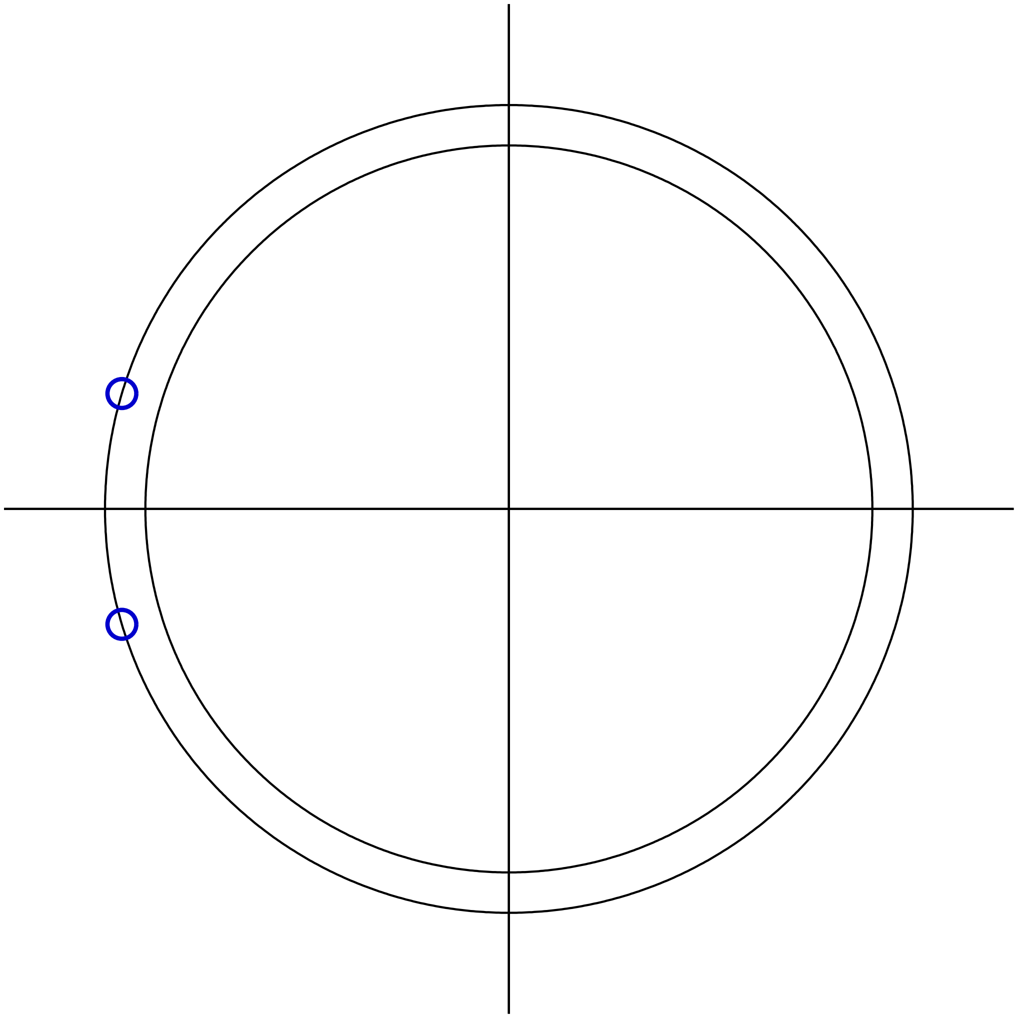
# Understanding PIC Generalized Linear Perturbation Analysis

J. Scott Berg
Brookhaven National Laboratory
Advanced Accelerator Group Meeting
7 June 2007

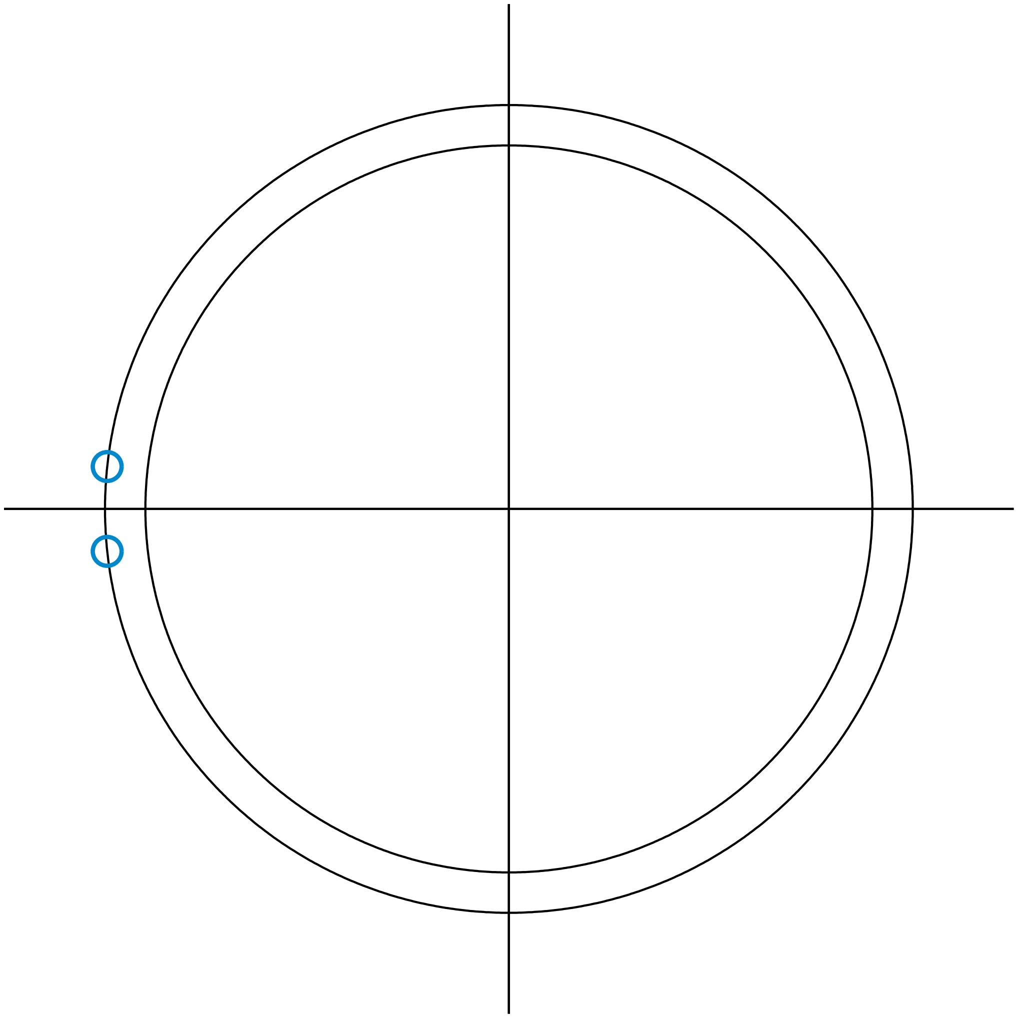
# Understanding Half-Integer Resonance

- Imagine varying a parameter, say it is momentum
- Have half-integer finite-width stopband
- Start stable
- Then metastable: eigenvalues -1
- Then unstable: one eigenvalue growing, other damping
- Returns

