Fixed Field Alternating Gradient Accelerators (FFAGs)

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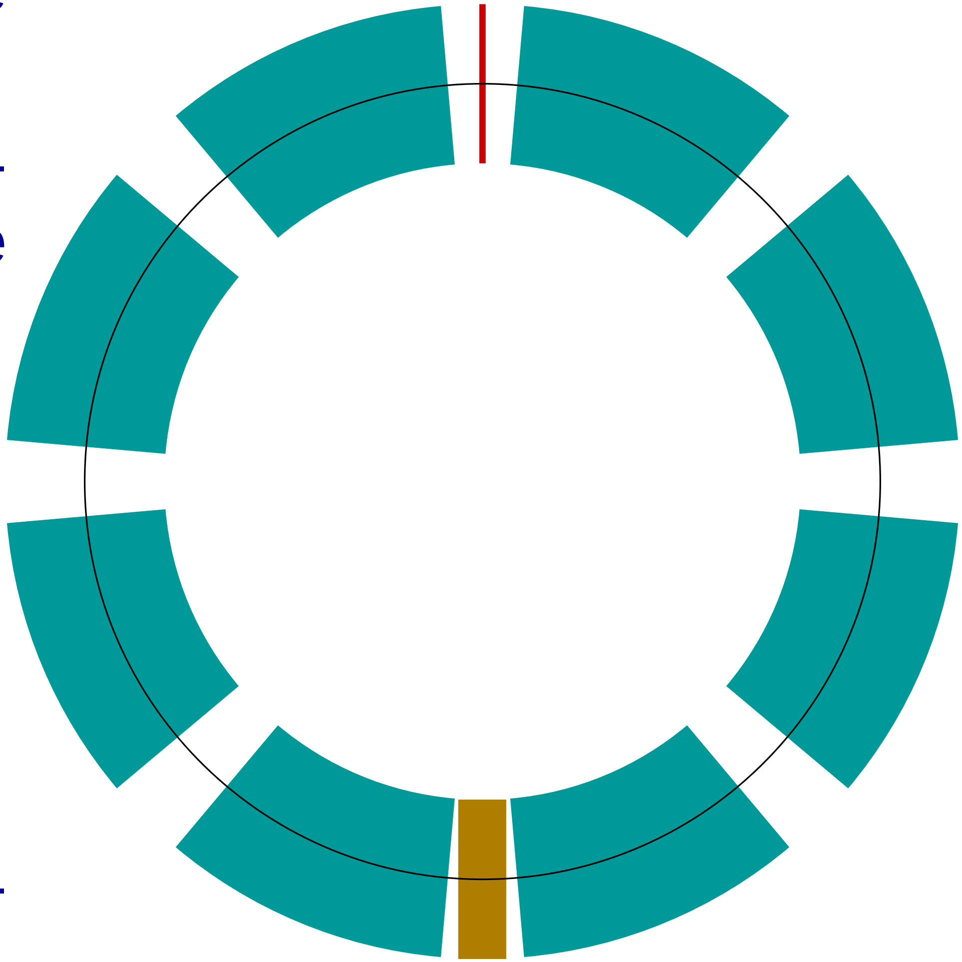
Outline

- Why do we want to make FFAGs?
- Tune and resonance: how they drive FFAG design.
- Time of flight: how it impacts FFAG design.

Accelerator Basics

- Guide particles around a more or less circular path
- Ask what the particle distribution looks like in a given plane in space
  - Position in plane
  - Momentum in the plane
  - Energy
  - Time
- Particles guided by magnets
- Electric fields (time varying) increase particles' energy



FFAG: Fixed Field
The Synchrotron

- You want to increase the energy of some charged particles
  - Ignore how the increase occurs for now
- Lorentz force equation, magnetic fields only

$$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

- Rewrite as (s is arc length along trajectory)

$$\frac{d}{ds}\left(\frac{\boldsymbol{p}}{|\boldsymbol{p}|}\right) = \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \times \left(\frac{q\boldsymbol{B}}{|\boldsymbol{p}|}\right)$$

- If $\boldsymbol{B}$ is raised in proportion to the magnitude of the momentum $|\boldsymbol{p}|$, orbit follows the same path in space regardless of energy
  - Time along trajectory varies with velocity magnitude
- Synchrotron: increase $\boldsymbol{B}$ in proportion to $|\boldsymbol{p}|$ as you accelerate

FFAG: Fixed Field

The Synchrotron: Ramping Rate

- Changing magnetic fields induce currents in the coils and other conductors (beam pipe)
  - These currents produce fields that misdirect the beam
  - Currents larger if rate of main field change higher
- Magnets have substantial stored energy
  - Power supply must not only supply steady-state current, must additionally supply increase in stored energy
- Limits number of acceleration cycles per second
  - Practical maximum around 50 Hz
  - If you want high fields with superconducting magnets, forced to minutes or hours per acceleration cycle

