Methods for Addressing the Problem of Time of Flight Dependence on Transverse Amplitude in Linear Non-Scaling FFAGs

J. Scott Berg
Brookhaven National Laboratory
FFAG06, KURRI
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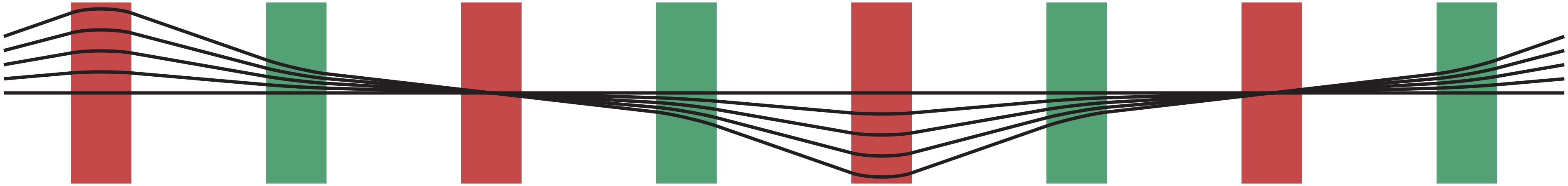
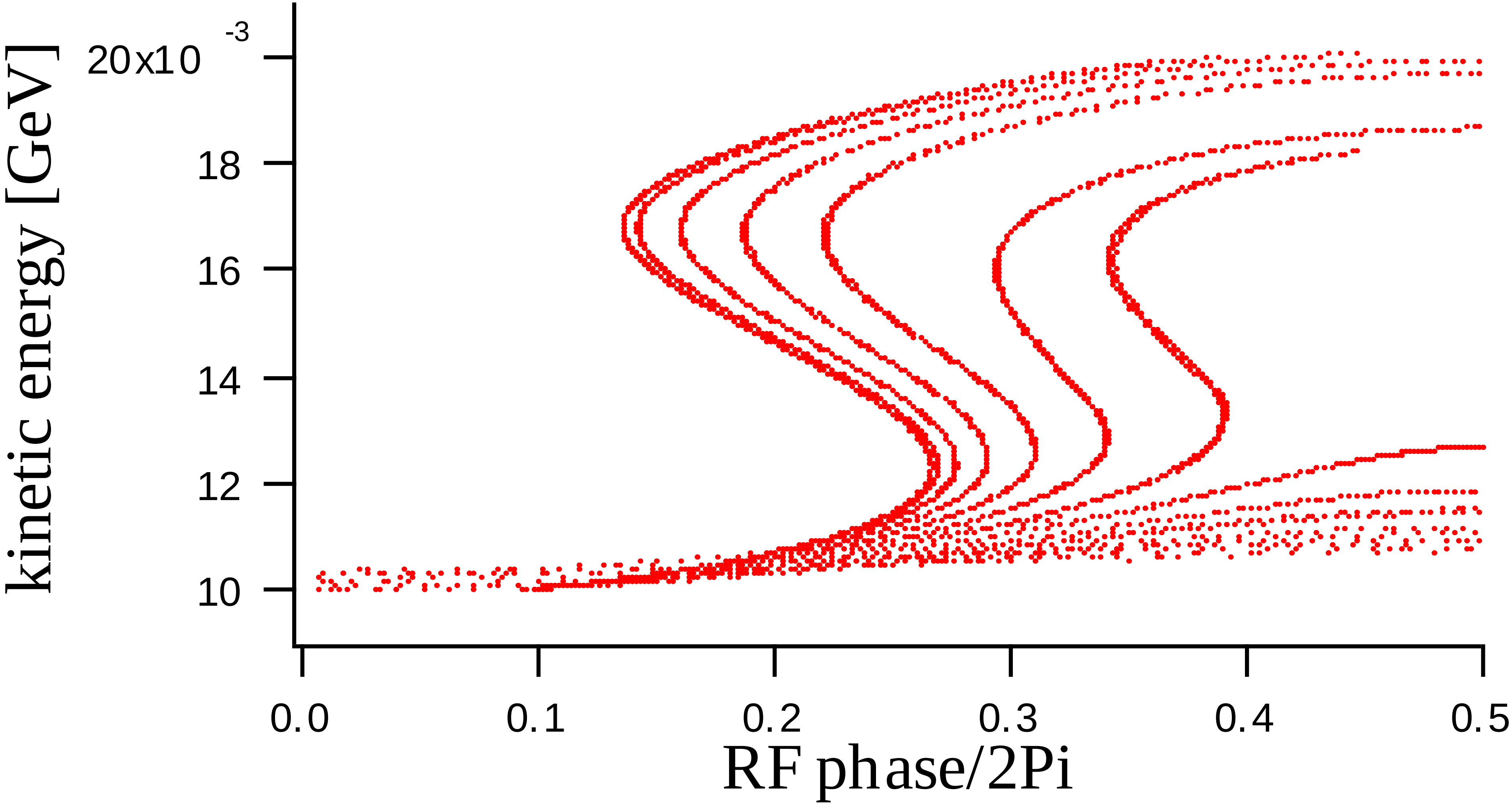
Description of the Problem