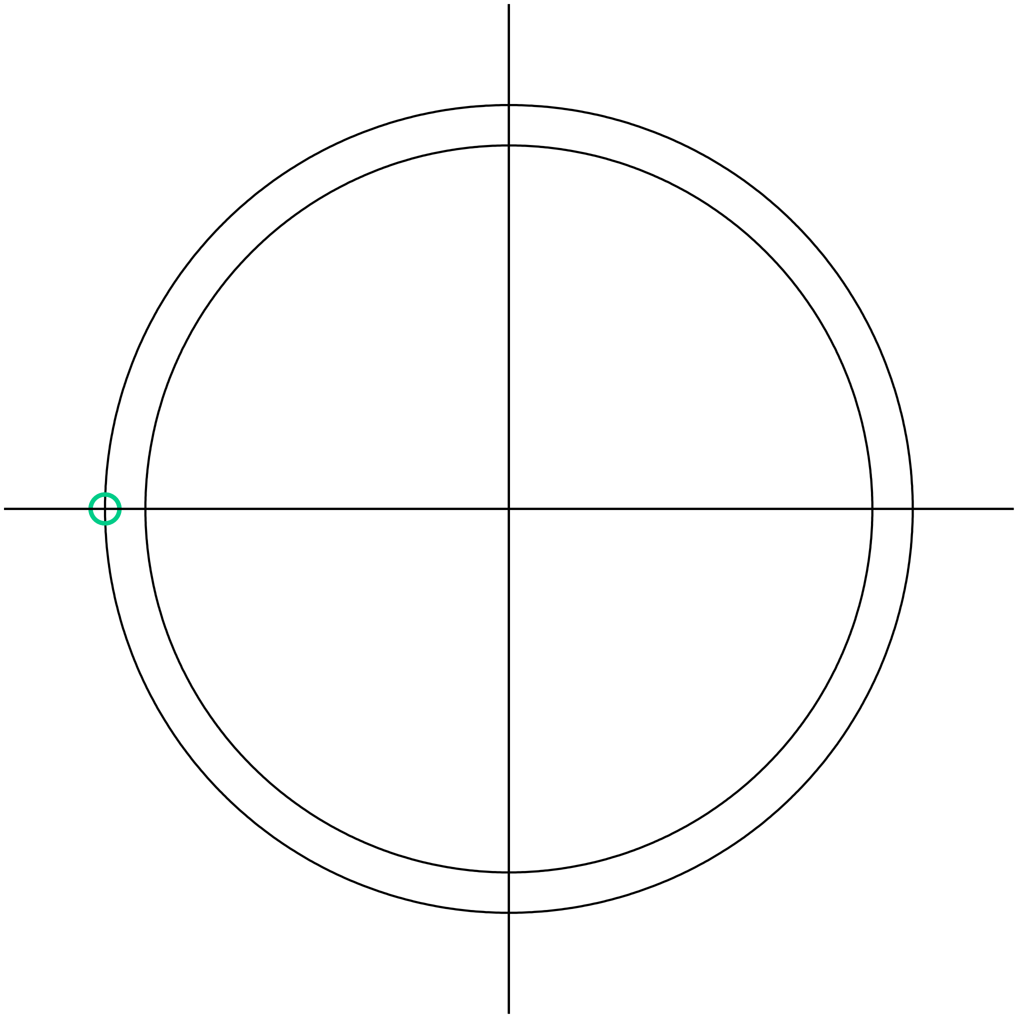
# Half Integer Resonance Crossing



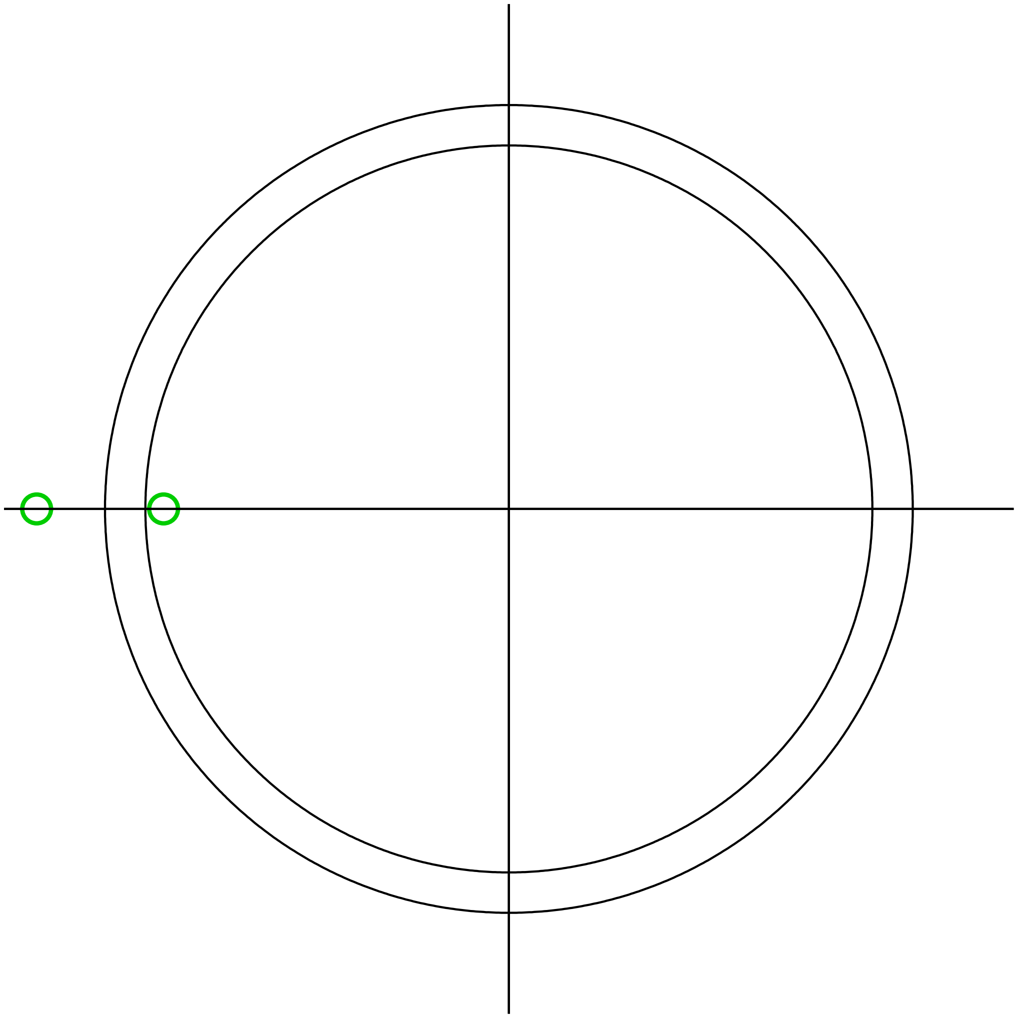
# Half Integer Resonance Crossing



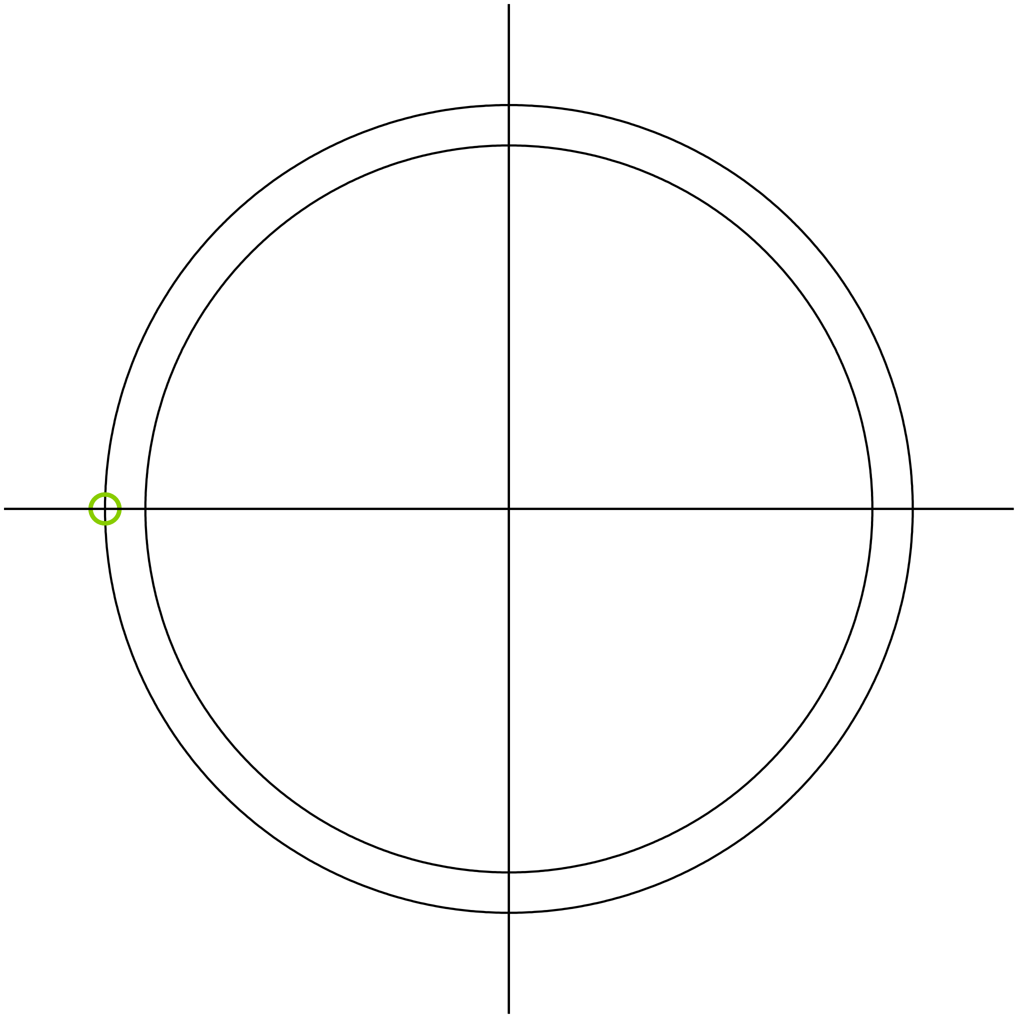