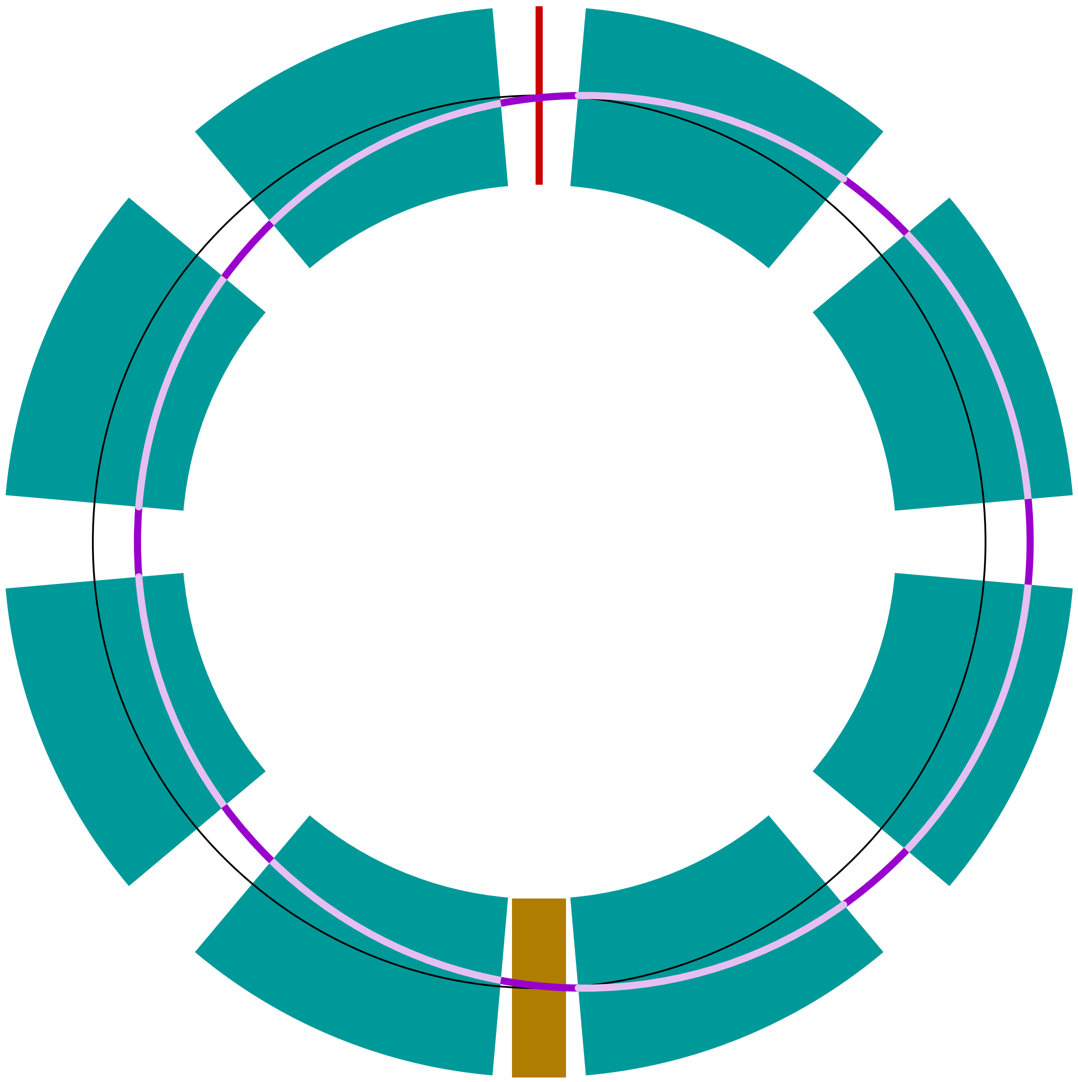
FFAG: Fixed Field

- If you want a higher repetition rate, you must keep the magnetic fields fixed
- Trajectories no longer independent of energy
  - Particles at different energies follow different paths
  - Dynamics (stability!) depend on energy (more on this later)
  - Larger magnet apertures required: cost
- Cost savings occur
  - No longer supply power to rapidly increase stored energy
  - Superconducting magnets possible for high repetition rate machines
- FFAGs can be better than synchrotrons when you need to accelerate particles rapidly

FFAG: Alternating Gradient
Weak Focusing

- Start with a circular orbit in a constant magnetic field
- Consider a particle next to that orbit
  - It follows a circular orbit of the same radius
  - The particle oscillates about the original orbit
  - This is called “weak focusing”
- Orbit radius is proportional to the momentum magnitude
- Vertical orbit stability requires reducing the field with radius
  - Orbits at different energies spread even further apart
- Large apertures required for large energy range
- This is the focusing method for many cyclotrons (which are fixed field machines)

FFAG: Alternating Gradient
Weak Focusing



FFAG: Alternating Gradient
Add a Field Gradient

- If field increases with radius, orbits will be more tightly packed together
- Example: cylindrical coordinates, field proportional to r^k, but uniform in θ

$$B_z = B_0 \left(\frac{r}{r_0}\right)^k \qquad r = r_0 \left(\frac{p}{p_0}\right)^{\frac{1}{k+1}} \qquad \Delta r \approx \frac{1}{k+1} r_0 \frac{\Delta p}{p_0}$$

- Larger field gradient, orbits more tightly packed together

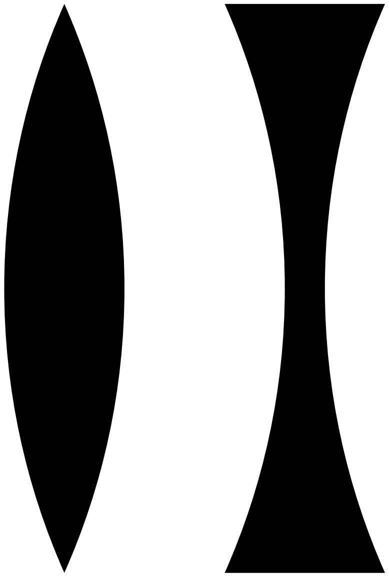
FFAG: Alternating Gradient Vertical Stability

- The catch: Maxwell's equations

$$\frac{dB_z}{dr} = \frac{dB_r}{dz}$$

- If B_z increases with r to pack orbits more compactly
  - B_r increases with z
  - Particle displaced vertically up feels an upward force
  - Motion is unstable
- To get vertical stability, need $dB_r/dz < 0$
  - For a small amount, orbits are spread further apart
  - For a larger amount, horizontal motion is unstable
  - I'm lying a little bit (spiral angle), but the general trends of the argument are correct. . .