- Particles with large transverse amplitudes aren't accelerated
- Time of flight depends on transverse amplitude
- Reason: larger amplitudes, angles make longer path length



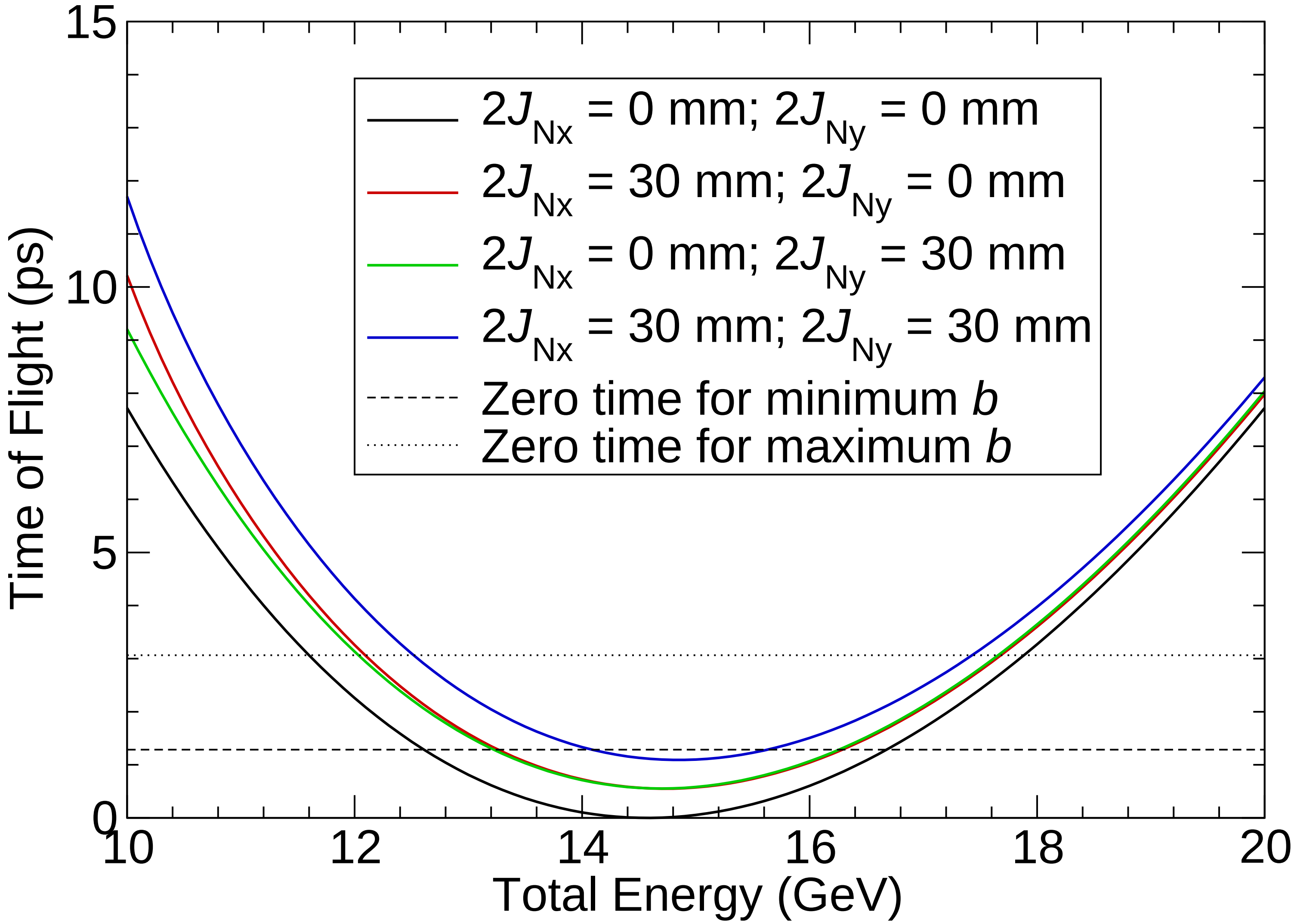
Acceleration of Particle Different Transverse Amplitudes (Machida)



