)^2\right)$$

β^* is like a depth of focus

As $z \to \infty$

$$\sigma_x \to \frac{\sigma_o \ z}{\beta^*}$$

giving an angular spread of

$$\theta = \frac{\sigma_o}{\beta^*}$$

as above in eq.13

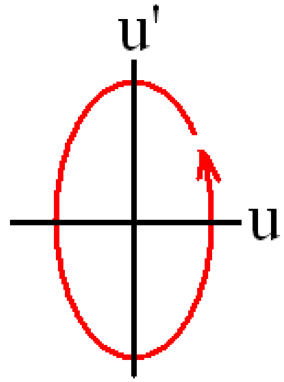
$\beta_\perp$ of a Lattice

$\beta_\perp$ above was defined by the beam, but a lattice or ring has a β_o that may or may not "match" the $\beta_\perp$ of the beam.

e,g. if a continuous inward focusing force, then there is a PERIODIC solution:

$$\frac{d^2u}{dz^2} = -k\ u$$

With possible solution:



$$u = A\ \sin\left(\frac{z}{\beta_o}\right) \qquad u' = \frac{A}{\beta_o}\ \cos\left(\frac{z}{\beta_o}\right)$$

where the lattice $\beta_o = 1/\sqrt{k} \qquad \lambda = 2\pi\ \beta_o$

In the u' vs. u plane, this motion is also an ellipse with

$$\frac{\text{width}}{\text{height}} \text{ of elliptical motion in phase space } = \frac{\hat{u}}{\hat{u'}} = \beta_o$$

If we have many particles with $\beta_\perp(\text{beam}) = \beta_o(\text{lattice})$ then all particles move arround the ellipse, the shape, and thus $\beta_\perp(\text{beam})$ remains constant, and the beam is "matched" to this lattice.

If the beam's $\beta_\perp(\text{beam}) \neq \beta_\text{o}$ of the lattice then $\beta_\perp(\text{beam}$ of the beam oscillates about $\beta_o(\text{lattice})$: often refered to as a "beta beat".

Useful Relativistic Relations

$$dE = \beta_v \, dp \tag{16}$$

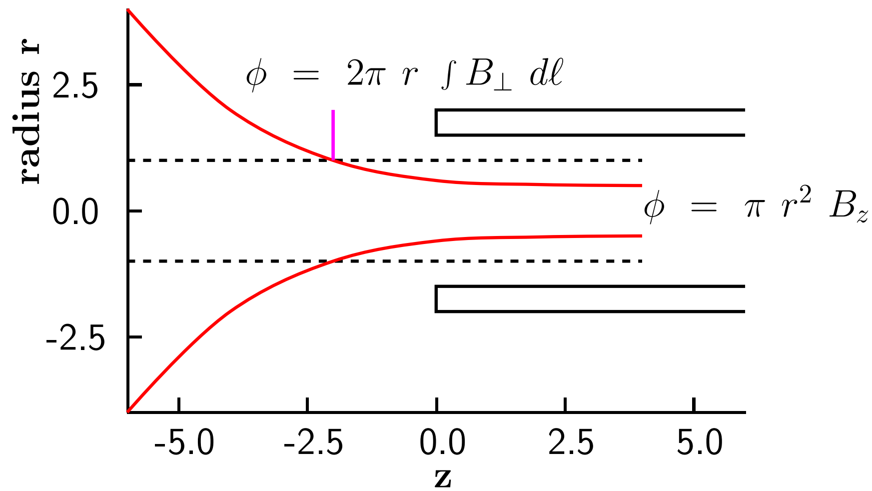
$$\frac{dE}{E} = \beta_v^2 \, \frac{dp}{p} \tag{17}$$

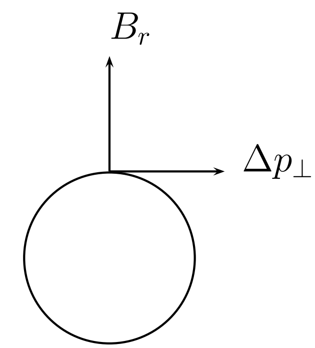
$$d\beta_v = \frac{dp}{\gamma^2} \tag{18}$$

I use β_v to denote v/c to distinguish it from the Courant-Schneider or Twiss parameters $\beta_\perp$

3 SOLENOID FOCUSING

Entering solenoid





Ignoring any p_r $$\Delta[pc/e]_\phi = \int B_r\ dz = -\frac{r\ c}{2}\ \Delta B_z \tag{19}$$

To be exact: Use $$\mathrm{DivB} = 0 = \frac{1}{r}\frac{\partial}{\partial r}(r\ B_r) + \frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}$$

$\frac{\partial B_\phi}{\partial \phi} = 0$ and assuming $B_r \propto r$ so: $B_r = r\ \frac{B_o}{r_o}$

$$\frac{1}{r}\frac{\partial}{\partial r}(r^2\ \frac{B_o}{r_o}) = \frac{2}{r} B_r = -\frac{\partial B_z}{\partial z}$$

$$\partial[pc/e]_\phi = c\ B_r = -\frac{rc}{2}\ \partial B_z \tag{20}$$

and the assumption $B_r \propto r$ was ok

Canonical angular momentum

Assuming no energy loss in material, then integrating:

$$[pc/e]_\phi \;=\; [pc/e]_{\phi o} - \frac{rc}{2}\, B_z \tag{21}$$

where $[pc/e]_{\phi o}$ is the initial value before, or in the absence of, the magnetic field. In a more general case $r\ [pc/e]_{\phi o}$ is the canonical angular momentum

$$\text{Angular momentum} \;=\; r\ [pc/e]_\phi \;=\; r\ [pc/e]_{\phi o} - \frac{r^2 c}{2}\, B_z \tag{22}$$

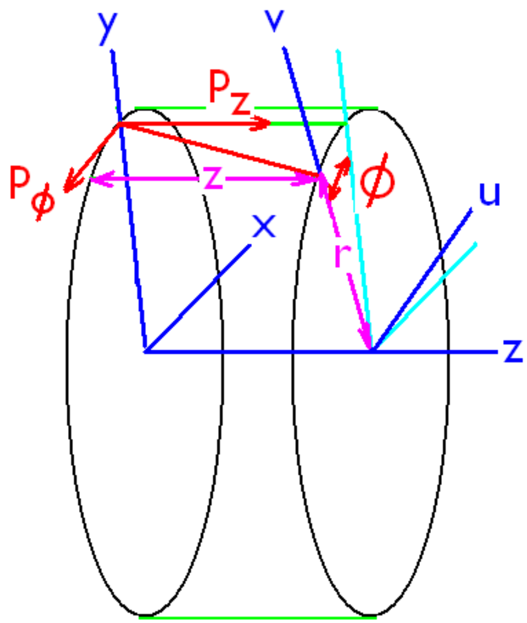
$$\text{Canonical Angular mome} \;=\; [pc/e]_{\phi o} \;=\; r\ [pc/e]_\phi \;+\; \frac{r^2 c}{2}\, B_z \tag{23}$$

If the initial canonical angular momentum is zero and there is no 'non-magnetic', changes in the angular momentum then the canonical angular momentum remains zero.

But as we will study in later homework, 'ionization cooling' in material reduces lab frame angular momentum thus increasing the canonical momentum. When the particle then leaves the field, the lab angular momentum reverts to the canonical and is now non-zero.

v,z motion in the Larmor frame for a fixed B_z

assuming $r\ [pc/e]_{\phi o}\ =\ 0$

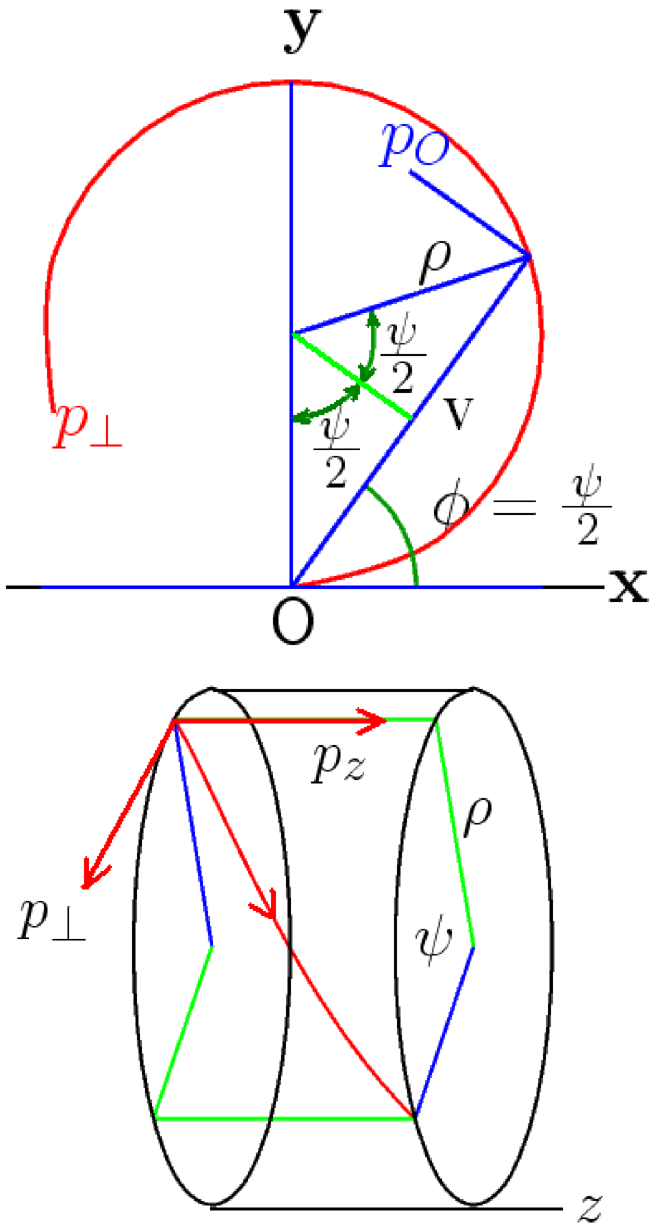


$$\frac{d\phi}{dz} = \frac{p_\phi}{p_z\ r} = -\frac{c\ B_z}{2\ [pc/e]_z} \tag{24}$$

If we define a coordinate system u, v that is rotated about the axis by the above angle ϕ, then in that frame angular momentum is conserved and a particle starting without angular momentum and $u = 0, \dot{u} = 0$ remains in the plane $u = 0$ plane. This is the Larmor frame.