FFAG: Alternating Gradient
Thin Lens Example



- Two lenses, one is focusing, the other is defocusing
- Separated by L, focal lengths f and $-f$
- Combination has focal length f^2/L
- Focal length positive: combination focuses

FFAG: Alternating Gradient
Alternating Gradient Focusing

- Same trick with accelerator magnets
  - Alternate magnets with $dB_z/dr > 0$ and $dB_z/dr < 0$
  - Combination gives orbit stability in both planes
  - Also keeps orbits with different energies packed closely
- FFAGs achieve smaller apertures then cyclotrons by using alternating gradient focusing

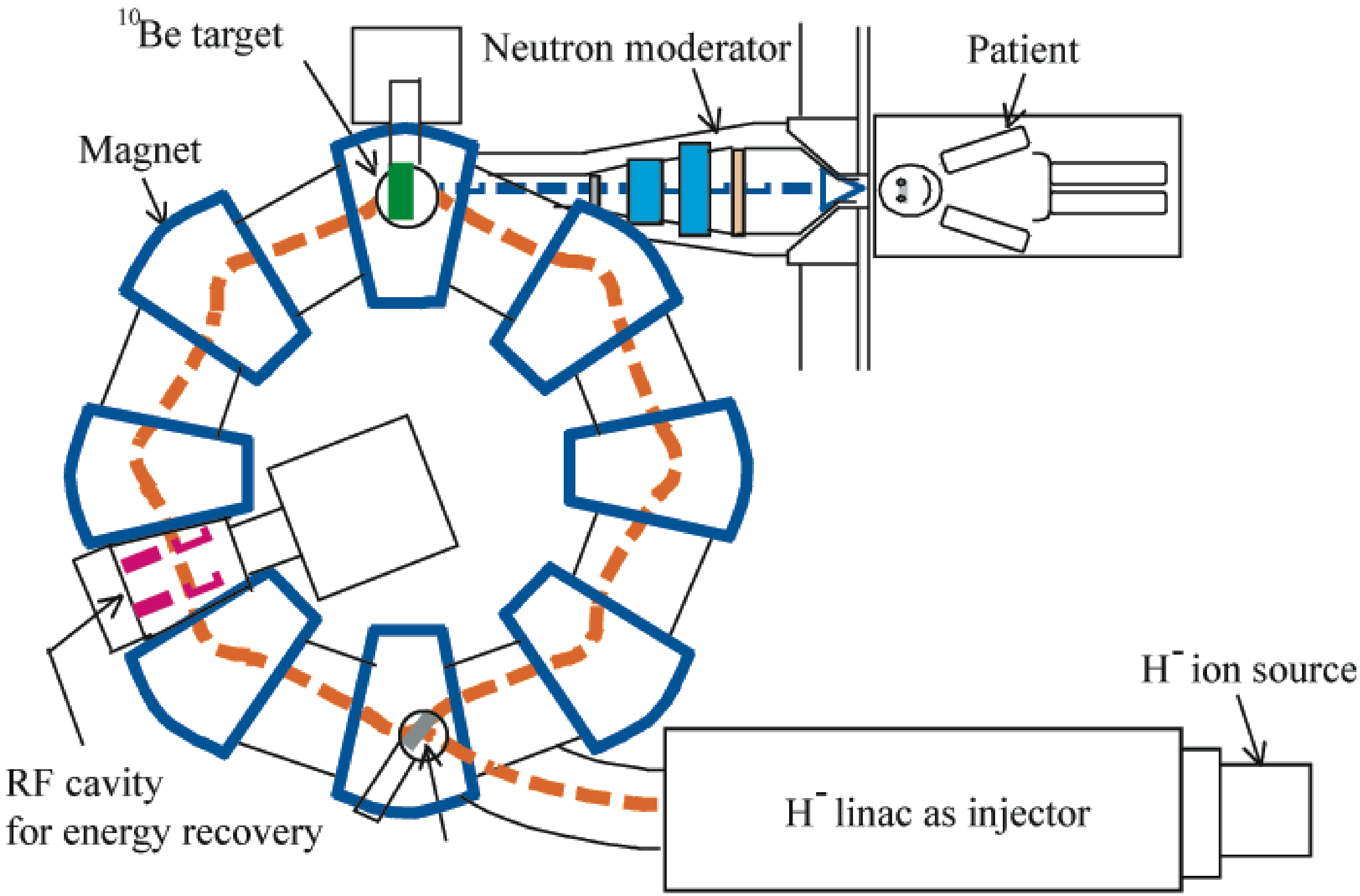
Summary: Motivations for FFAG Use

- FFAGs are useful and even necessary when you need rapid acceleration
- FFAGs allow one to have a wide range of energies in a machine yet (relatively) small magnet apertures (when compared to a weak focusing machine)
- There are more which we'll get to a bit later. . .

FFAG Applications

- Applications that require rapid acceleration
- High power beams hitting targets want high repetition rate
  - Need high average beam power
  - Target can only handle so much energy per pulse
  - Space charge (forces of beam particles on each other) limits current one can accelerate
- Medical applications
- Muon machines (decay!)
- Machines requiring large energy spread but no acceleration
  - PRISM (captures large energy spread muon bunch)
  - Boron neutron capture therapy: production target increases beam energy spread