# Half Integer Resonance Crossing



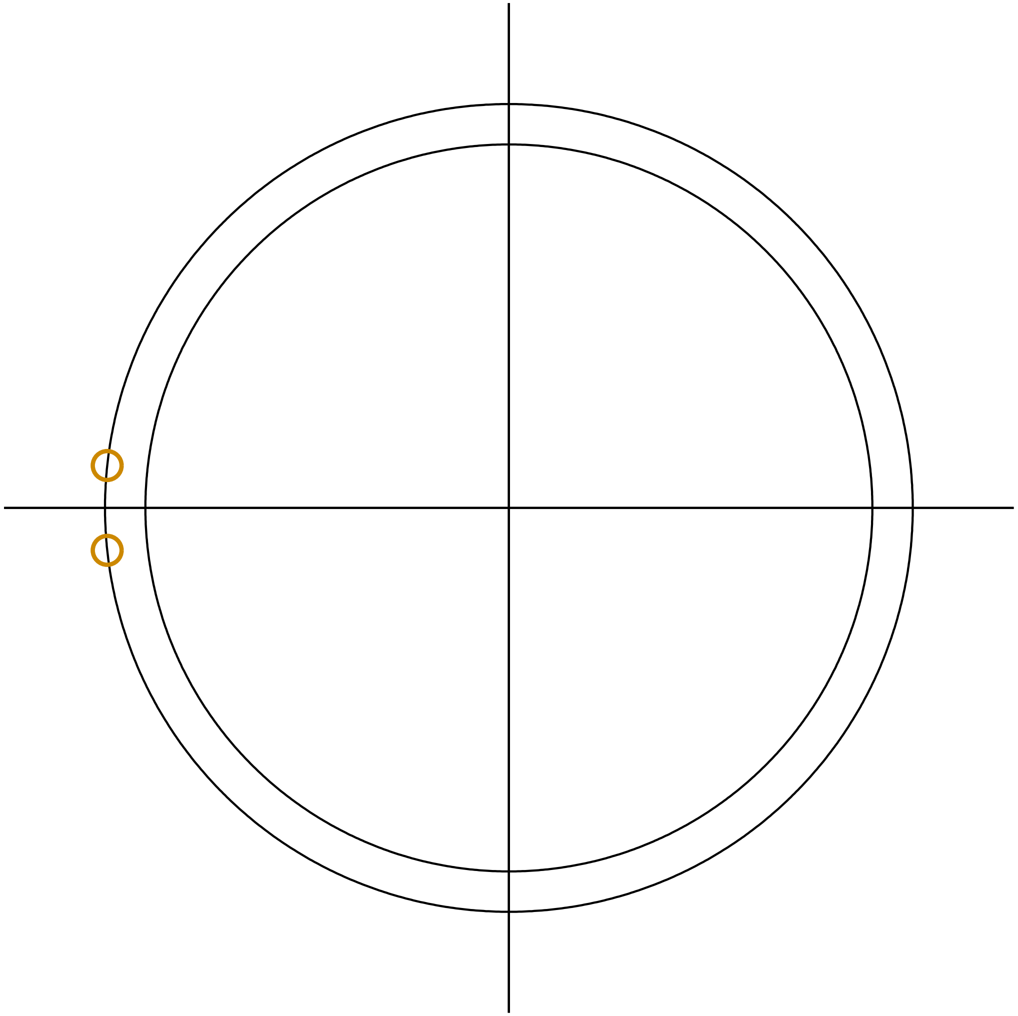
# Half Integer Resonance Crossing



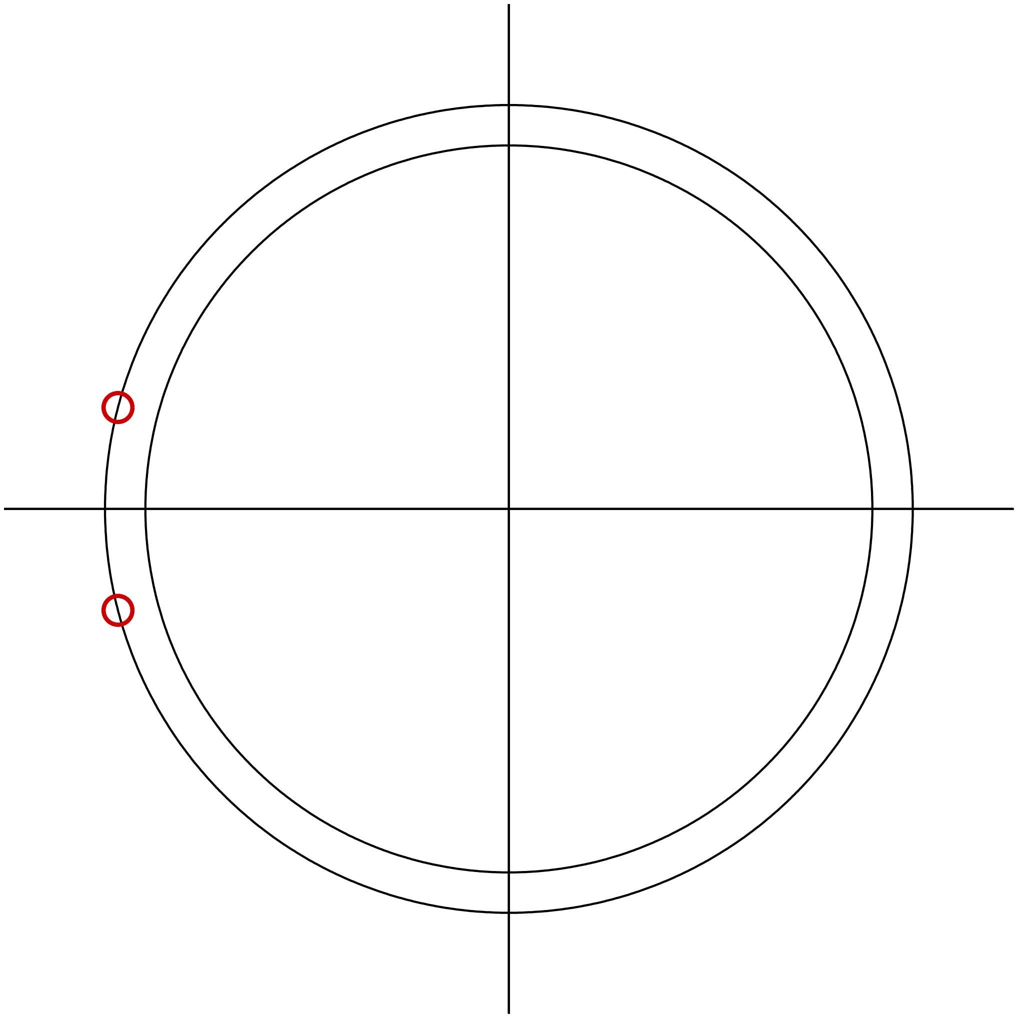
# Half Integer Resonance Crossing



# Half Integer Resonance Crossing



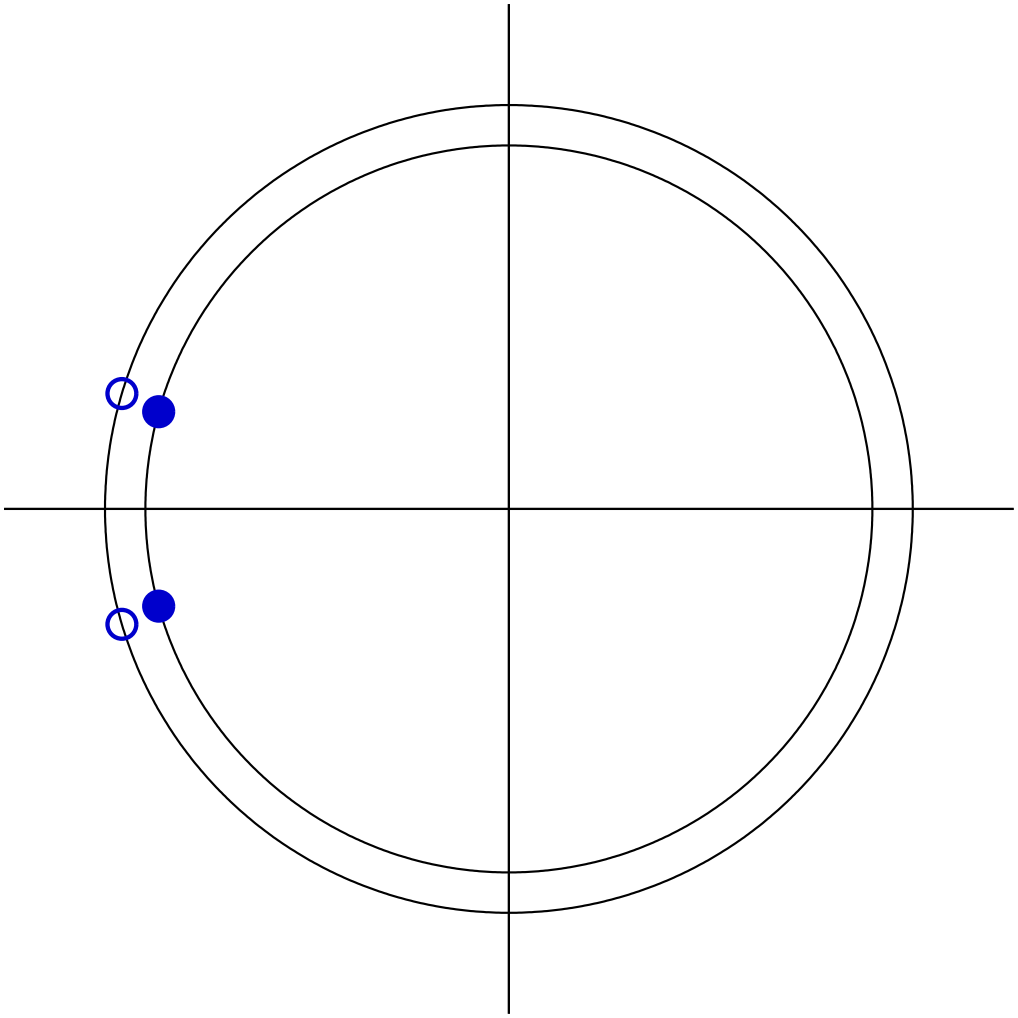