But we have not yet included changes in ϕ due to the action of B_z on the particle motion.

x, y motion in Long Solenoid (B_Z =constant)



Consider motion in a fixed axial filed B_z, starting on the axis O with finite transverse momentum $p_\perp$ i.e. with initial angular momentum=0.

$$\rho = \frac{[pc/e]_\perp}{c\ B_z} \tag{25}$$

$$x = \rho \sin(\psi)$$

$$y = \rho\ (1 - \cos(\psi))$$

$$v = 2\rho\ \sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi) \tag{26}$$

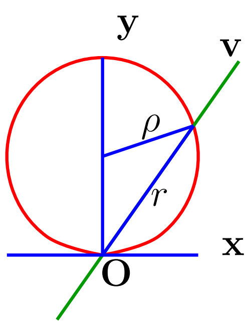
$$\frac{d\psi}{dz} = \frac{[pc/e]_\perp}{\rho\ [pc/e]_z} = \frac{[pc/e]_\perp}{[pc/e]_z} \frac{c\ B_z}{[pc/e]_\perp}$$

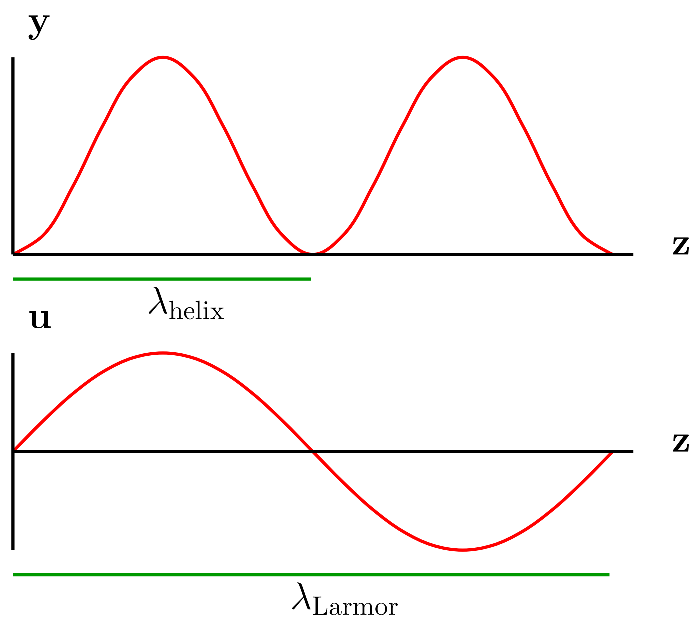
$$\frac{d\phi}{dz} = \frac{d\psi}{2\ dz} = -\frac{c\ B_z}{2\ [pc/e]_z} \tag{27}$$

Which is same as eq.24. So this is true even including effects of the axial field on a particle

Larmor Plane

If The center of the solenoid magnet is at O, then consider a plane that contains this axis and the particle. This, for a particle with initially no angular momentum, is the 'Larmor Plane:





$$y = \rho\,(1 - cos(\psi)) \tag{28}$$

$$v = 2\rho\ \sin(\phi) \tag{29}$$

$$\lambda_{\mathrm{Helix}} = 2\pi\,\frac{dz}{d\psi} = 2\pi\ \rho\ \frac{p_z}{p_\perp} = 2\pi\ \frac{[pc/e]_z}{c\ B_z} \tag{30}$$

$$\lambda_{\mathrm{Larmor}} = 2\pi\,\frac{dz}{d\phi} = 2\pi\ 2\rho\ \frac{p_z}{p_\perp} = 4\pi\ \frac{[pc/e]_z}{c\ B_z} \tag{31}$$

The lattice parameter β_o is defined in the Larmor frame, so

$$\beta_o = \frac{\lambda_{Larmor}}{2\pi} = \frac{2\ [pc/e]_z}{c\ B_z} \tag{32}$$

Focusing Force

In this constant B_z case, the observed sinusoidal motion in the $u = 0$ plane is generated by a restoring force towards the axis O.

The momentum p_O about the axis O (perpendicular to the Larmor plane), using eq.26 and eq.27:

$$v \;=\; 2\rho\ \sin(\phi) \qquad \text{and} \qquad \phi \;=\; z\left(\frac{cB_z}{[pc/e]_\perp}\right)$$

Differentiating twice gives inward bending with radius η

$$\frac{1}{\eta} \;=\; \frac{d^2v}{dz^2} \;=\; -\left(\frac{cB_z}{2\,[pc/e]}\right)^2\ v \tag{33}$$

This inward bending $\propto\ v$, the distance from the axis is an ideal focusing force

Eq.33 is true, even for varying fields

Note: the focusing "Force" $\propto\ B_z^2$ so it works the same for either sign of B_z or the particle charge.

And it is $\propto\ 1/p_z^2$. Whereas in a quadrupole the force $\propto\ 1/p$
So solenoids are not good for high p, but beat quads at low p.

Larmor Theorem

Motion of a charged particle in any axial symmetric solenoid fields $B_z(z)$ is given by that of a particle moving with the same p_z in a $u,\ v$ frame rotating about that axis by

$$\frac{d\phi}{dz} = -\frac{c\ B_z}{2\ [pc/e]_z}$$

under a focusing 'force' towards the axis giving bending

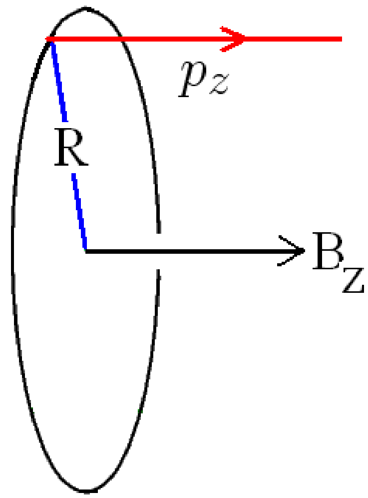
$$\frac{1}{\eta} = \frac{d^2R}{dz^2} = -\left(\frac{cB_z}{2\ [pc/e]}\right)^2\ R$$

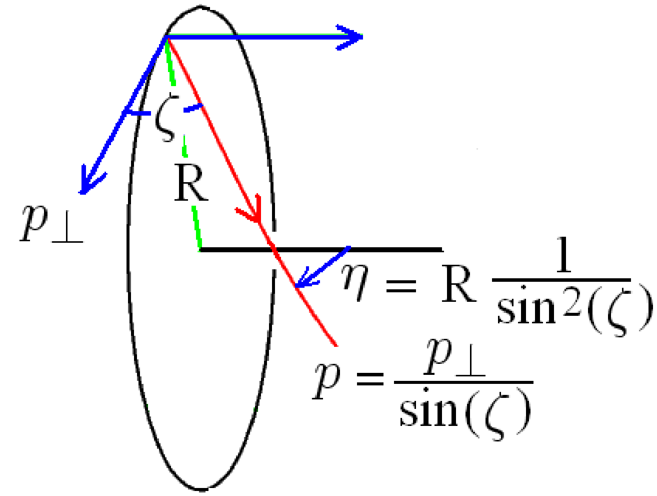
where R is the distance to the axis and $[pc/e]$ is the momentum component perpendicular to R

This being true with any initial angular momentum and thus motions unconfined by either $u = 0$ or $v = 0$ planes

Example with initial Canonical momentum

A particle at fixed $x = 0$ and $y = R$ traveling down a fixed axial field B_z





In the $u,\ v$ frame the particle is executing a helix with

$$\frac{d\phi}{dz} = \frac{c\ B_z}{2\ [pc/e]_z} \qquad [pc/e]_{\phi,\ \mathrm{effective}} = \frac{d\phi}{dz}\,[pc/e]_z\ R = \left(\frac{R\ c\ B_z}{2}\right)$$

Bending radius η from effective force towards axis

$$\frac{1}{\eta} = \left(\frac{c\ B_z}{2[pc/e]_s}\right)^2 R = \left(\frac{c\ B_z\ \sin(\zeta)}{2[pc/e]_\phi}\right)^2 R = \left(\frac{c\ B_z\ \sin(\zeta)}{2\ \left(\frac{R\ c\ B_z}{2}\right)}\right)^2 R = \frac{\sin^2(\zeta)}{R}$$

as required

Conclusion on solenoid focusing

- In a uniform solenoid field a particle moves in a helix of wavelength $\lambda_{\rm helix}$
- But in the rotating larmor plane it oscillates with wavelength $\lambda_{\rm larmor} = 2\,\lambda_{\rm helix}$
- Even with non uniform fields, motion in the larmor plane:
  - Focus is always towards the axis
  - With a 'force' $\propto B^2/p^2$
  - If a particle starts in the Larmor plane, it stays in that plane
- Since a solenoid focuses with a 'force' $\propto B^2/p^2$, compared with a quadrupole 'force' $\propto B/p$, the solenoid is always stronger at a low enough momenta and Solenoids focus in both planes, whereas quadrupoles focus in one and defocus in the other
- A solenoid can focus very large transverse emittances, with angles of a radian or more, which makes solenoids the preferred focusing in ionization cooling

4 SOLENOID HOMEWORK

1. Consider a 200 MeV/c particle starting on the axis with a transverse momentum of 20 MeV/c in an axial solenoidal field of 3.33 T.