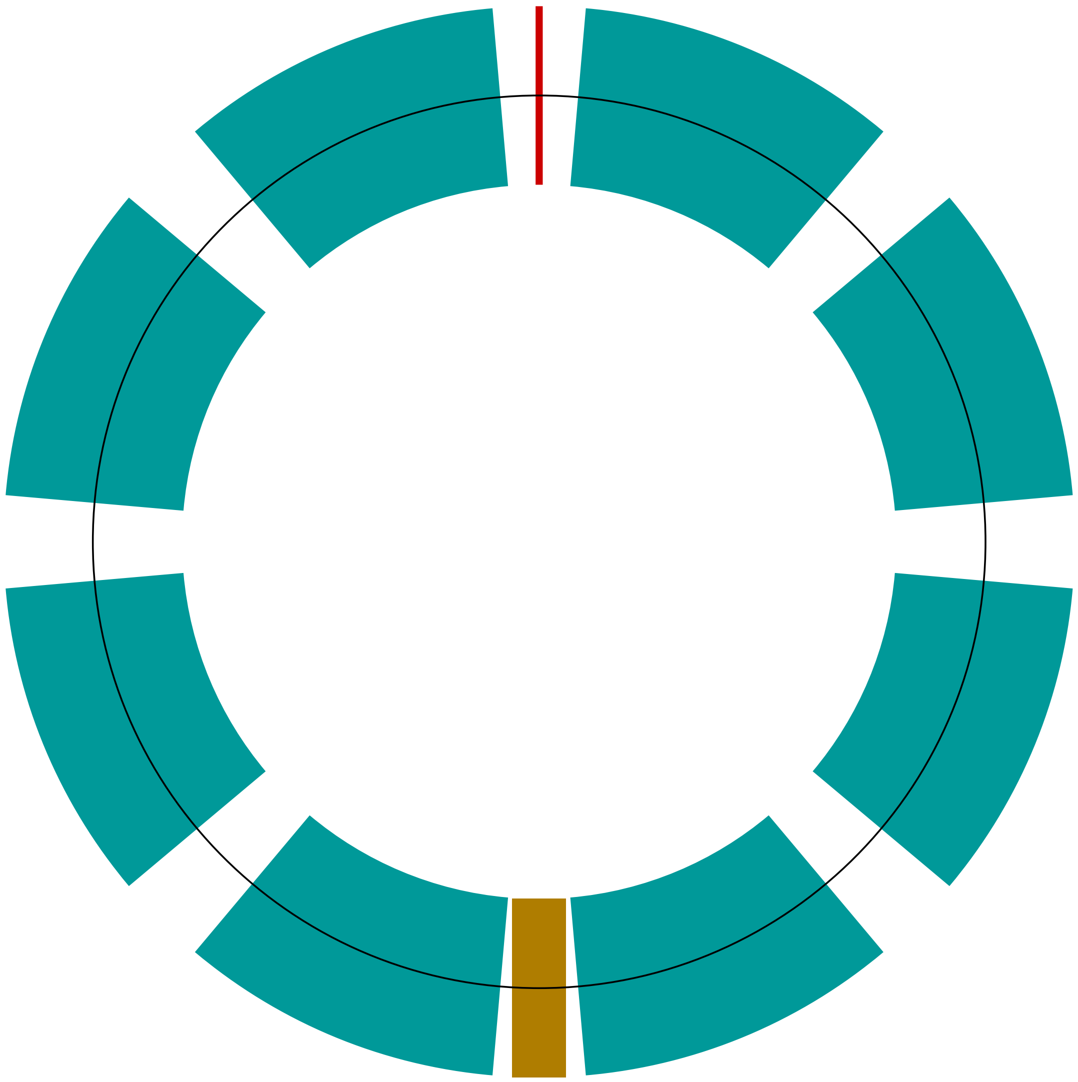
FFAG Applications
Boron Neutrino Capture Therapy



Accelerator Concepts
Closed Orbit

- For a given particle energy
- Particle starts in the plane (Ponicaré Surface) described before
- Arrives back at the same plane one turn later
- Find a particle trajectory which returns to the plane with the same
  - Position
  - Transverse momentum
- This is called the closed orbit
- If no such orbit exists, the accelerator probably won't work
- There is an amount of time that it takes a particle following the closed orbit to come back to the plane
  - Analyzed later

Accelerator Concepts
Poincaré Surface



Small Deviations from Closed Orbit

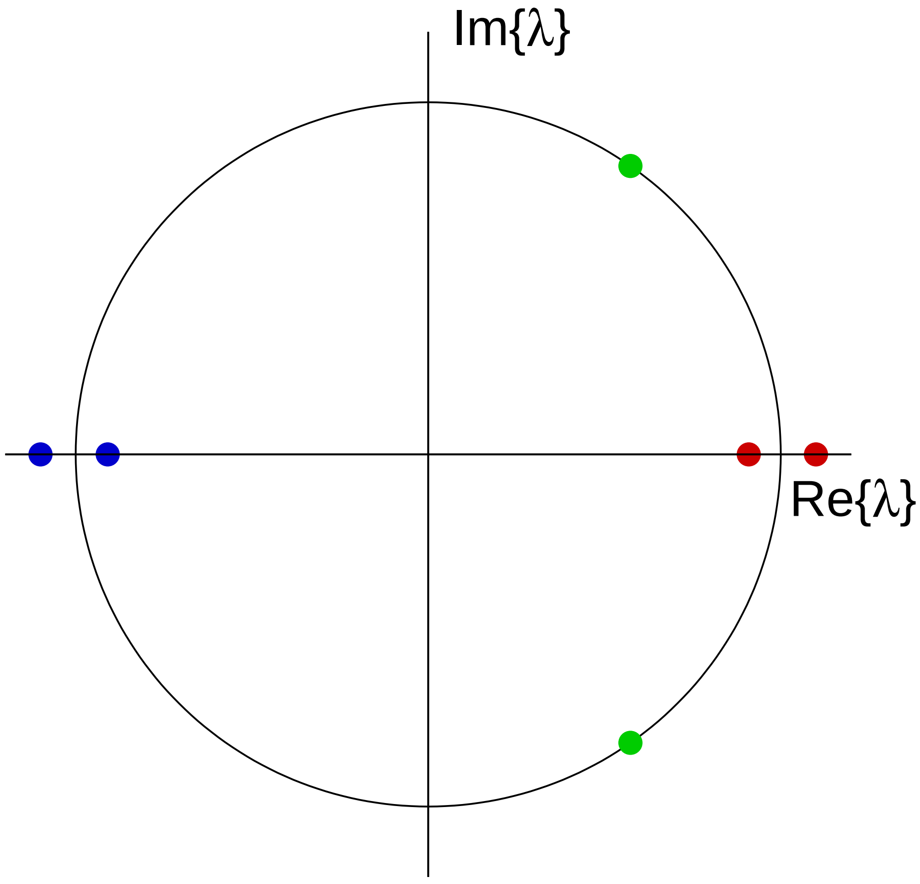
- Analyze small deviations about the closed orbit
- Deviations at plane in terms of deviations at plane one turn earlier
- Can be linearized

$$\boldsymbol{z}_1 = M\boldsymbol{z}_0$$

  - $\boldsymbol{z}_0$ vector of phase space variables at plane
  - $\boldsymbol{z}_1$ phase space variables one turn later
  - M linear transfer matrix