# Half Integer Resonance Crossing



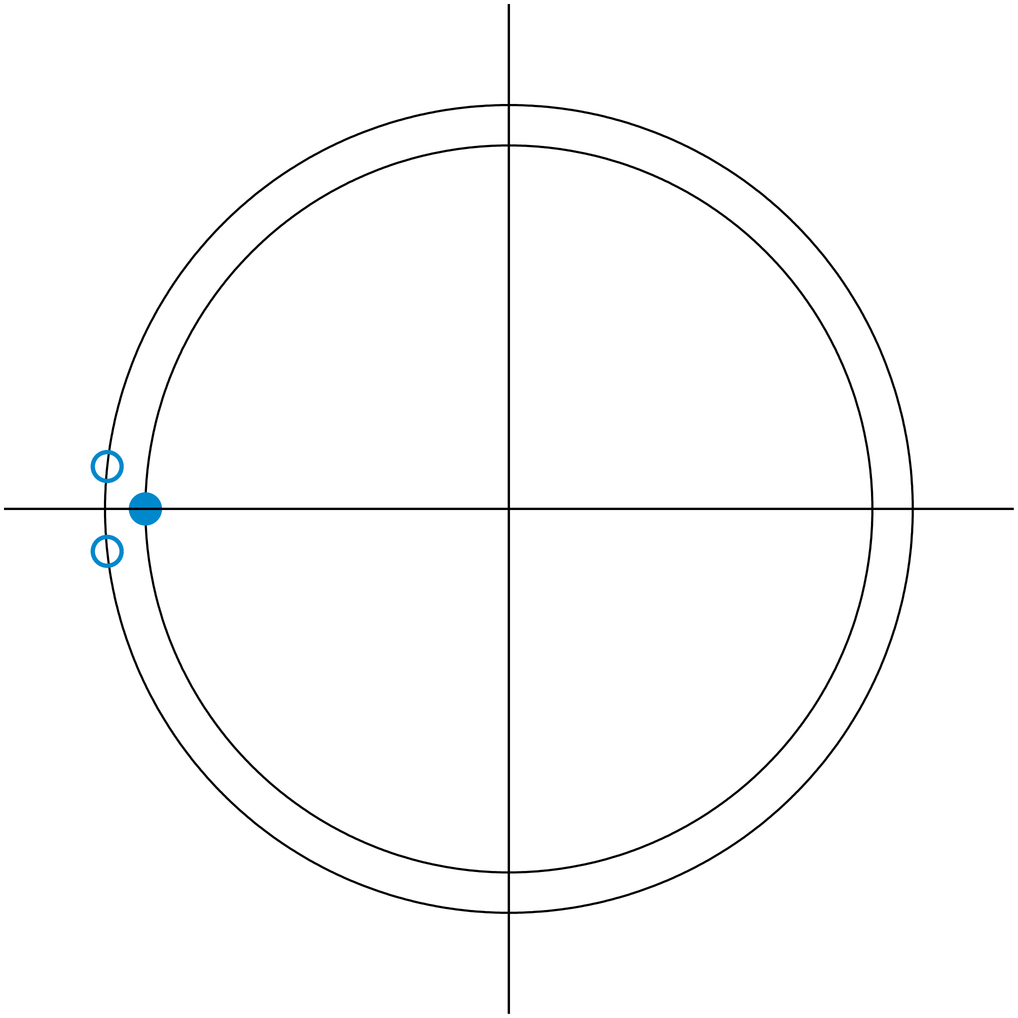
# Half-Integer Resonance With Damping

- Single thin absorber
- Similar behavior to undamped
- Damped eigenvalues collide when undamped still stable