Effect on Linear Non-Scaling FFAGs

- Different times of flight for different amplitudes create acceleration problems in FFAGs
- Small range of phase space that is accelerated at all amplitudes
- Low amplitude and high amplitude particles accelerated in different channels
- Particles start out together, end up spread out in time
- Different paths require different number of cells to complete: energy spread

Time of Flight Depends on Transverse Amplitude



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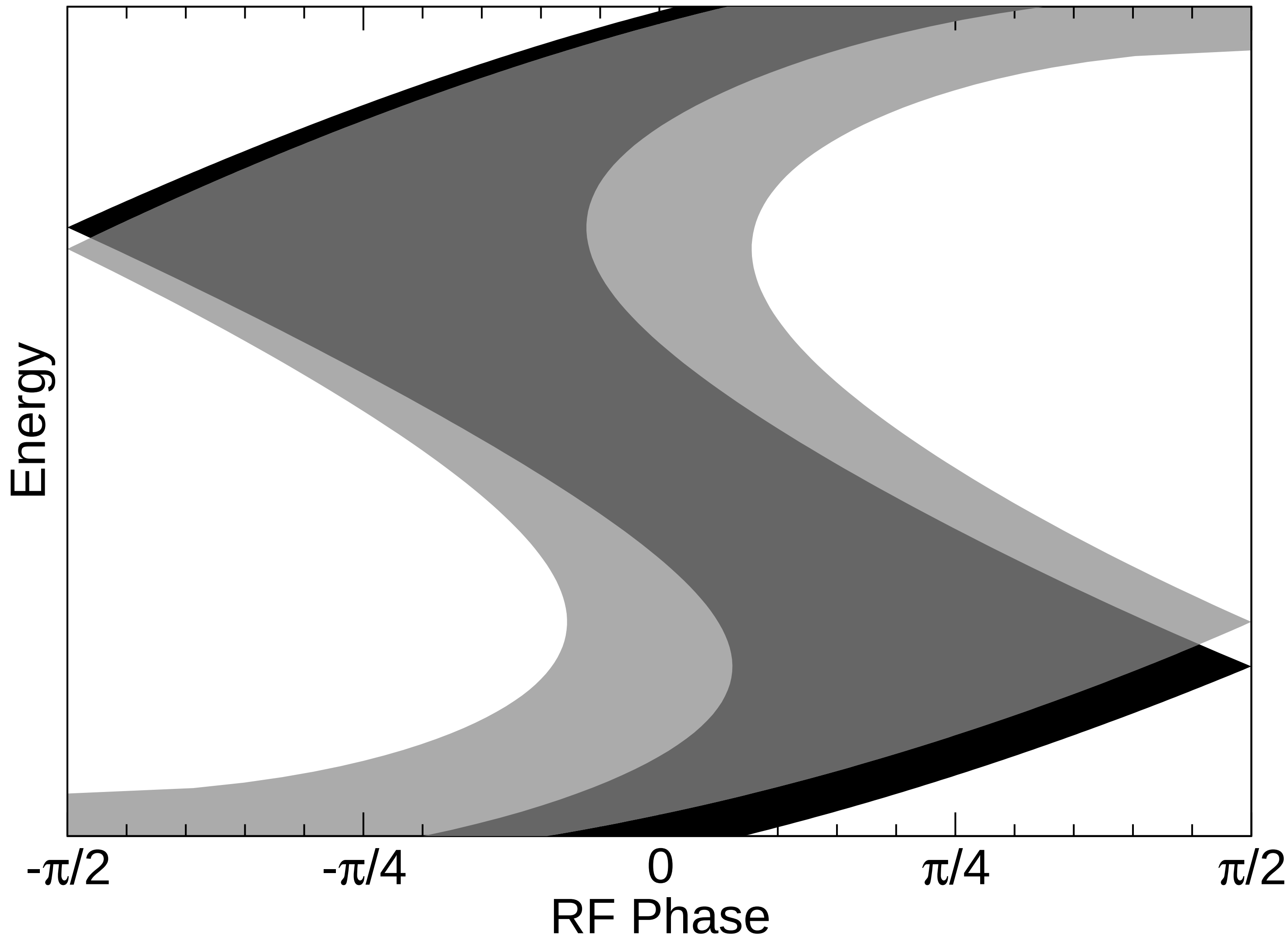
Acceleration Channels in FFAGs