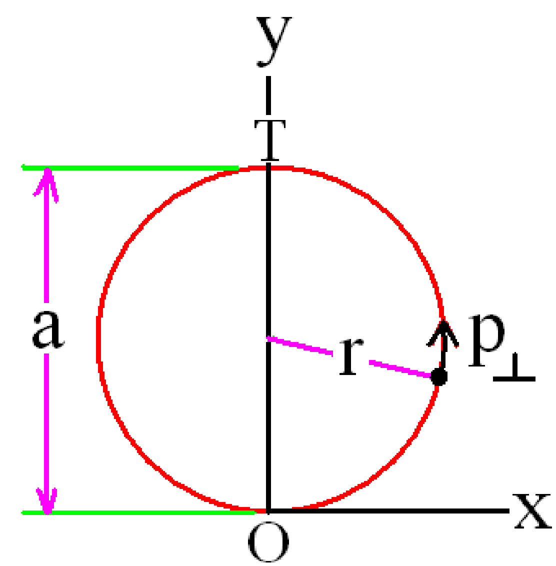
   (a) What is its motion in the lab frame and out to what transverse distance from the axis does it get.

   (b) What is the distance along the axis before it first returns to that axis?

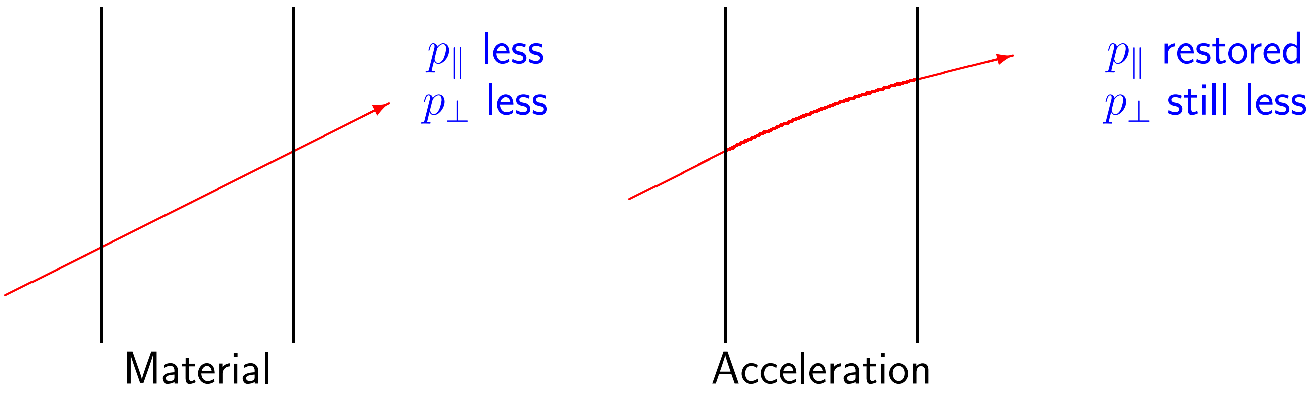
   (c) What is the wavelength λ in the Larmor frame?

   (d) what is the lattice parameter $\beta_\perp$ for that particle

2. Consider again a 200 MeV/c particle starting on the axis with a transverse momentum of 20 MeV/c in an axial solenoidal field of 3.33 T. After a distance
A) corresponding to 1/2 a helix rotation, or
B) corresponding to a full helix rotation determine,
the field abruptly doubles to 6.66 T. In the two cases determine:
   (a) The shape of the motion projected onto the x, y plane
   (b) The following length in z for one helix rotation (λ_{helix})



5 TRANSVERSE IONIZATION COOLING



Cooling rate vs. Energy

$$\text{(eq 10)} \quad \epsilon_{x,y} \;=\; \gamma\beta_v\; \sigma_\theta\; \sigma_{x,y}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\ /\epsilon$ is (using eq.17):

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{34}$$

which, for a given energy change, strongly favors cooling at low energy.

Note that the energy loss reduces not only stochastic transverse momenta (contributing to emittance), but also average momenta such as the angular momenta generated by entering a solenoid. So if cooling is done in a solenoid, the canonical momenta are also reduced

Heating Terms

$$\epsilon_{x,y} \;=\; \gamma\beta_v\;\sigma_\theta\;\sigma_{x,y}$$

Between scatters the drift conserves emittance (Liouiville).
When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\begin{aligned}
\Delta(\epsilon_{x,y})^2 &= \gamma^2\beta_v^2\;\sigma_{x,y}^2\Delta(\sigma_\theta^2) \\
2\epsilon\;\Delta\epsilon &= \gamma^2\beta_v^2\left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right)\;\Delta(\sigma_\theta^2) \\
\Delta\epsilon &= \frac{\beta_\perp\gamma\beta_v}{2}\;\Delta(\sigma_\theta^2)
\end{aligned}$$

e.g. from Particle data booklet $\quad \Delta(\sigma_\theta^2) \approx \left(\frac{14.1\;10^6}{[pc/e]\beta_v}\right)^2\;\frac{\Delta s}{L_R}$

$$\Delta\epsilon \;=\; \frac{\beta_\perp}{\gamma\beta_v^3}\;\Delta E\;\left(\left(\frac{14.1\;10^6}{2[mc^2/e]_\mu}\right)^2\;\frac{1}{L_R dE/ds}\right)$$

Defining

$$C(mat,E) \;=\; \frac{1}{2}\;\left(\frac{14.1\;10^6}{[mc^2/e]_\mu}\right)^2\;\frac{1}{L_R\;d\gamma/ds} \tag{35}$$

$$\text{then}\qquad \frac{\Delta\epsilon}{\epsilon} \;=\; dE\;\frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\;C(mat,E) \tag{36}$$

Equilibrium emittance

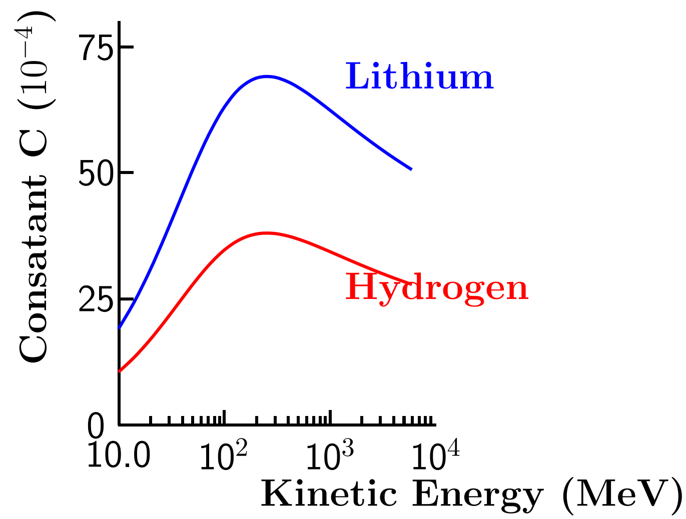
Equating this with equation 34
$$dE\,\frac{1}{\beta_v^2\ E} = dE\,\frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

gives the equilibrium emittance ϵ_o :
$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{\beta_v}\ C(mat,E) \quad (37)$$

At energies for minimum ionization loss:

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

As a function of energy:



Clearly Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows which will significantly degrade the performance. At lower energies C is much lower but there is then longitudinal (dp/p) heating.

Rate of Cooling

$$\frac{d\epsilon}{\epsilon} = \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right) \frac{dp}{p} \tag{38}$$

One would assume that one would keep $\epsilon_{\min} \ll \epsilon$, but this generally gives problems from non-linearities with the required large beam divergence angles σ_θ required.

Beam Divergence Angles

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \; \beta_v \gamma}}$$

so, from equation 37, for a beam in equilibrium

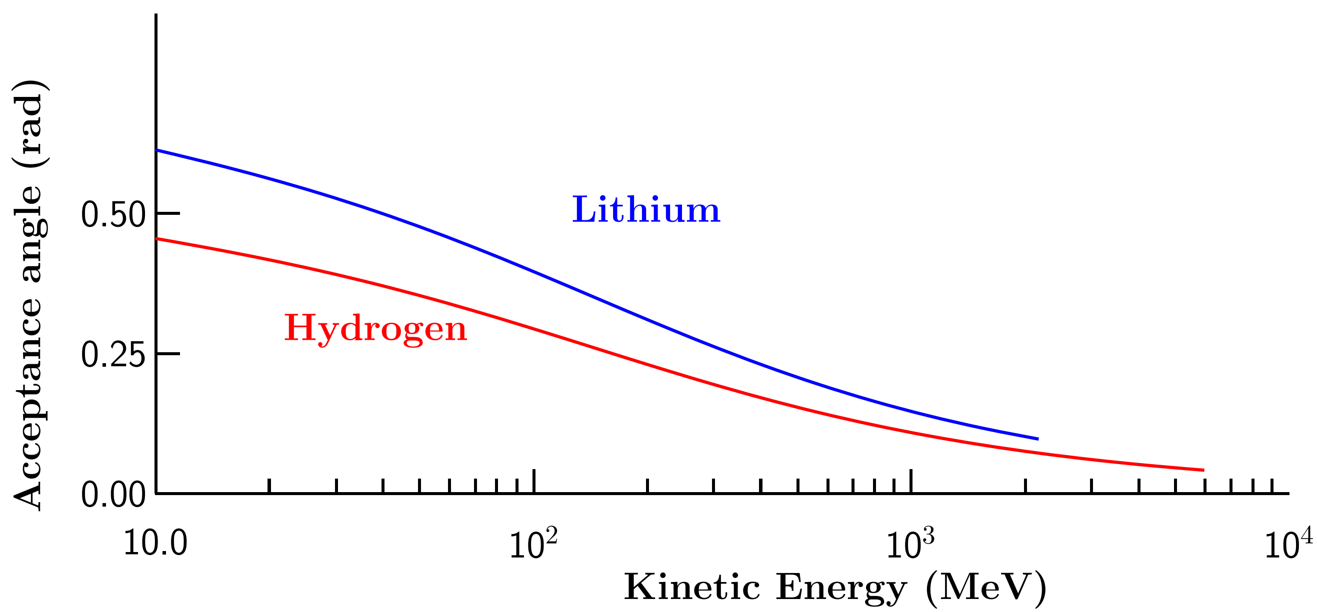
$$\sigma_\theta = \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{39}$$

and for 50 % of maximum cooling rate and an aperture at 3 σ, the angular aperture $\mathcal{A}$ of the system must be