- Undamped eigenvalues are 1, one damped eigenvalue is 1
- If unstable without damping, damping cannot stabilize

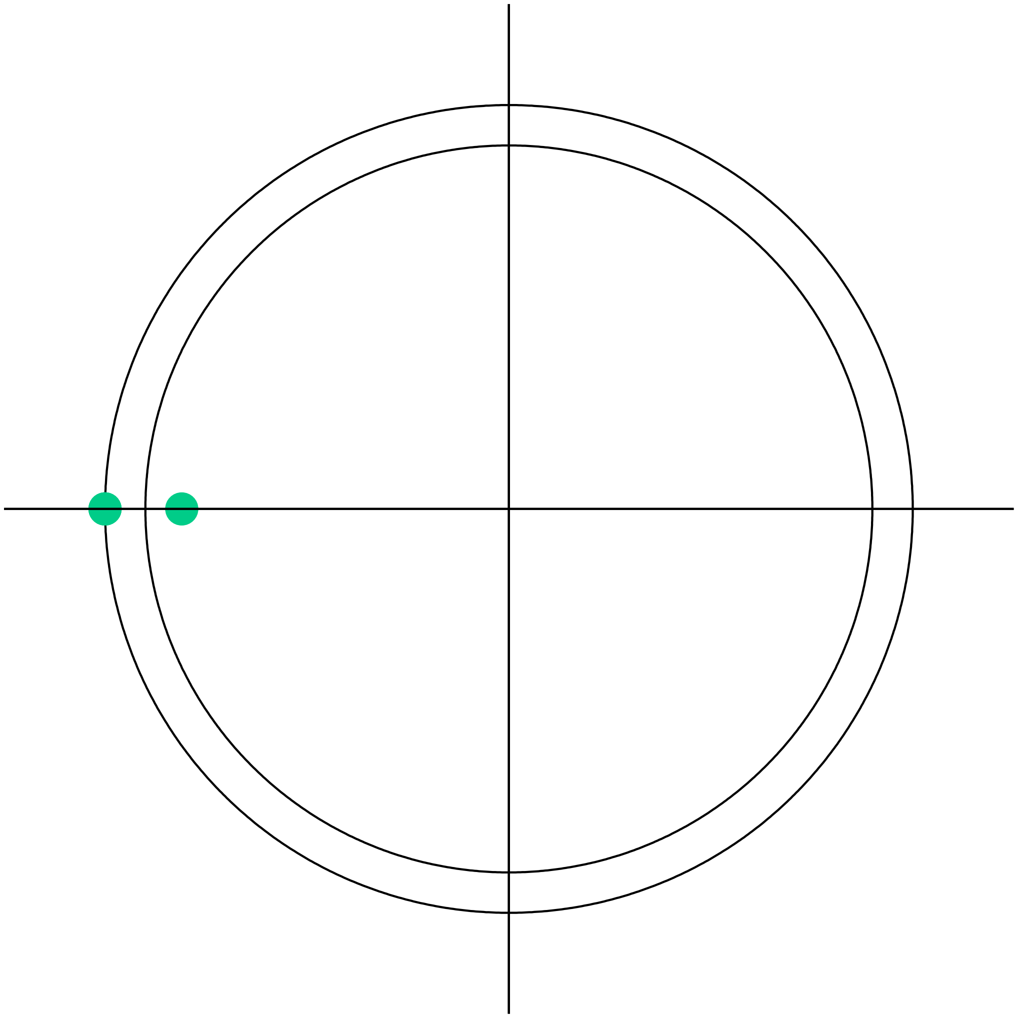
# Half Integer Resonance Crossing With Damping