Theory

- Time of flight dependence on amplitude related to chromaticity

$$\frac{d\bar{t}}{ds} = -\partial_E H_T - \frac{2\pi(\partial_E \boldsymbol{\nu}) \cdot \boldsymbol{J}_n}{L} + O(\boldsymbol{J}_n^{3/2}).$$

  - $\boldsymbol{J}_n$ is normalized transverse amplitude (in eV-s)
  - $\boldsymbol{\nu}$ is tune in cell of length L
- Time of flight difference at end for uniform acceleration

$$-2\pi\Delta\boldsymbol{\nu} \cdot \boldsymbol{J}_n/(\Delta E)$$

  - $\Delta\boldsymbol{\nu}$ is tune difference from beginning to end of acceleration, per cell
  - ΔE is energy gain per cell

Addressing the Problem

- Time of flight difference: $-2\pi\Delta\boldsymbol{\nu}\cdot\boldsymbol{J}_n/(\Delta E)$
- Reduce tune range during acceleration
  - Lower tune
    - Increases aperture
    - Increases time of flight range
  - Correct chromaticity
    - Nonlinearities reduce dynamic aperture
- Increase energy gain per cell
  - More cavities per cell
- A couple more mentioned later. . .

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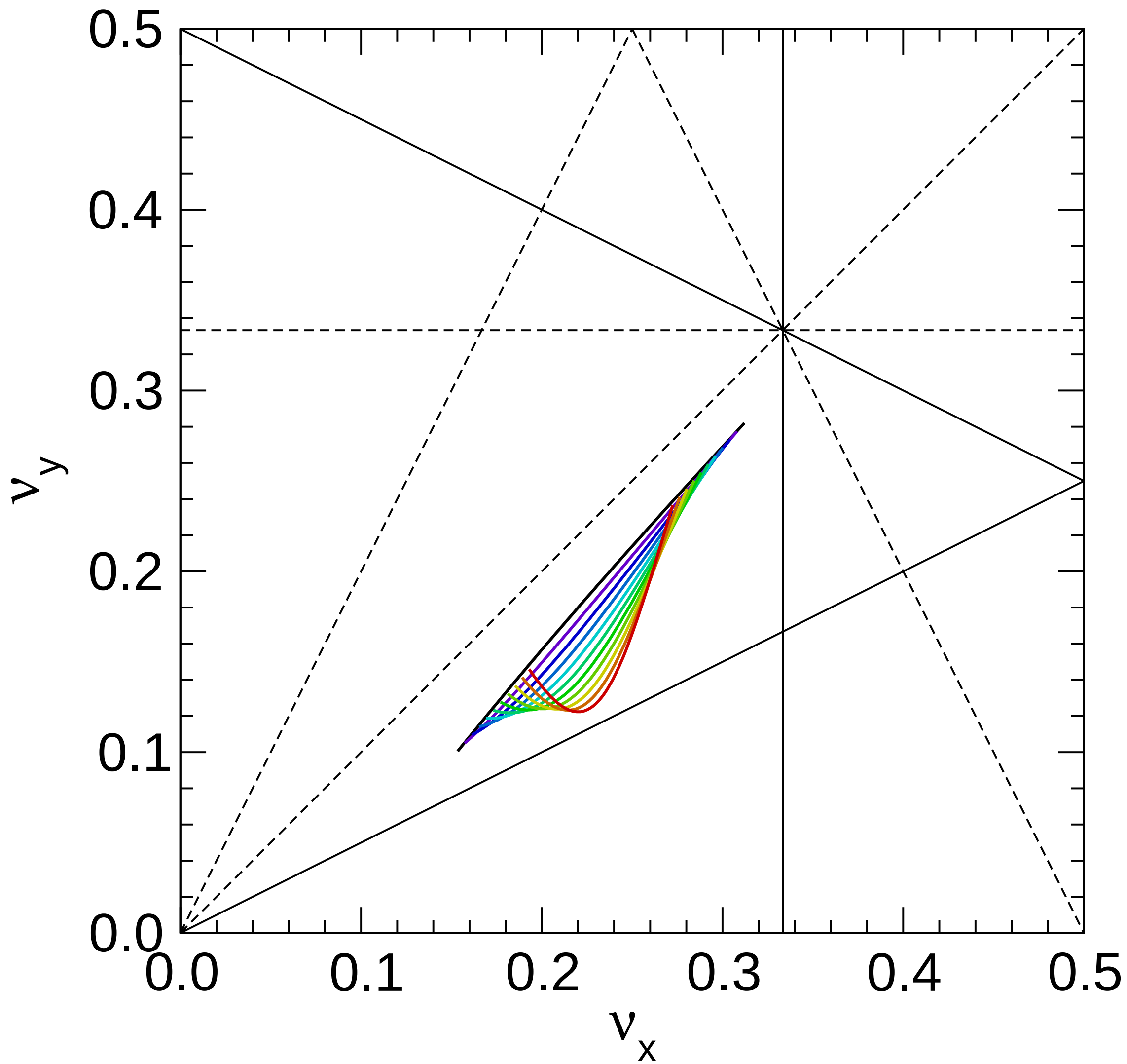
Chromaticity Correction Method