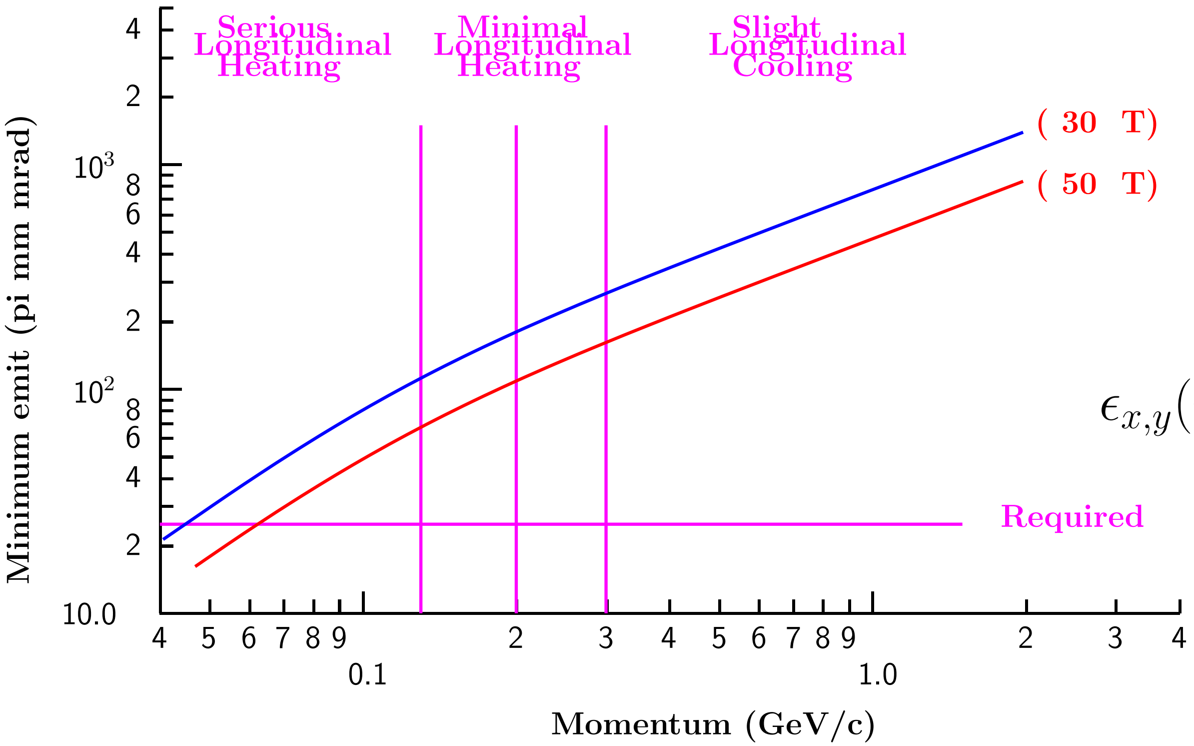
$$\mathcal{A} = 3\sqrt{2}\,\sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{40}$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are very large angles, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV ($\approx$ 170 MeV/c) for Lithium, and about 25 MeV ($\approx$ 75 MeV/c) for hydrogen.

$\theta = 0.3$ may be about as large as is possible in a lattice, but larger angles may be sustainable in a continuous focusing system such as a lens or solenoid. is optimistic, as we will see in the tutorial.



Focusing as a function of the beam momentum



From eq.32

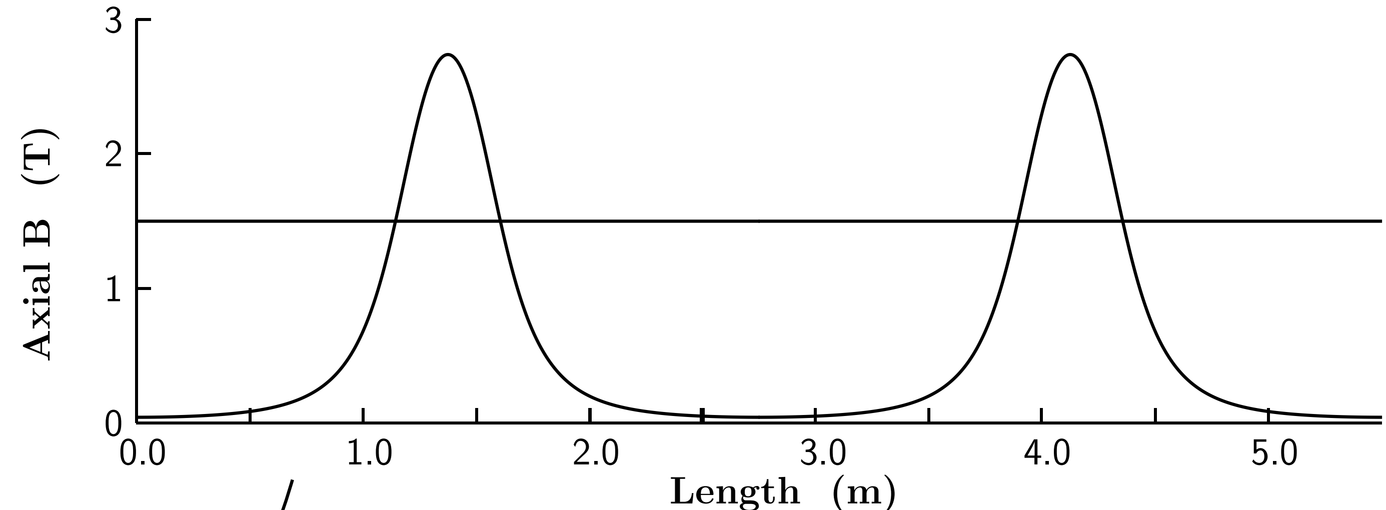
$$\beta_\perp = \frac{2\ [pc/e]}{c\ B_{sol}}$$

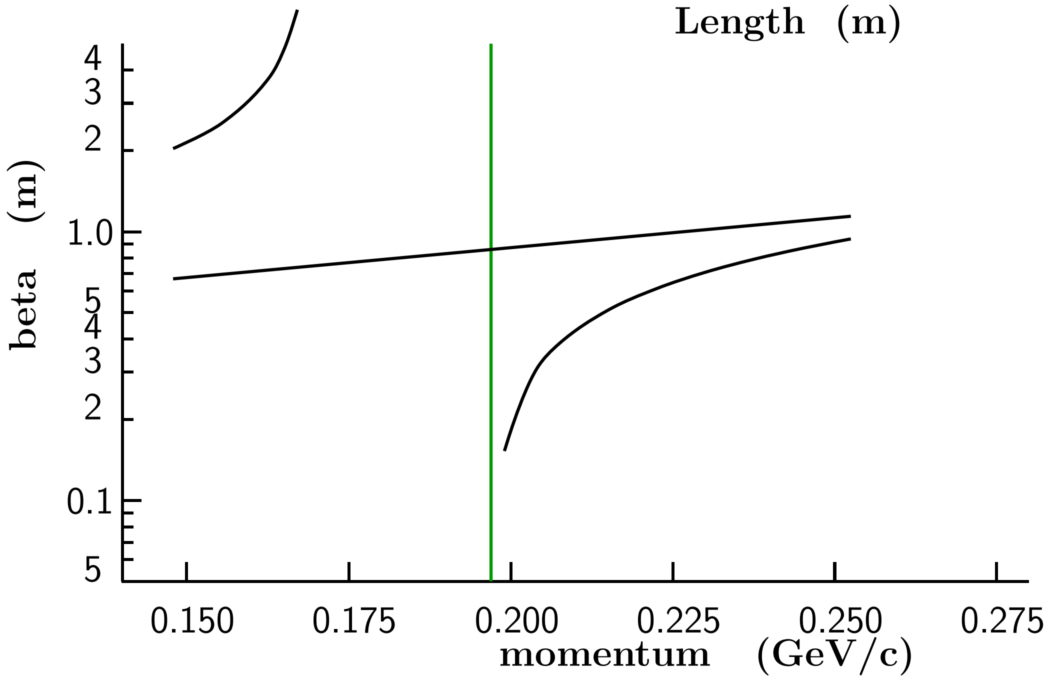
$$\epsilon_{x,y}(min) = C(mat, E)\ \frac{2\ \gamma\ [mc^2/e]_\mu}{B_{sol}\ c} \tag{41}$$

We see that at momenta where longitudinal emittance is not blown up ($\approx$ 200 MeV/c) then even at 50 T the minimum emittance is $\approx$ 100 μm $>>$ required 25 μm

But if we allow longitudinal heating and use very low momenta (45-62 MeV/c or 9-17 MeV) the muon collider requirements can be met

Decreasing beta in Solenoids by adding periodicity



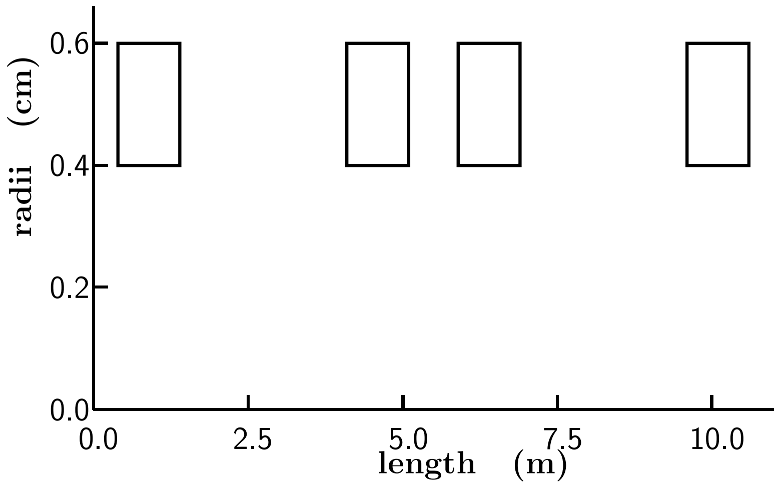


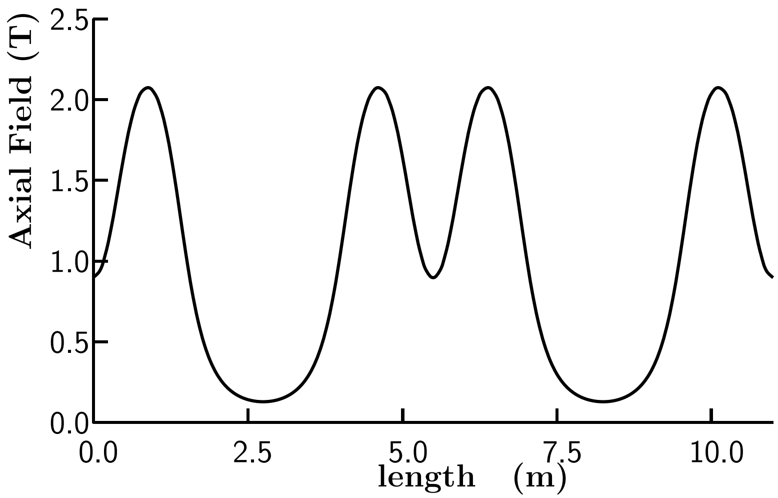
- Determination of lattice betas
  - Track single near paraxial partic through many cells
  - plot θ_x vs x after each cell
  - fit ellipse: $\beta_{x,y} =$ A((x) / A(θ_x)
- Resonances introduced
- Betas reduced locally
- Momentum acceptance small