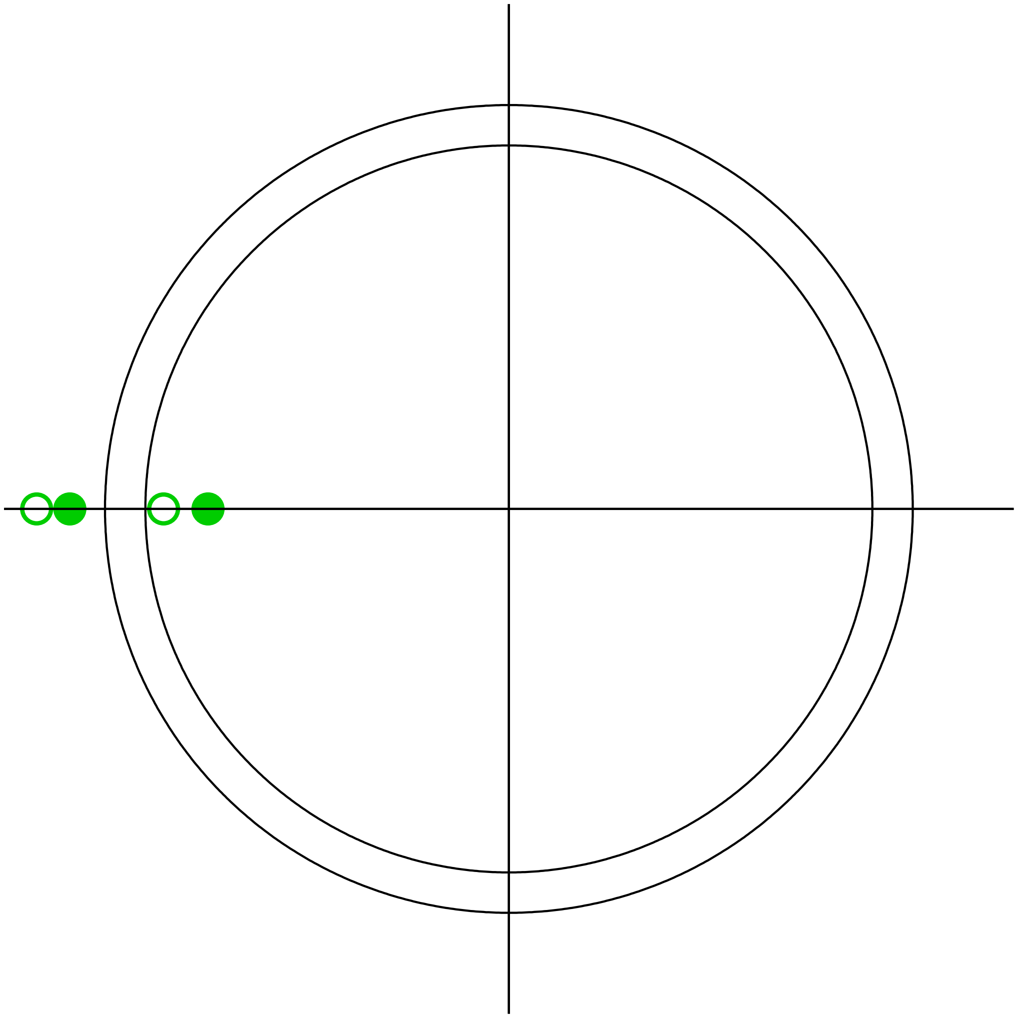
# Half Integer Resonance Crossing With Damping



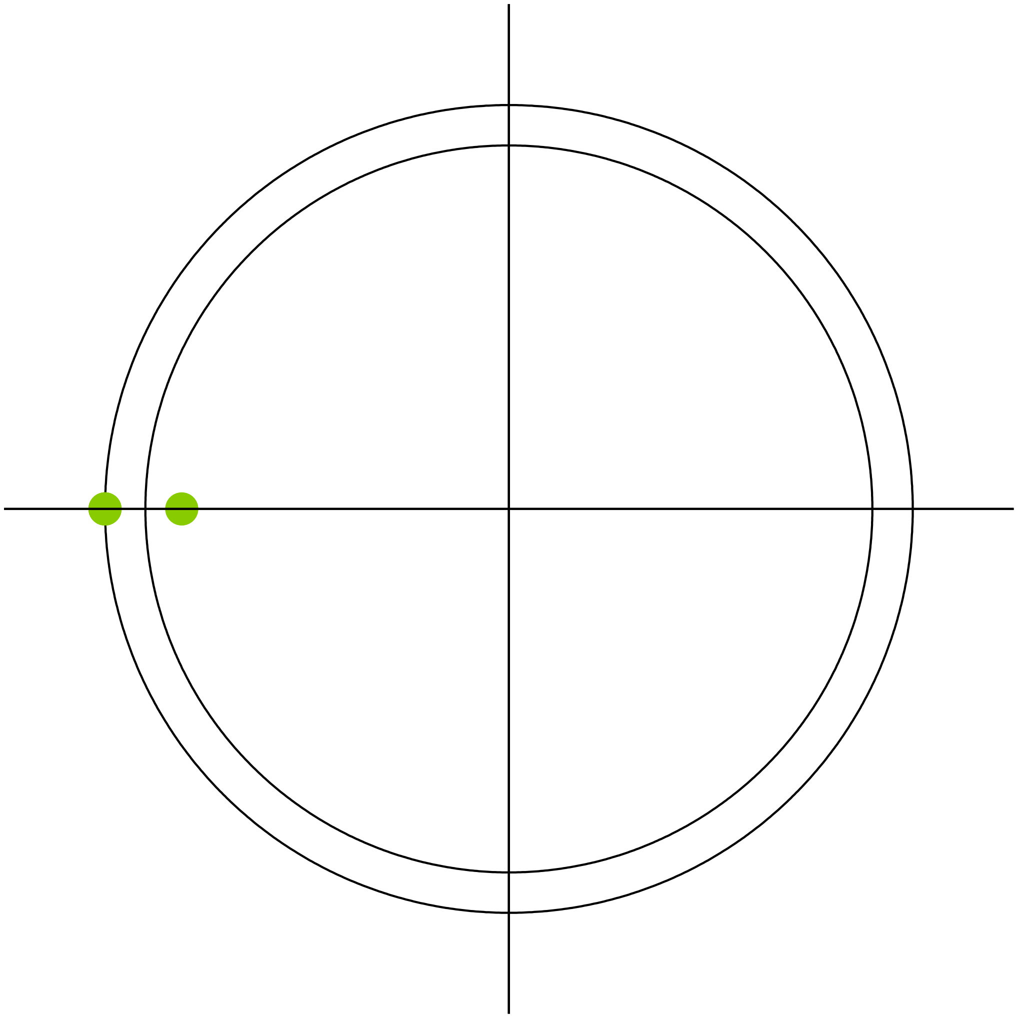
# Half Integer Resonance Crossing With Damping