Accelerator Concepts
Eigenvalues

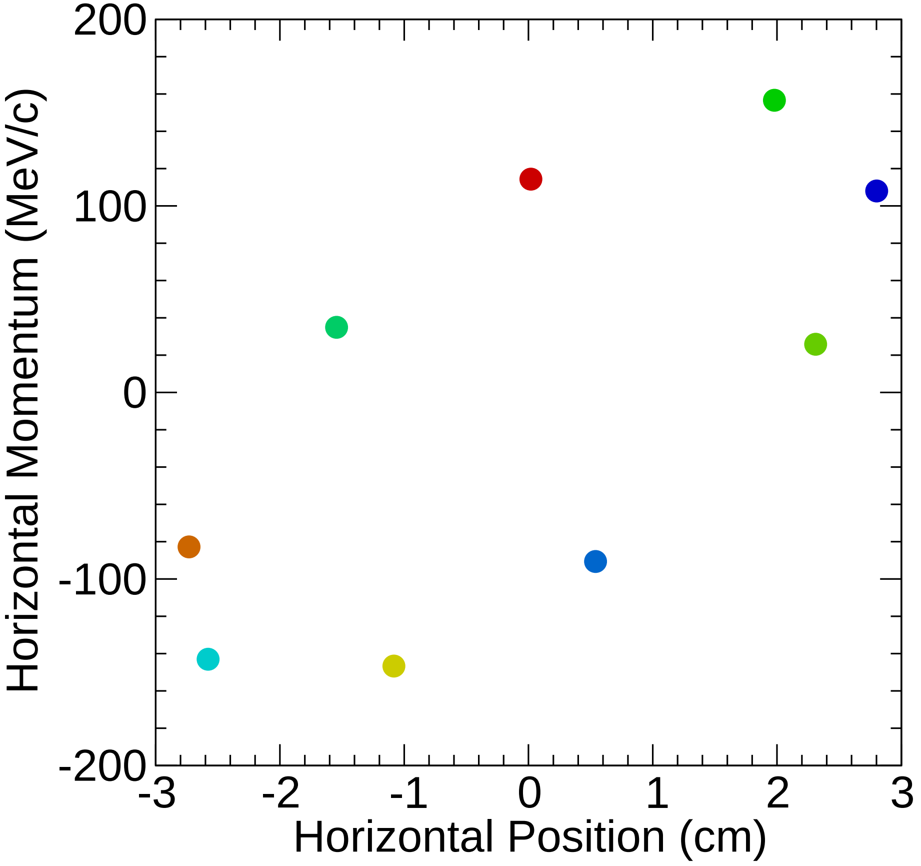
- Matrix M has eigenvalues
  - If λ is an eigenvalue, so is λ^{-1}
  - If λ is an eigenvalue, so is λ^*
- Two-dimensional phase space
  - Two possible types of solution
    - ⋆ λ real, λ and λ^{-1}
      - ➢ Either $|\lambda| > 1$ or $|\lambda^{-1}| > 1$
      - ➢ Motion is unstable
    - ⋆ $\lambda = e^{-i\mu}$
      - ➢ $|\lambda| = 1$
      - ➢ Motion is stable
  - $\mathrm{Tr}\, M$ characterizes stability
    - ⋆ $|\mathrm{Tr}\, M| \leqslant 2$ for stable motion