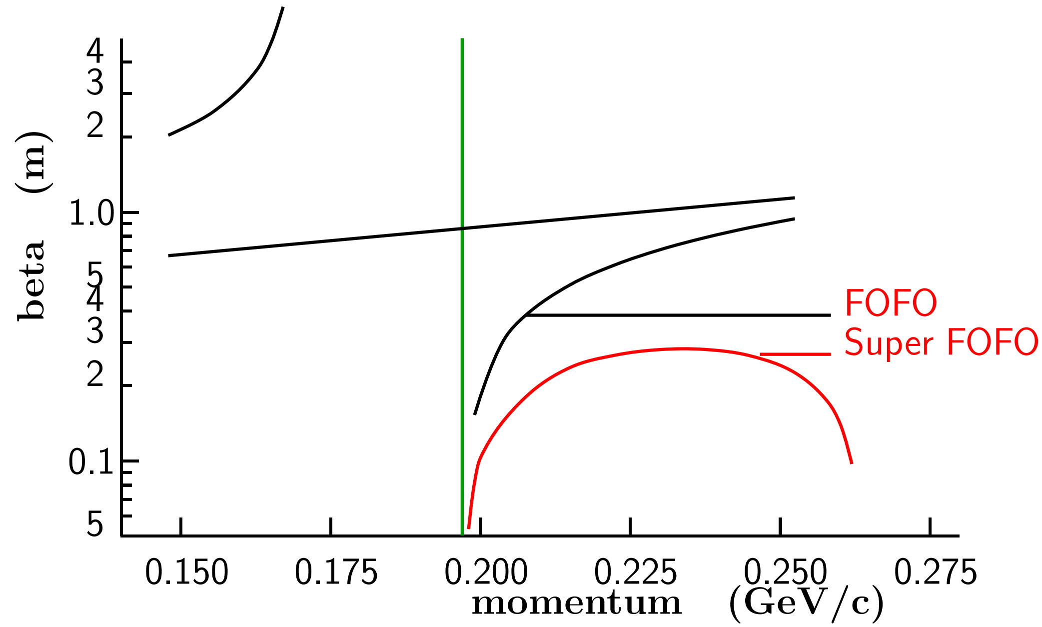
In practice, the solenoid fields are usually altering to avoid a buildup of angular momentum - our homework will show how this occurs

Super FOFO

Double periodicity

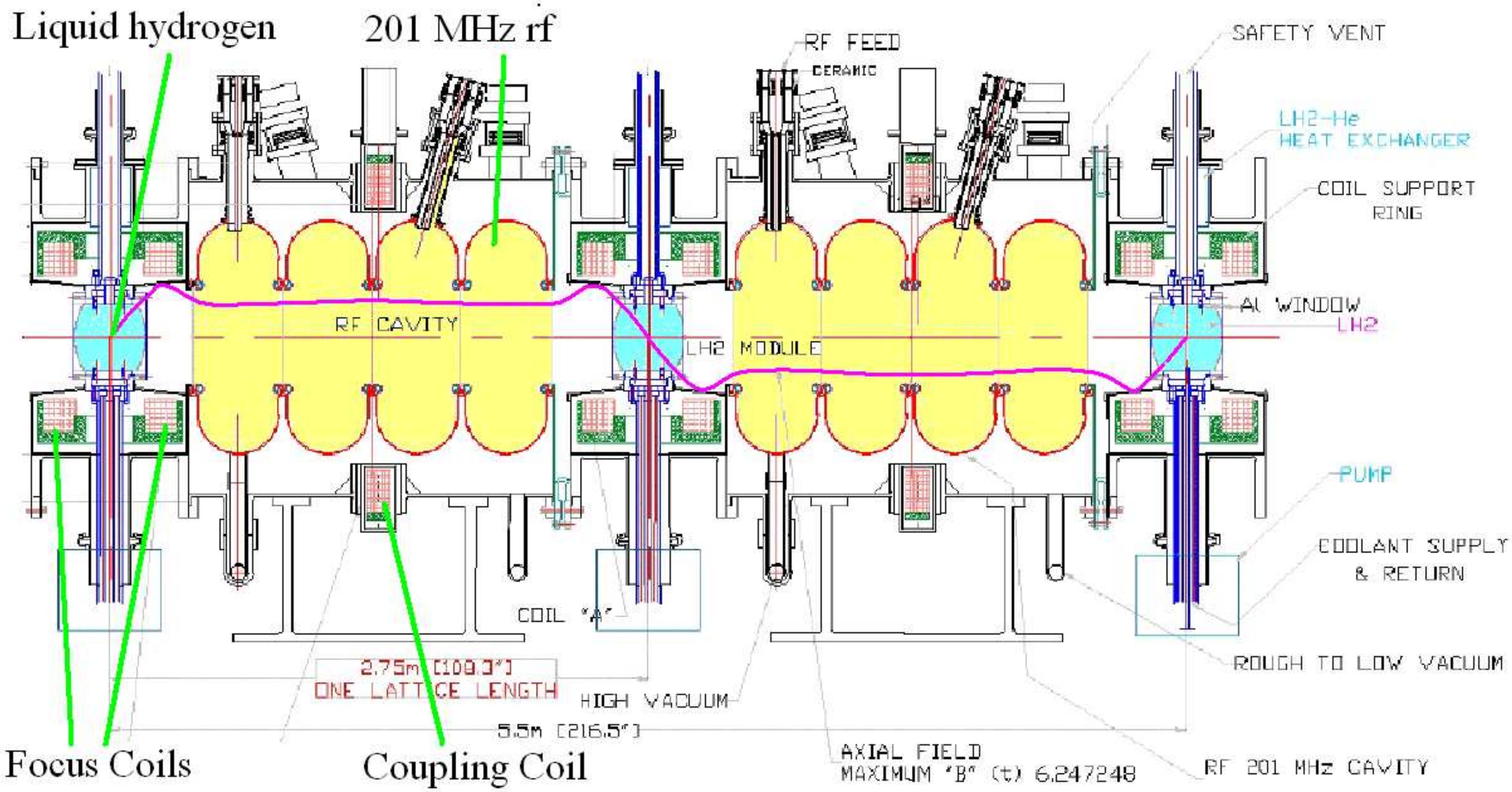






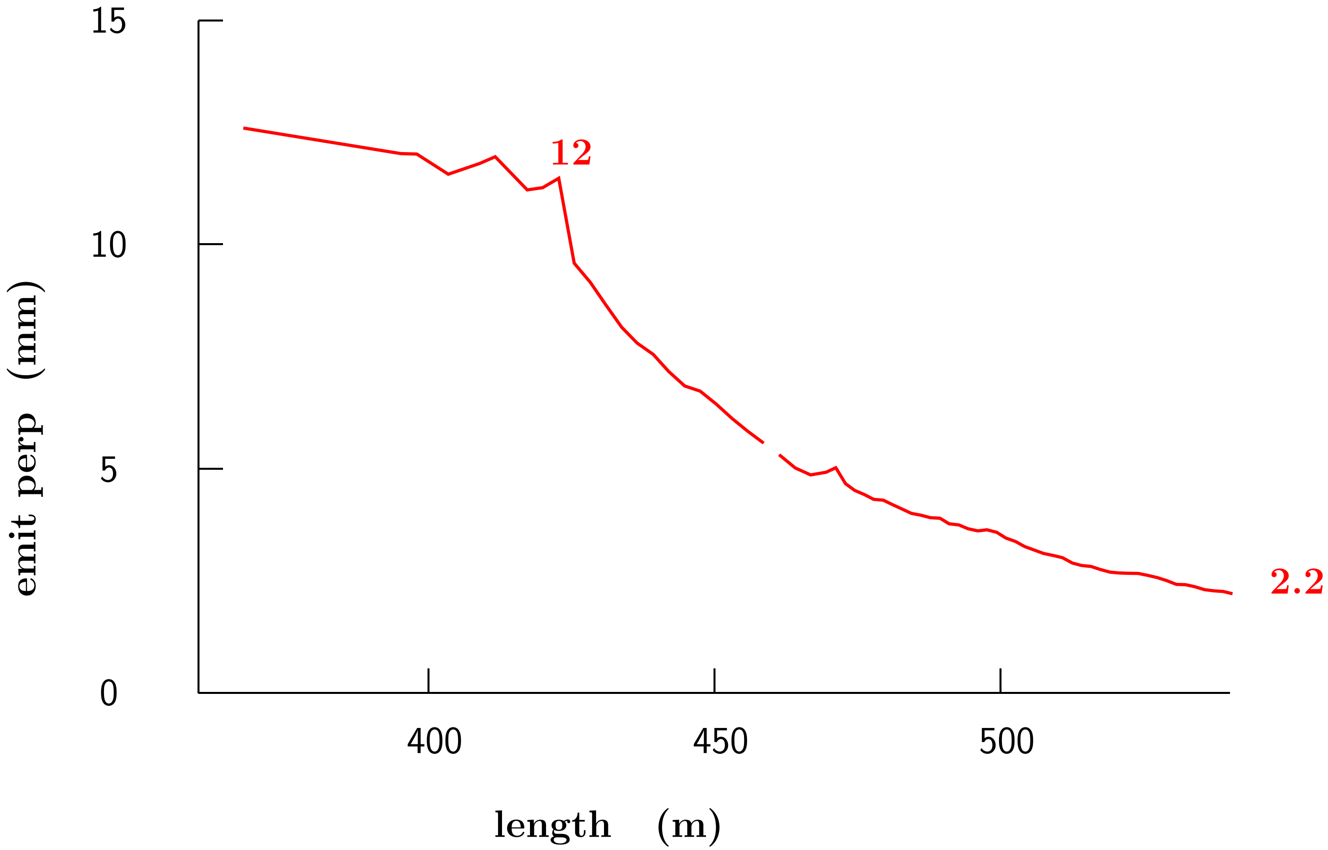
- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