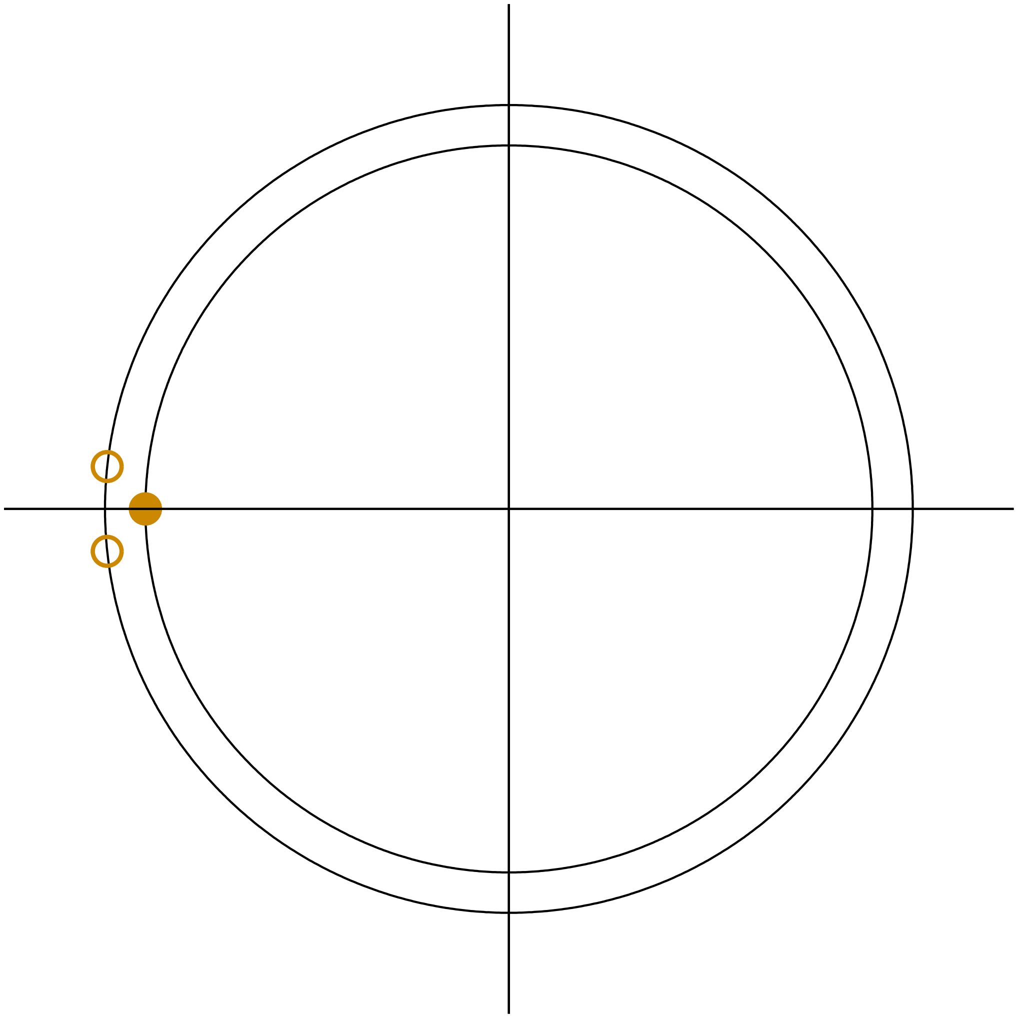
# Half Integer Resonance Crossing With Damping