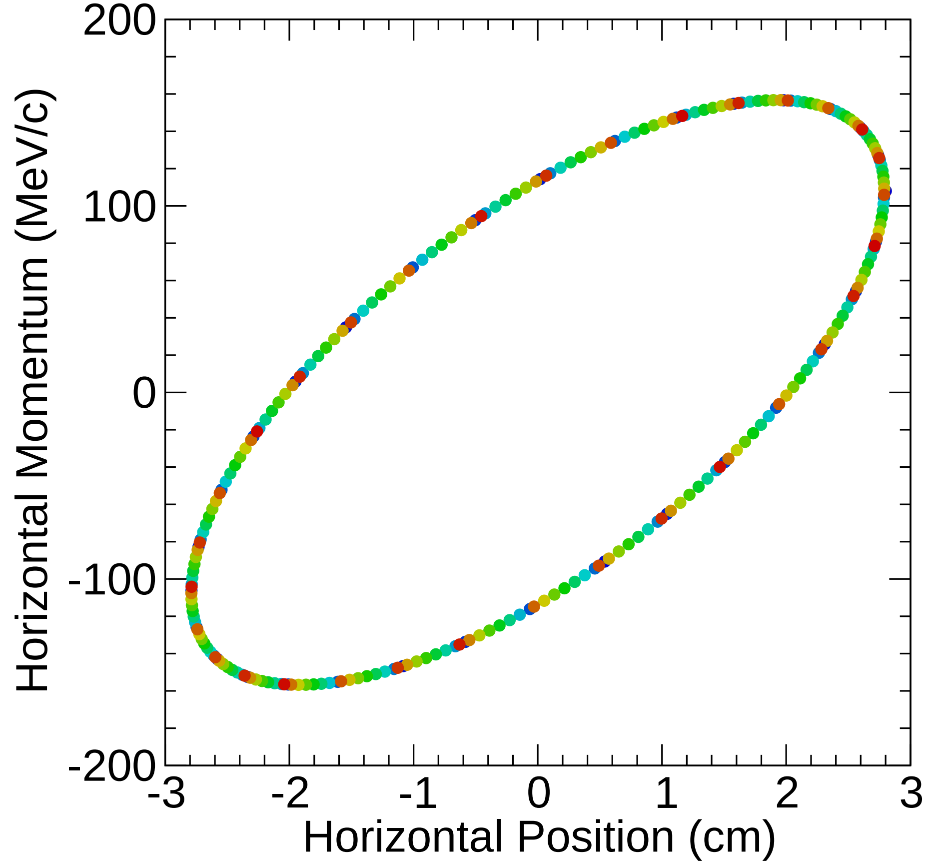


Accelerator Concepts
Stable Motion

- Near the closed orbit, stable motion means particles move on ellipses in "phase space"
  - One dimensional motion: coordinate and its conjugate momentum
- Transform to "normalized variables" so that motion is on a circle
  - Useful way to view phase space
  - Define tune to be $\mu/2\pi$, where μ is the angle a particle move on the circle for one turn
  - Eigenvalue is $e^{\pm i\mu}$, $\mathrm{Tr}\, M = 2\cos\mu$
- Tunes indicate stability
  - Near 0.5 or 0 potentially leads to instability
  - Trace of matrix approaching -2 and 2 respectively

Accelerator Concepts
Phase Space Elliptical Motion





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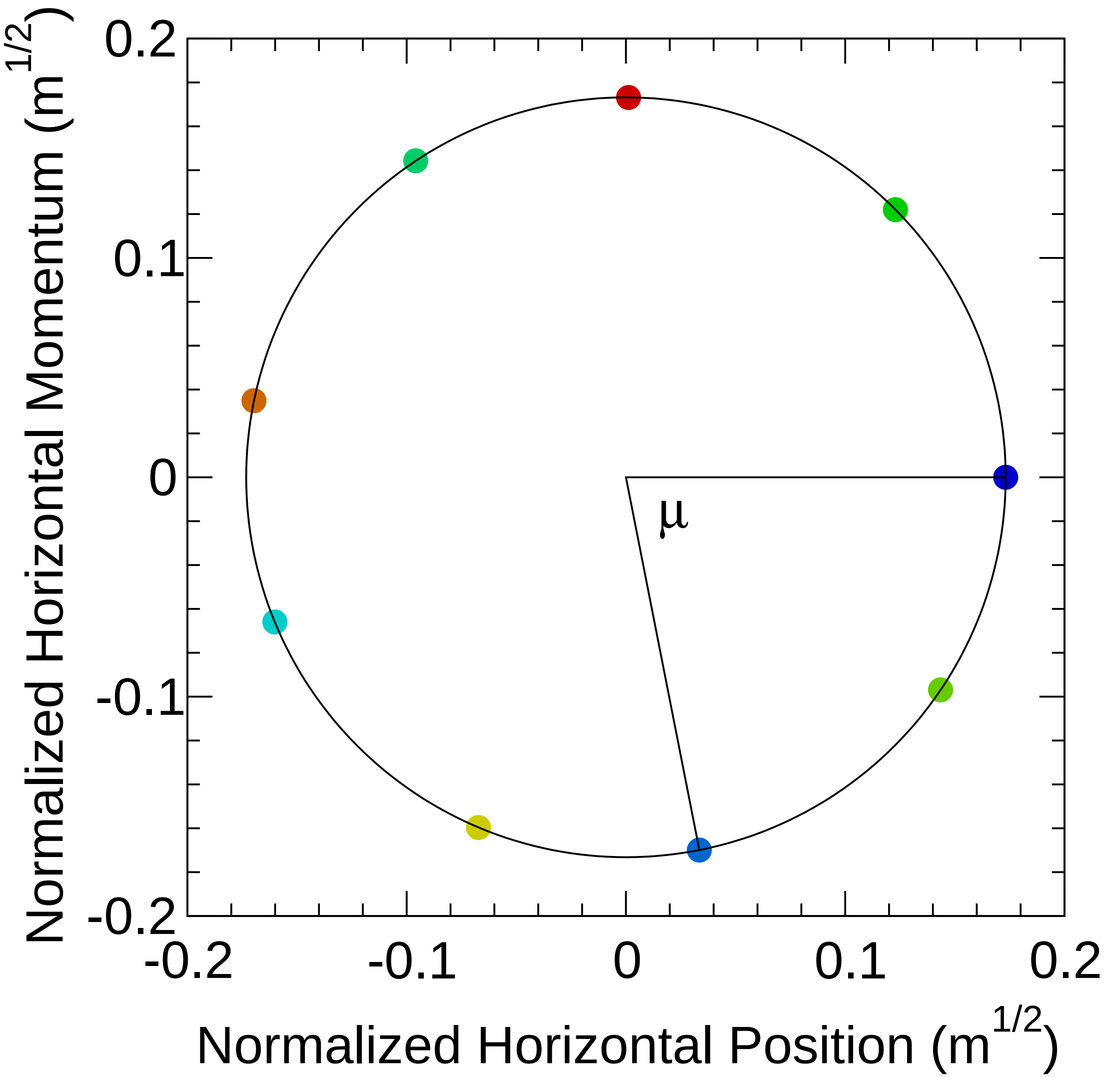
Accelerator Concepts
Normalized Phase Space