SFOFO Lattice Engineering
Study 2 at Start of Cooling



- This is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- In study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls
  This keeps σ_θ and ϵ/ϵ_o more or less constant, thus maintains cooling rate

Performance



Conclusion on transverse cooling

- Hydrogen (gas or liquid) is the best material to use
- Cooling requires very large angular acceptances -
- Only realistically possible in solenoid focused systems
- Adding periodicity lowers the $\beta_\perp$ for a given solenoid field
- But periodicity does reduce momentum acceptance
- Final cooling to 25 μm possible at 50 T and low energies
  but longitudinal emittance then rises

6 LONGITUDINAL IONIZATION COOLING

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} = \frac{\frac{\Delta\ (\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{42}$$

$$J_6 = J_x + J_y + J_z \tag{43}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In electron synchrotrons, with no gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In muon ionization cooling, $J_x = J_y = 1$, but J_z is negative or small.

Transverse cooling with $J_{x,y} \neq 1$

From last lecture:

$$\frac{\Delta \sigma_{p\perp}}{\sigma_{p\perp}} = \frac{\Delta p}{p}$$

and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta \epsilon_{x,y}}{\epsilon_{x,y}} = \frac{\Delta p}{p}$$

and thus

$$J_x = J_y = 1 \tag{44}$$

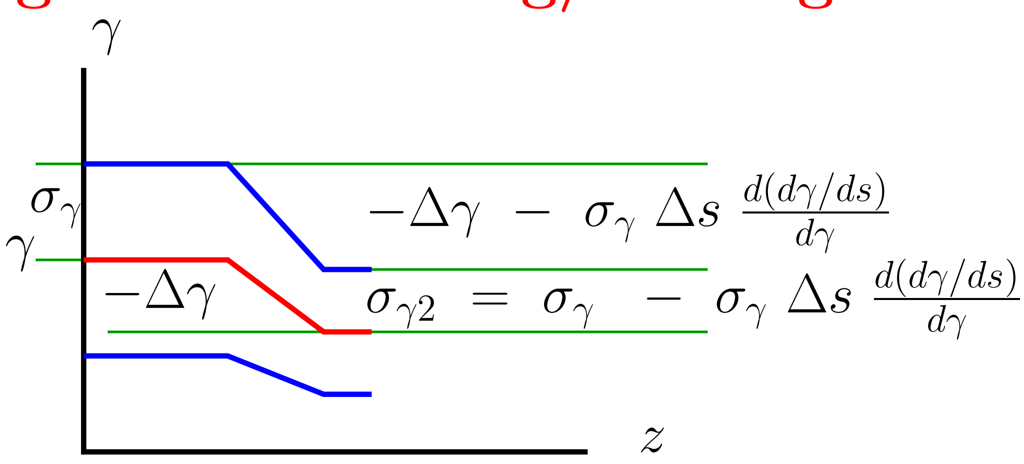
But if $J_{x,y} \neq 1$

$$\frac{\Delta \epsilon_{x,y}}{\epsilon_{x,y}} = \frac{1}{J_{x,y}} \frac{\Delta p}{p} \tag{45}$$

and

$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{J_{x,y}\ \beta_v}\ C(mat, E) \tag{46}$$

Longitudinal cooling/heating from shape of dE/dx



The emittance in the longitudinal direction ϵ_z is (eq.11):

$$\epsilon_z \;=\; \gamma\beta_v\,\frac{\sigma_p}{p}\,\sigma_z \;=\; \frac{1}{m}\,\sigma_p\sigma_z \;=\; \frac{1}{m}\,\sigma_E\sigma_t \;=\; c\,\sigma_\gamma\,\sigma_t$$

where σ_t is the rms bunch length in time, and c is the velocity of light. Drifting between interactions will not change emittance (Louville), and an interaction will not change σ_t, so emittance change is only induced by the energy change in the interactions:

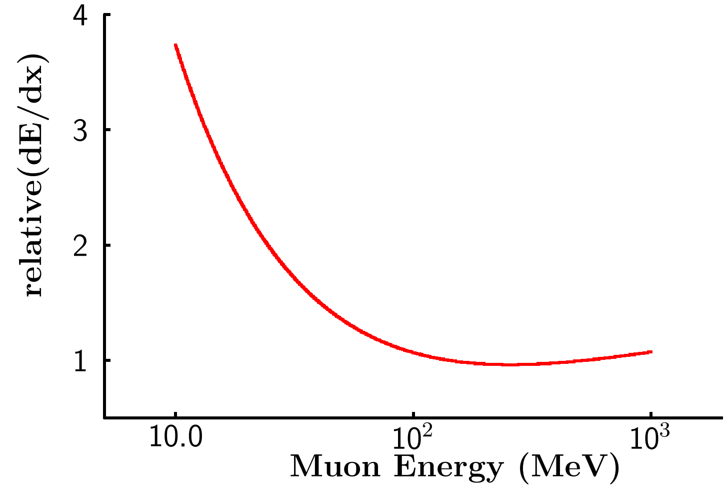
$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \;=\; \frac{\sigma_\gamma\,\Delta s\,\frac{d(d\gamma/ds)}{d\gamma}}{\sigma_\gamma} \;=\; \Delta s\,\frac{d(d\gamma/ds)}{d\gamma}$$

and

$$\frac{\Delta p}{p} \;=\; \frac{\Delta\gamma}{\beta_v^2\gamma} \;=\; \frac{\ell}{\beta_v^2\gamma}\,\left(\frac{d\gamma}{ds}\right)$$

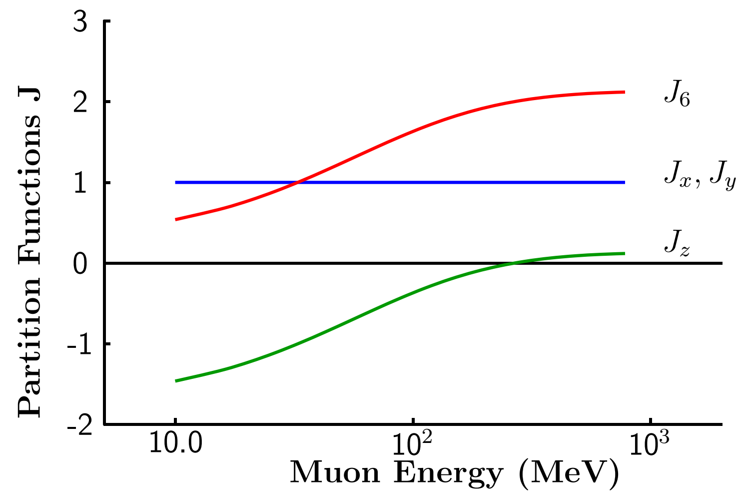
So from the definition of the partition function J_z:

$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\Delta s \ \frac{d(d\gamma/ds)}{d\gamma}\right)}{\frac{\Delta s}{\beta_v^2\gamma}\ \left(\frac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2 \ \frac{d(d\gamma/ds)}{d\gamma/\gamma}\right)}{\left(\frac{d\gamma}{ds}\right)} \tag{47}$$



A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It is given approximately by:

It is seen that J_z is strongly negative at low energies (longitudinal heating), and is only barely positive at momenta above 300 MeV/c. In practice there are many reasons to cool at a moderate momentum around 250 MeV/c, where $J_z \approx 0$. However, the 6D cooling is still strong $J_6 \approx 2$.