# Half Integer Resonance Crossing With Damping

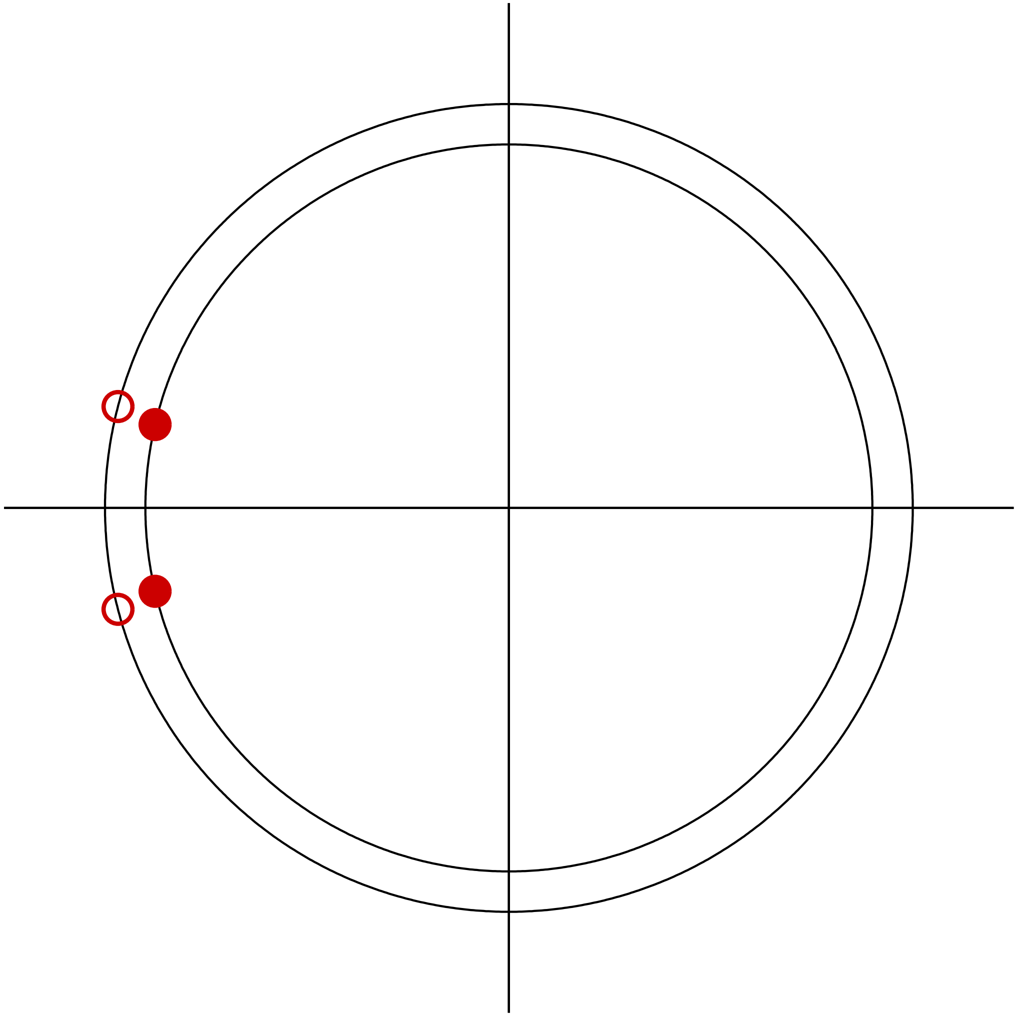


# Half Integer Resonance Crossing With Damping



BROOKHAVEN NATIONAL LABORATORY

# Half Integer Resonance Crossing With Damping



# PIC Speculations

- Has a finite bandwidth
- Center of band is most unstable point
- Extends between points where damped eigenvalues collide
- One direction always damps
- Other direction may damp or grow

# Perturbation Computation Basic Equations

- Only linear here

$$z(s) = M(s;0)z(0) + z_0(s)$$

- Homogeneous linear equation of motion

$$\frac{dM(s;0)}{ds} = A(s)M(s;0) \qquad M(0;0) = I$$

# Perturbation Expansion

- $A(s) = A_0(s) + \epsilon A_1(s)$, $M(s;0) = M_0(s;0) + \epsilon M_1(s;0)$
- $M_0(s;0)$ satisfies

$$\frac{dM_0(s;0)}{ds} = A_0(s)M_0(s;0) \qquad M_0(0;0) = I$$

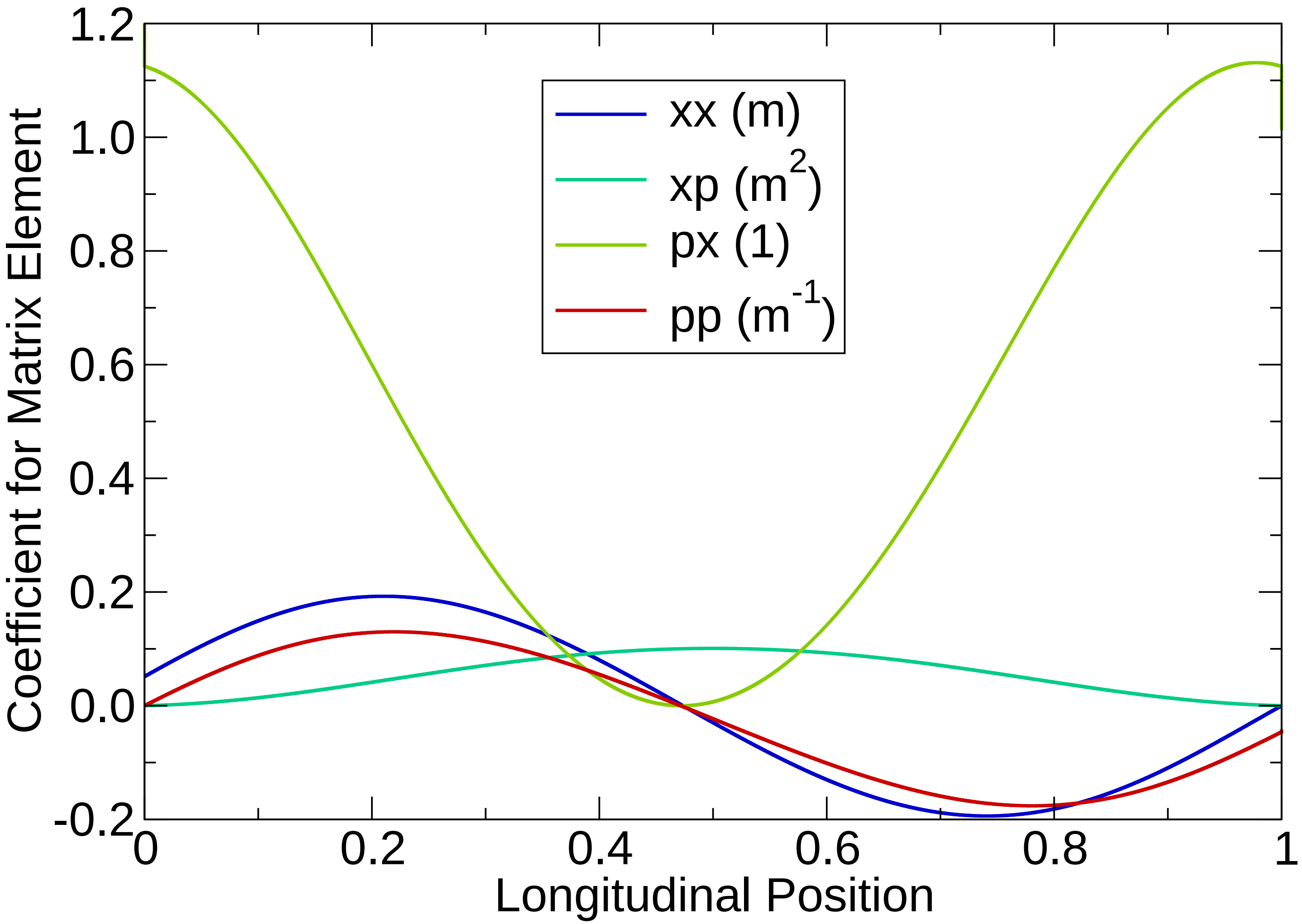
- Thus, solution for $M_1(s;0)$ is

$$M_1(s;0) = \int_0^s M_0(s;\bar{s})A_1(\bar{s})M_0(\bar{s};0)\,d\bar{s}$$

# Gives Everything, Works Everywhere

- Not just tunes: full transform
- Symplectic not necessary
- Lattice need not be stable

# Example: Focusing Perturbation



# Propagation of Covariance Matrix

- General rule for covariance matrix

$$\Sigma(s) = M(s;0)\Sigma(0)M^T(s;0)$$

- Perturbation: $\Sigma(s) = \Sigma_0(s) + \epsilon\Sigma_1(s)$, $\Sigma_1(0) = 0$

$$\Sigma_1(s) = M_0(s;0)\Sigma_0(0)M_1^T(s;0) + M_1(s;0)\Sigma_0(0)M_0^T(s;0)$$

- Integral from 0 to $s$, linear in $A_1(s)$