- Correct chromaticity with a sextupole component to magnets as follows
  - Construct a linear lattice where
    - Magnet lengths, drift lengths, and the number of cells are fixed
    - Time of flight is the same at low and high energy
    - The following three distances in the tune plane are equal
      - Low energy tune ($\boldsymbol{\nu}_{\mathrm{lo},0}$) to $3\nu_x = 1$ line
      - Low energy tune to $\nu_x - \nu_y = 0$ line
      - High energy tune ($\boldsymbol{\nu}_{\mathrm{hi},0}$) to $\nu_x - 2\nu_y = 0$ line

Chromaticity Correction Method

- Chromaticity correction procedure (cont.)
  - Add sextupole components, and modify dipole and gradient components so that
    - Magnet lengths, drift lengths, and the number of cells are fixed
    - Time of flight is the same at low and high energy
    - If x is the fraction of chromatic correction
      - $\boldsymbol{\nu}_{\mathrm{lo}} = (1 - x/2)\boldsymbol{\nu}_{\mathrm{lo},0} + (x/2)\boldsymbol{\nu}_{\mathrm{hi},0}$
      - $\boldsymbol{\nu}_{\mathrm{hi}} = (x/2)\boldsymbol{\nu}_{\mathrm{lo},0} + (1 - x/2)\boldsymbol{\nu}_{\mathrm{hi},0}$
- Choice of tune range to avoid third order resonances which sextupole will drive
- Plot shows to $x = 0.5$