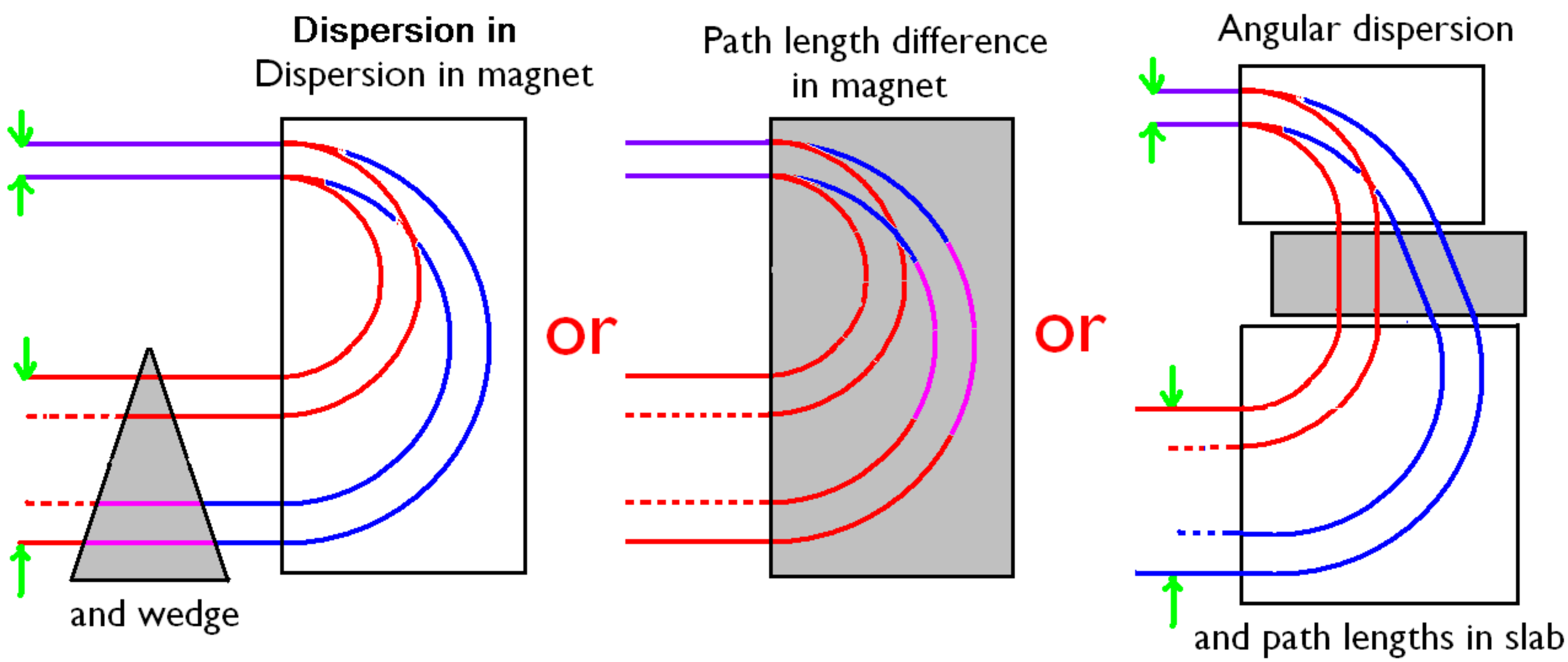
Emittance Exchange

What is needed is a method to exchange cooling between the transverse and longitudinal direction s. This is done in synchrotron cooling if focusing and bending is combined, but in this case, and in general, one can show that such mixing can only increase one J at the expense of the others: J_6 is conserved.

$$\Delta J_x + \Delta J_x + \Delta J_x = 0 \tag{48}$$

and for typical operating momenta:

$$J_x \; + \; J_y \; + \; J_z \; = \; J_6 \; \approx \; 2.0 \tag{49}$$

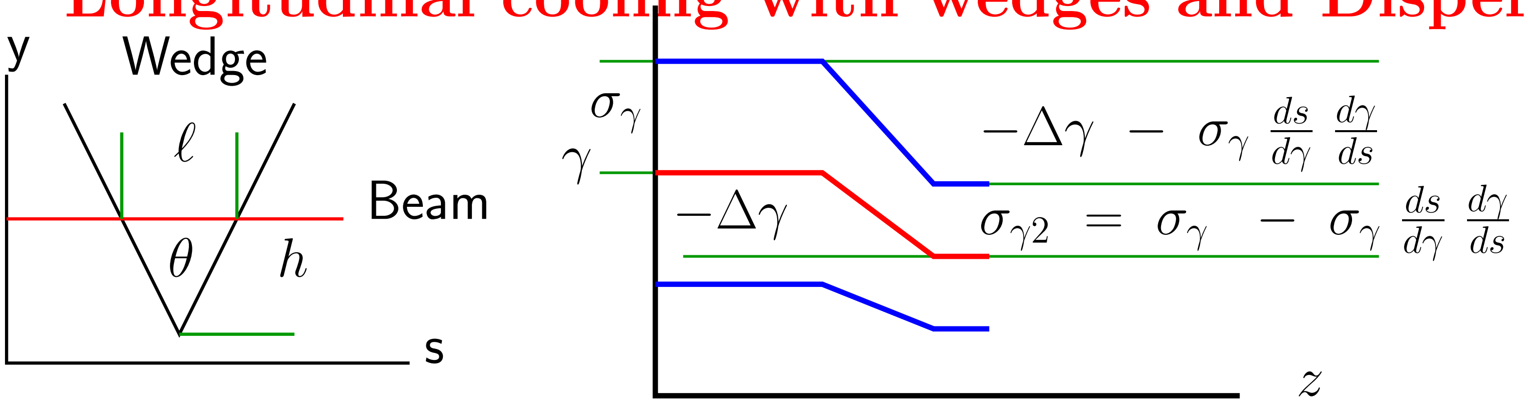


dp/p reduced But σ_y increased

Long Emit reduced Trans Emit Increased

"Emittance Exchange"

Longitudinal cooling with wedges and Dispersion



For a wedge with center thickness ℓ and height from center h ($2h\ \tan(\theta/2) = \ell$), in dispersion D ($D = \frac{dy}{dp/p}$, or with eq.17: $D = \beta_v^2 \frac{dy}{d\gamma/\gamma}$) (see fig. above):

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right)}{\sigma_\gamma} = \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right) = \left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\left(\frac{d\gamma}{ds}\right)$$

and

$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$\Delta J_z(\mathrm{wedge}) = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)} = \frac{D}{h} \qquad (\text{for simple bend \& gas } \Delta J_z(\mathrm{wedge}) = 1) \quad (50)$$

$$J_z = J_z(\text{no wedge}) + \Delta J_z(\text{wedge}) \qquad (51)$$

But from eq.48, for any finite $J_z(\mathrm{wedge})$, J_x or J_y will change in the opposite direction.

Longitudinal Heating Terms

Since $\epsilon_z \;=\; \sigma_\gamma\ \sigma_t\ c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

Straggling:

$$\Delta(\sigma_\gamma) \approx \frac{\Delta\sigma_\gamma^2}{2\sigma_\gamma} \approx \frac{1}{2\sigma_\gamma}\ 0.06\ \frac{Z}{A}\ \left(\frac{m_e}{m_\mu}\right)^2\ \gamma^2\ \left(1-\frac{\beta_v^2}{2}\right)\ \rho\ \Delta s$$

$\Delta E \;=\; E\ \beta_v^2\ \frac{\Delta p}{p}$, so:

$$\Delta s = \frac{\Delta E}{dE/ds} = \frac{1}{dE/ds} E\ \beta_v^2\ \frac{\Delta p}{p}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{0.06}{2\sigma_\gamma^2}\ \frac{Z}{A}\ \left(\frac{m_e}{m_\mu}\right)^2\ \gamma^2\ \left(1-\frac{\beta_v^2}{2}\right)\ \rho\ \frac{\beta_v^2\ E}{dE/ds}\ \frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} = -\ J_z\ \frac{dp}{p}$$

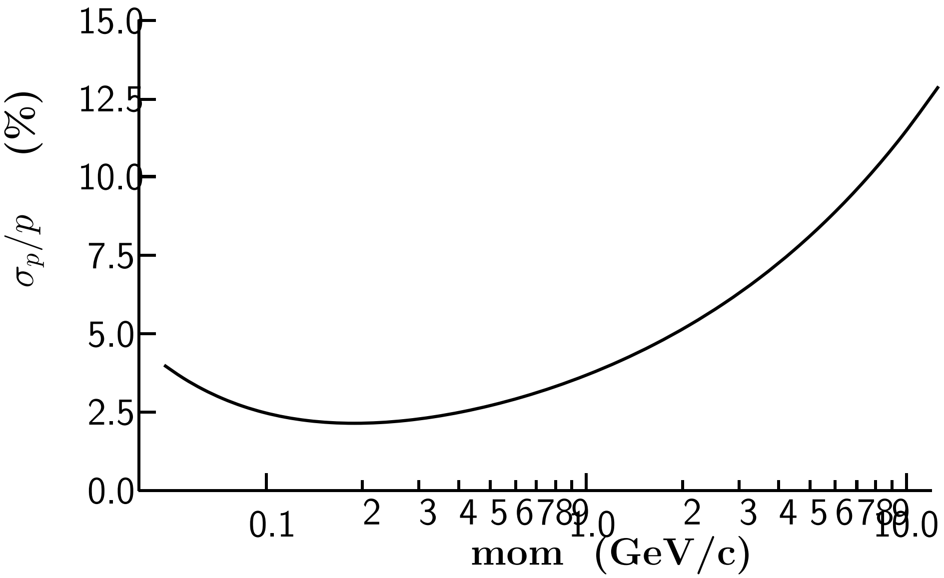
giving an equilibrium:

$$\frac{\sigma_p}{p} = \left(\left(\frac{m_e}{m_\mu}\right)\ \sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right)\ \sqrt{\frac{\gamma}{\beta_v^2}\ \left(1-\frac{\beta_v^2}{2}\right)\quad \frac{1}{J_z}} \tag{52}$$

For Hydrogen, the value of the first parenthesis is ≈1.36 %.

Without coupling, J_z is small or negative, and the equilibrium does not exist. But with equal partition functions giving $J_z \approx 2/3$ then this expression, for hydrogen, gives: the values plotted below.