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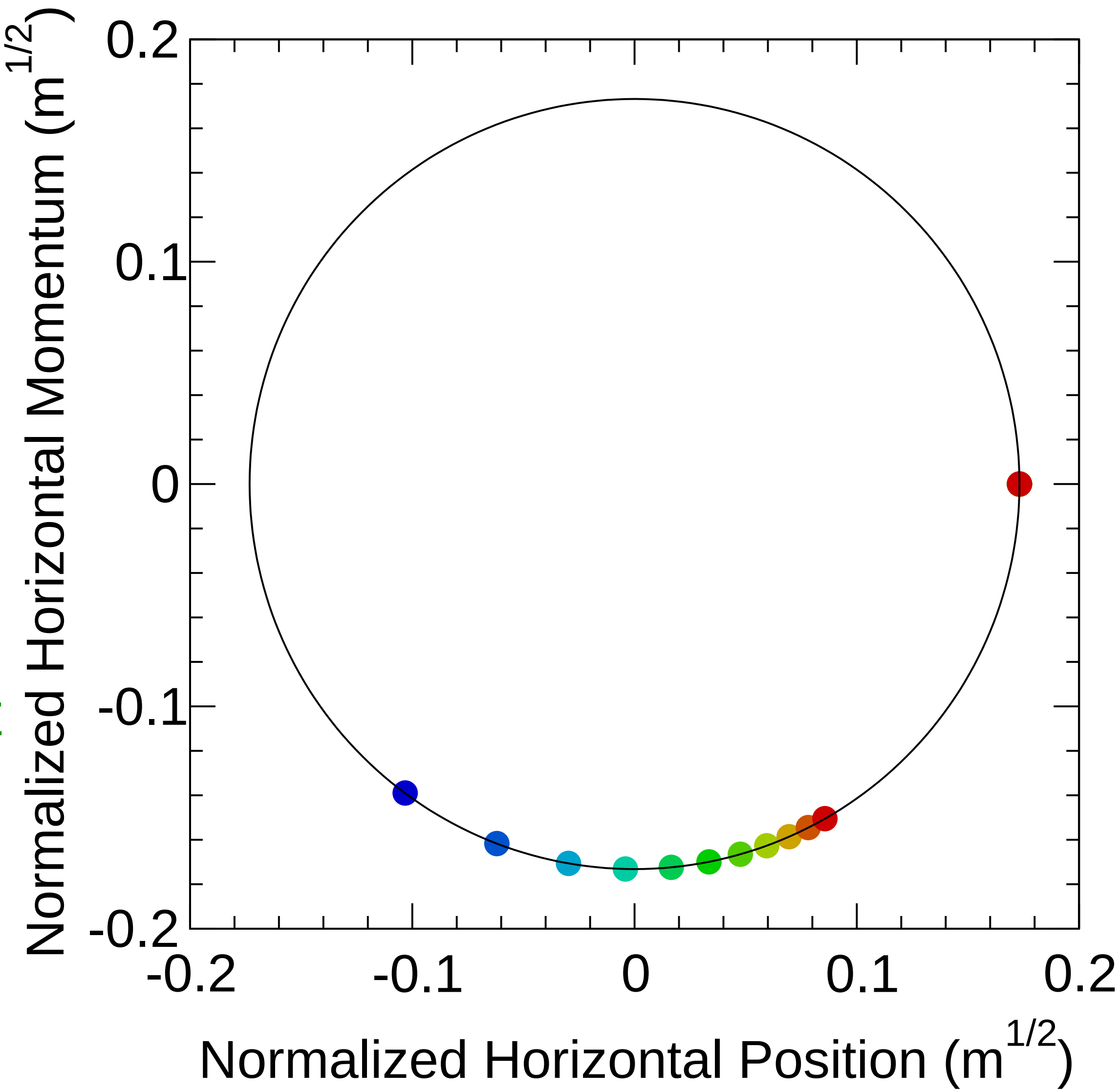
Accelerator Concepts

Tune Variation with Energy: Chromaticity

- The tune depends on the energy
- Reason: angular deviations by magnet inversely proportional to momentum
- Plot
  - Each color a different energy
  - Each energy normalized separately