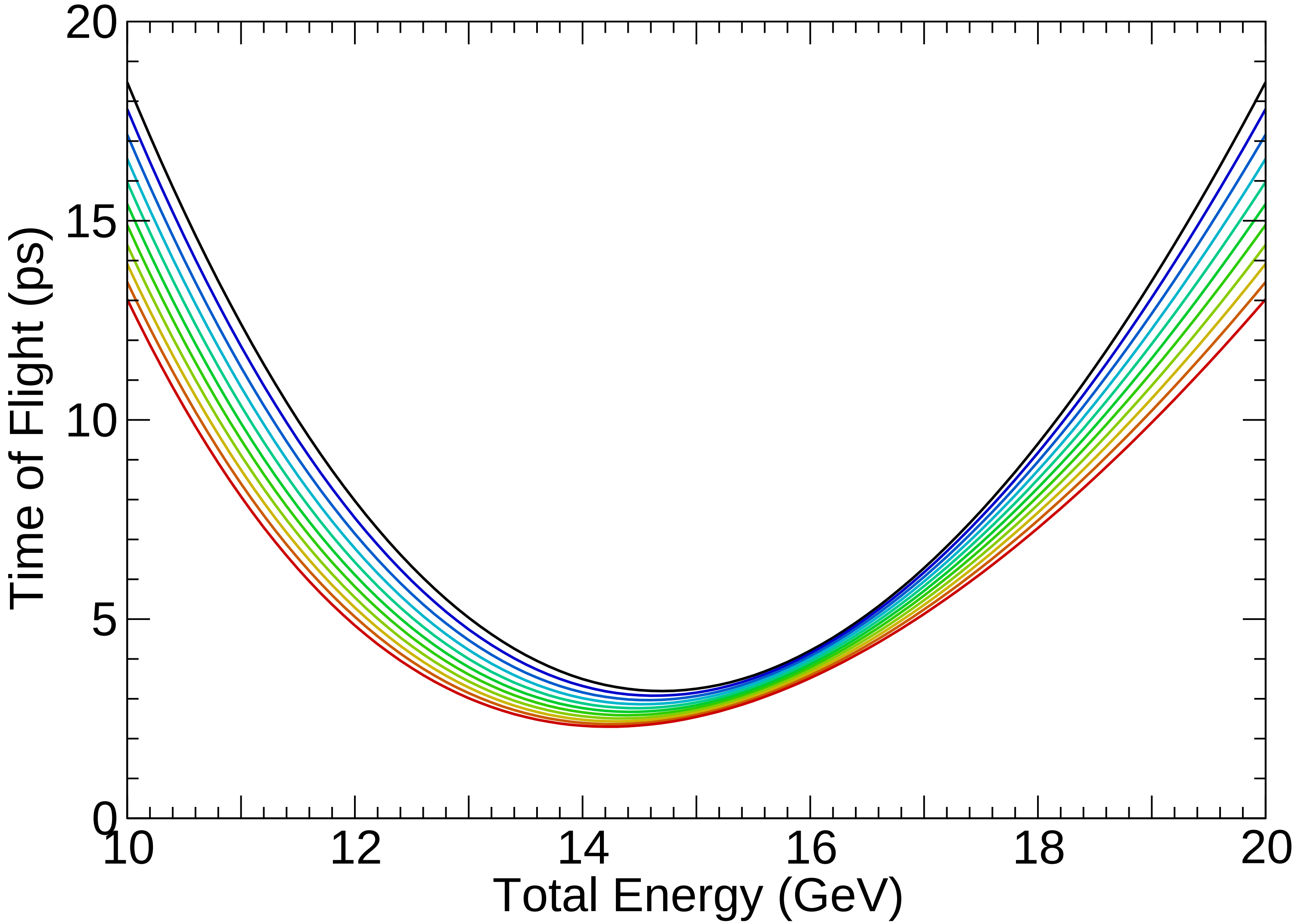
Tune Range with Chromaticity Correction



Observations

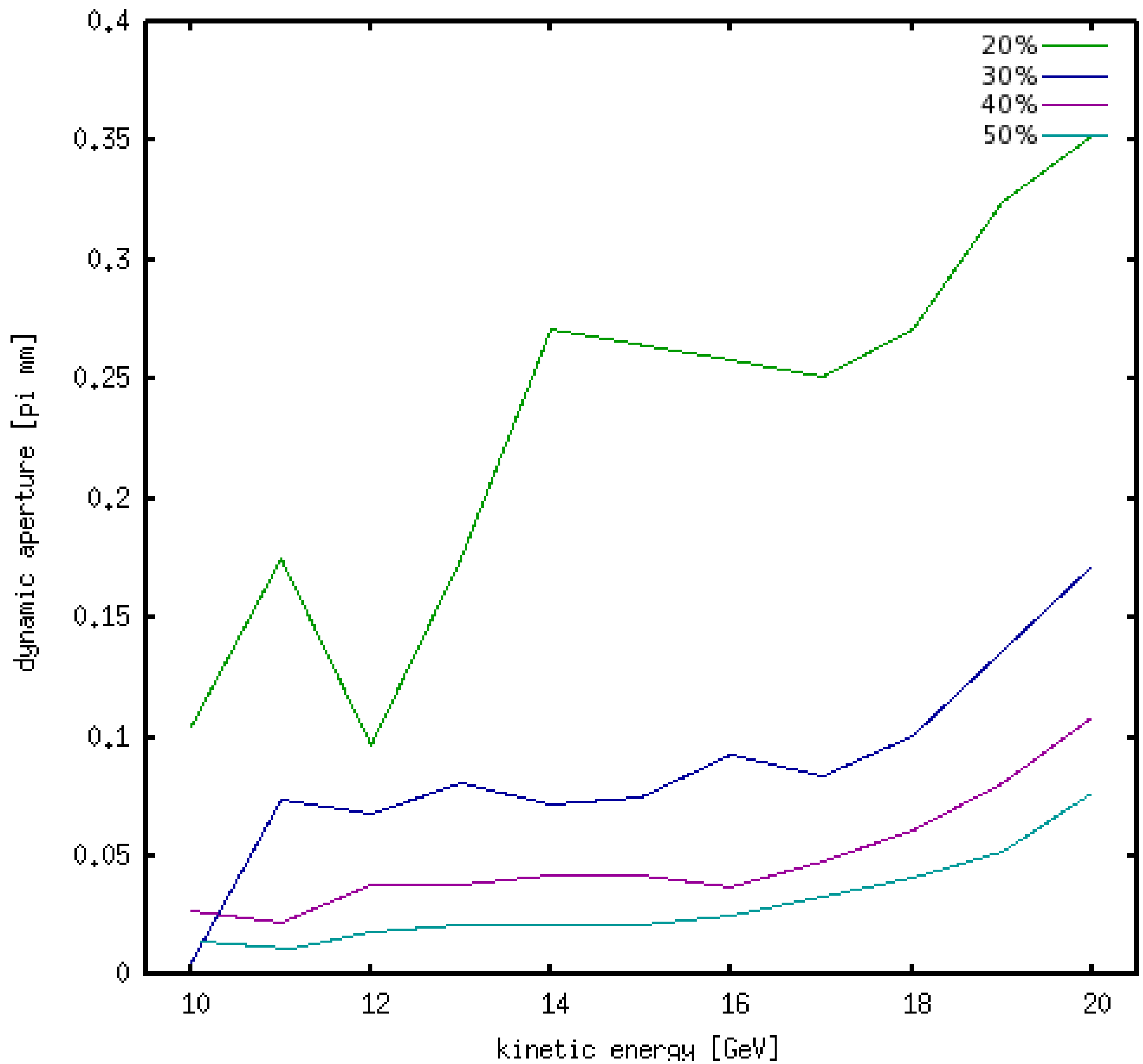
- Note chromaticity is locally higher!
- However, for uniform acceleration, what matters is the total change in tune
  - However, increased chromaticity may affect phase space locally!
- Time of flight range actually improves with more sextupole
- Must determine if dynamic aperture is sufficient
  - Losses likely on $4\nu_x = 1$ resonance
  - Should ascertain if we have decent dynamic aperture except for that