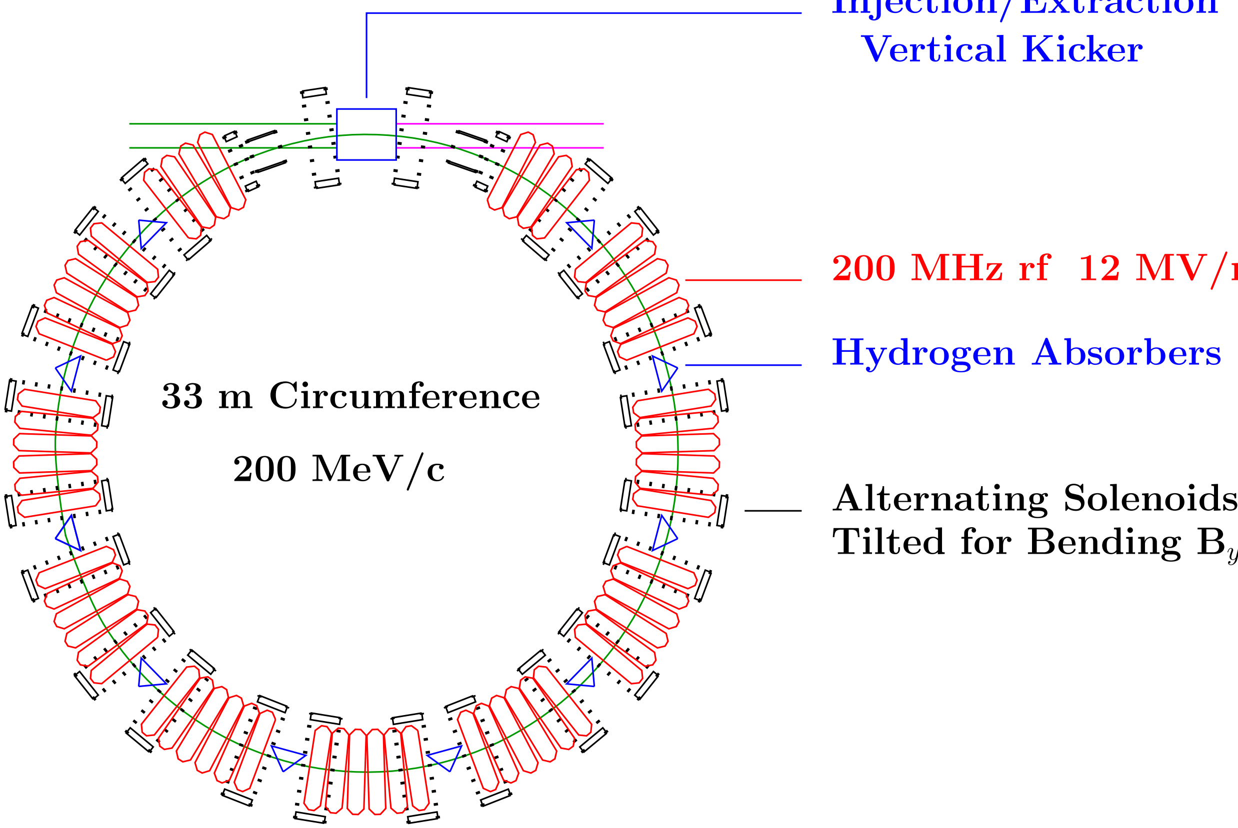
The following plot shows the dependency for hydrogen

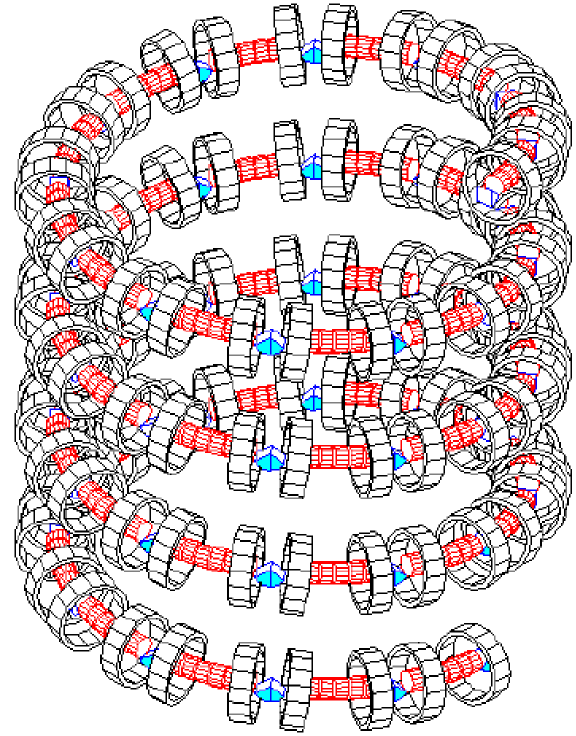


It is seen to favor cooling at around 200 MeV/c, but has a broad minimum.

Example: RFOFO Ring

R.B. Palmer R. Fernow J. Gallardo[1], and Balbekov[2]





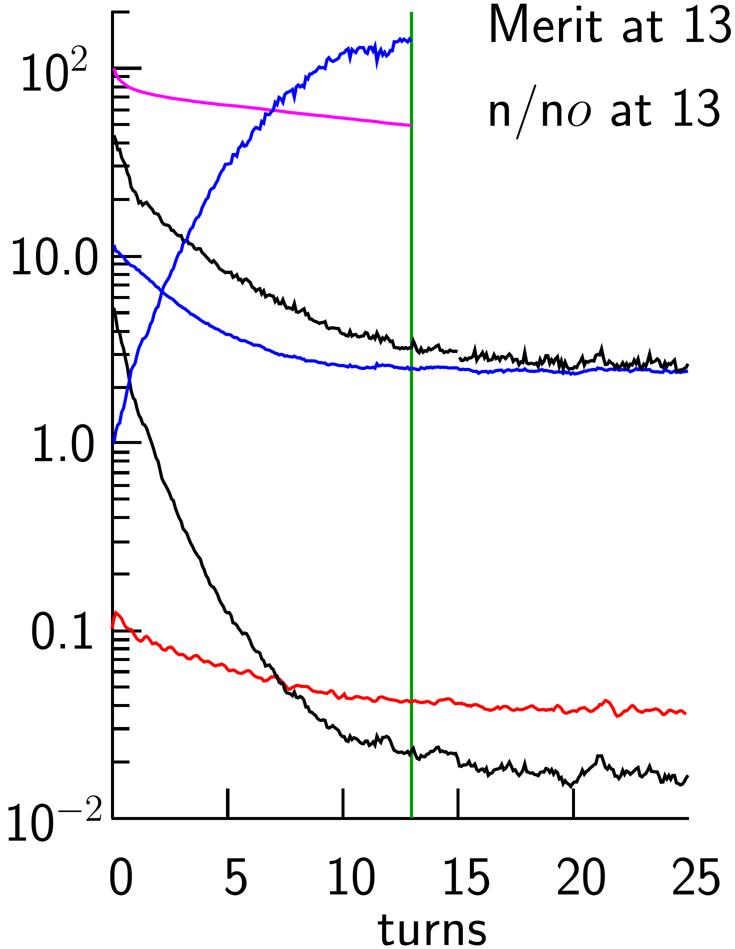
[1] Fernow and others: MUC-232, 265, 268, & 273
[2] V.Balbekov "Simulation of RFOFO Ring Cooler with Tilted Solenoids" MUC-CONF–0264

Performance

Using Real Fields, but no windows or injection insertion

$$\text{Merit} = \frac{n}{n_o}\,\frac{\epsilon_{6,o}}{\epsilon_6} = \frac{\text{Initial phase density}}{\text{final phase density}}$$

$n/n_o =$ 1543 / 4494



Merit at 13 turns 139 Falls after 13 turns from decay loss

n/no at 13 turns 0.50

$\epsilon \parallel$ 43.9 to 2.65 ($\pi\ mm$)
$\epsilon \perp$ 11.4 to 2.43 ($\pi\ mm$)

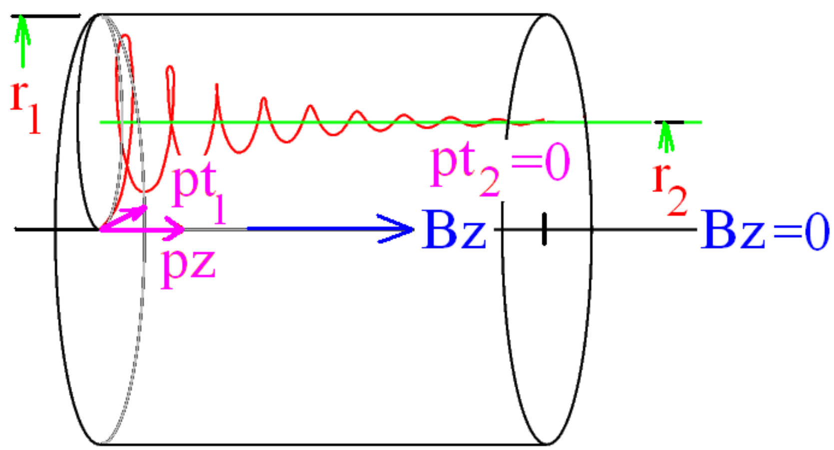
dp/p 10.2 to 3.6 %
$\epsilon 6$ 5.3 to 0.017 $(\pi\ mm)^3$

Longitudinal Cooling Conclusion

- Good cooling in 6 D in a ring
  - But injection/extraction difficult
  - Requires short bunch train
- Also good 6D cooling in HP Gas Helix (not discussed here)
  - But difficult to introduce appropriate frequency rf
  - And questions about use of gas with an ionizing beam
- Converting Ring cooler to a large Helix (Guggenheim)
  - Solves Injection/extraction problem
  - Solves bunch train length problem
  - Allows tapering to improve performance
  - But more expensive than ring

7 IONIZATION COOLING HOMEWORK

1. Again consider a solenoid with $B_z = 3.33$ T and a muon with starting on axis with $p_t = 20$ MeV/c and $p_z = 200$ MeV/c. Imagine an ideal transverse cooling system with continuous energy loss and re-acceleration so that all transverse momenta are reduced to near zero, then the above particle will settle at half its maximum distance from the axis $r_2 = r_1/2$ and pass straight down the field lines at p_z.



(a) What now is its motion in the Larmor frame?

(b) If now the field B_z suddenly stops, what is the further motion of the muon?

(c) Taking the initial phase space to be $\Phi_1 = pt_1\ r_1$, what is the final phase space if it is defined as $\Phi_2 = pt_3\ r_3$?

(d) This (from Bush's Law) is not good. What could one do to avoid it ?

2. In the longitudinal cooling section, we describe a cooling ring that, with emittance exchange in wedges cools all 6 dimensions. Assume $\beta_\perp = 0.4$ m, dispersion at the hydrogen wedge $D = 7$ cm, the length of the wedge on axis $\ell = 28.6$ cm, and the height from the axis to the apex of the wedge $h = \frac{\ell}{2\ \tan(100^o/2)} = 12 \quad \mathrm{cm}.$ Assume that the sum of partition functions $\Sigma J_i \approx 2.0$, $C(mat, E) = 38\ 10^{-4}$, and assume good mixing between x and y. As before, β_v=0.85.

a) What are the three partition functions in this case?

b) What is the expected equilibrium transverse emittance?

8 PAC09 PAPER & References