Time of Flight Variation with Chromaticity Correction



Dynamic Aperture (Machida)

- Dynamic aperture less for higher chromaticity
- Some dynamic aperture reduction on $4\nu_x = 1$ or $4\nu_y = 1$
- 20–30% may be tolerable



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Increasing Energy Gain per Cell

- Previous baseline had left many cells empty
  - Making the ring longer reduced its aperture and fields, reducing magnet costs
  - Filling every cell with cavities would be very expensive, and decay cost didn't justify this
- Now we want to increase average gradient as much as practical
  - Fill every cell with single-cell cavities
  - Instead, use two-cell cavities to get even more gradient
    - Requires longer drifts

More Energy Gain per Cell
Lattice Parameters

Energy (GeV)	6.25–12.5			12.5–25		
Method	Empty	1/Cell	2/Cell	Empty	1/Cell	2/Cell
Cells	69	61	50	93	78	63
Cavities	48	55	44	58	72	57
Turns	10.8	9.3	5.8	18.2	14.6	9.2
Cost	80.7	82.3	116.8	95.0	98.7	140.2
Avg. Gradient (MV/m)	1.9	2.4	3.3	1.6	2.1	3.1

- Cost reduced to account for fewer decays
- Filling every cell with cavities gives substantial increase in average gradient for very little cost
- Two cavities per cell gives even more gradient, but at a substantial increase in cost
- FODO with 2 single cell cavities slightly better than 2/cell doublet

Other Techniques
Higher Harmonic Cavities

- Don’t attack the time of flight problem directly
- Instead reduce some of its ill effects
  - Increase region of phase space successfully accelerated
  - Reduce energy spread
  - Reduces ellipse distortion
- Higher harmonic cavities have less stored energy than main cavities
  - Could be problem when there are many turns
  - Lower fill times may allow use of NC cavities in multiple-train schemes
- Will reduce average gradient from main cavities

Other Techniques

- Don't attempt large longitudinal phase space transmission at high transverse amplitudes (ellipsoidal distribution)
- Choose initial initial conditions approproately (work in progress)

The Overall Plan

- Add some chromaticity, as much as dynamic aperture will tolerate.
- Fill every cell with cavities. Probably use two cells per cavity.
- Study higher harmonic RF
  - Seems to help substantially
  - May have practical concerns
- Make the best use of what we have