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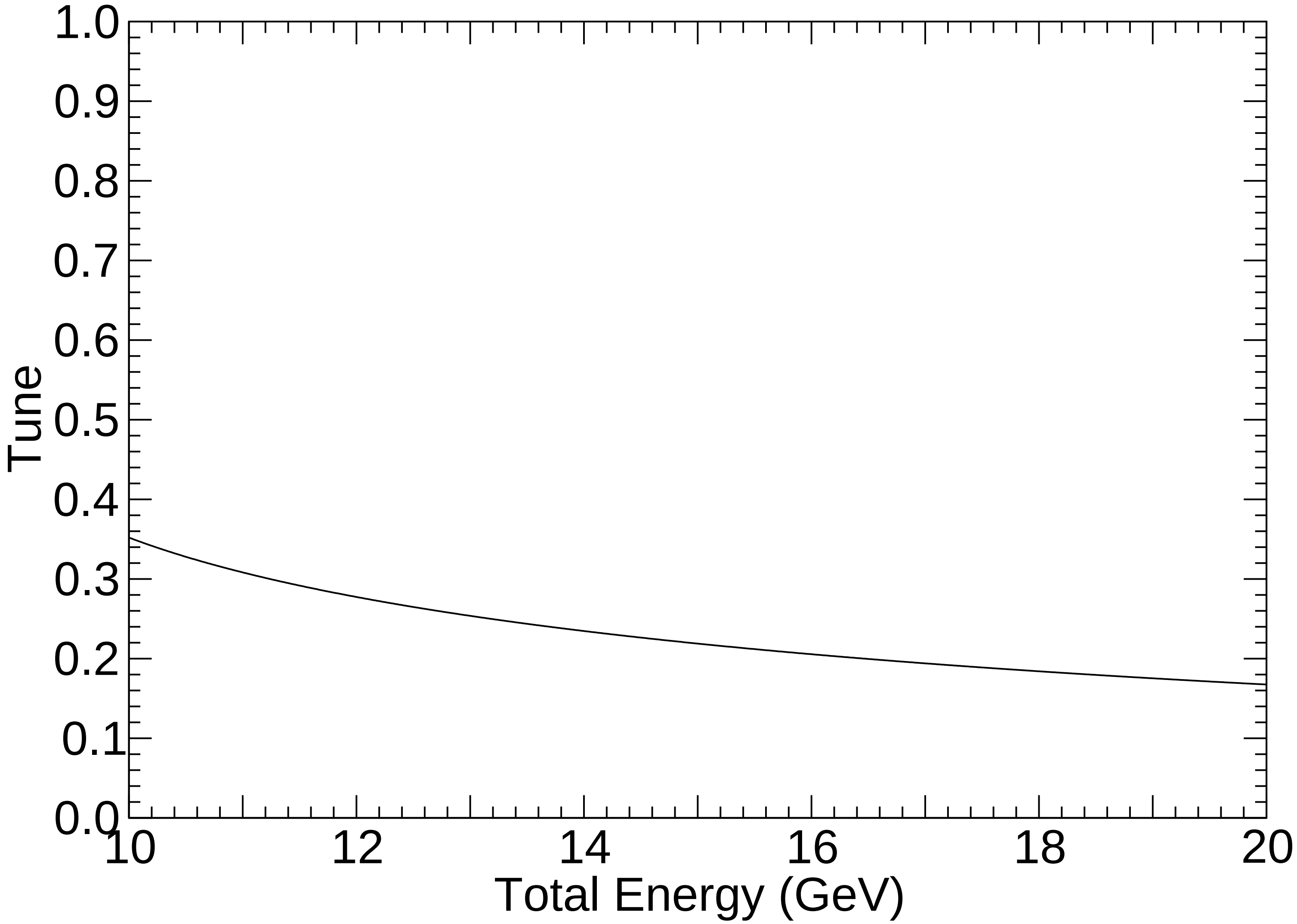
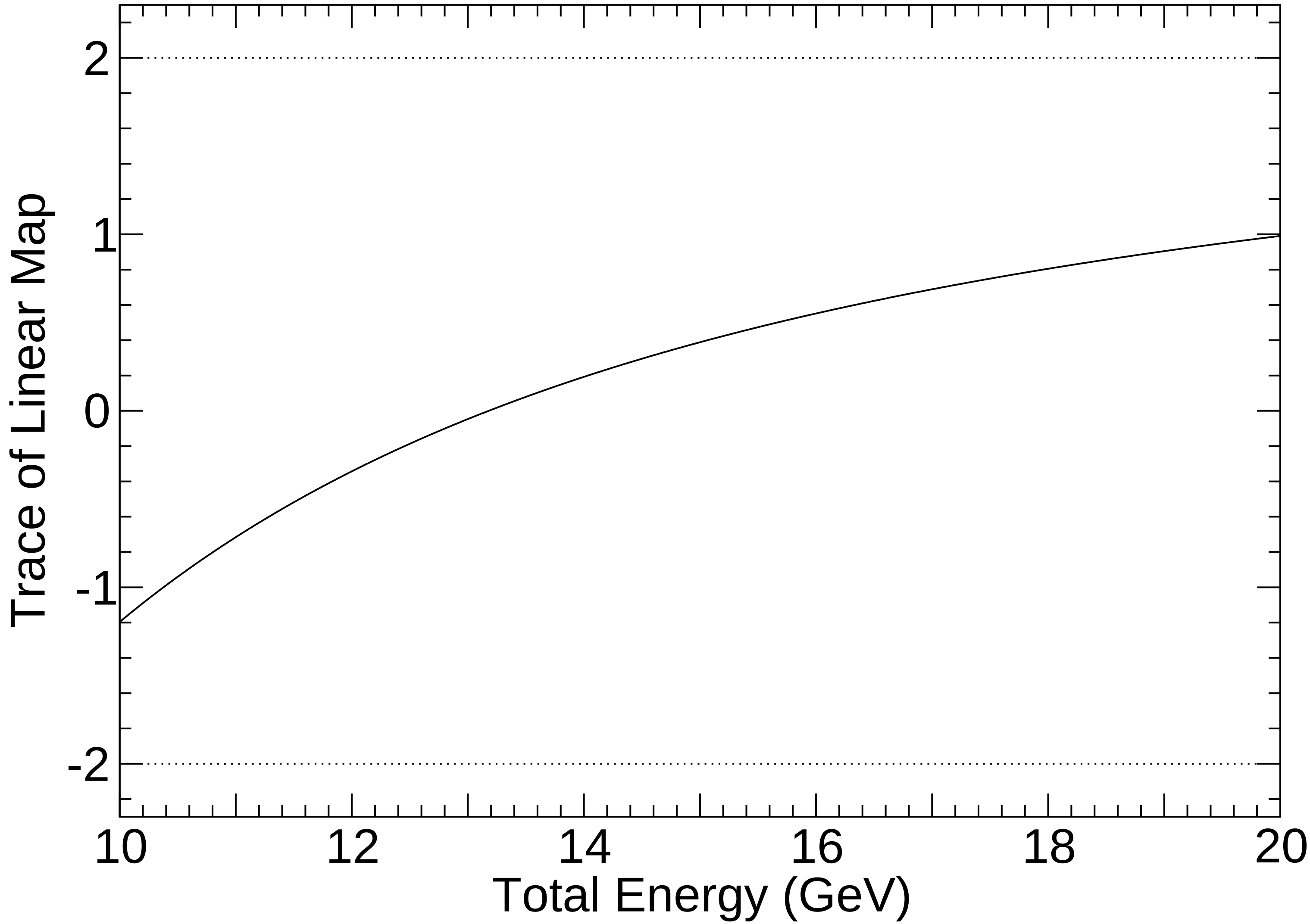
Symmetry of the Ring
Single Cell

- Imagine the ring is a sequence of many short, identical cells
- Behavior for one cell tells us what occurs for the entire ring
- Plot tune and trace as a function of energy for the single cell

Symmetry of the Ring
Tune for a Single Cell



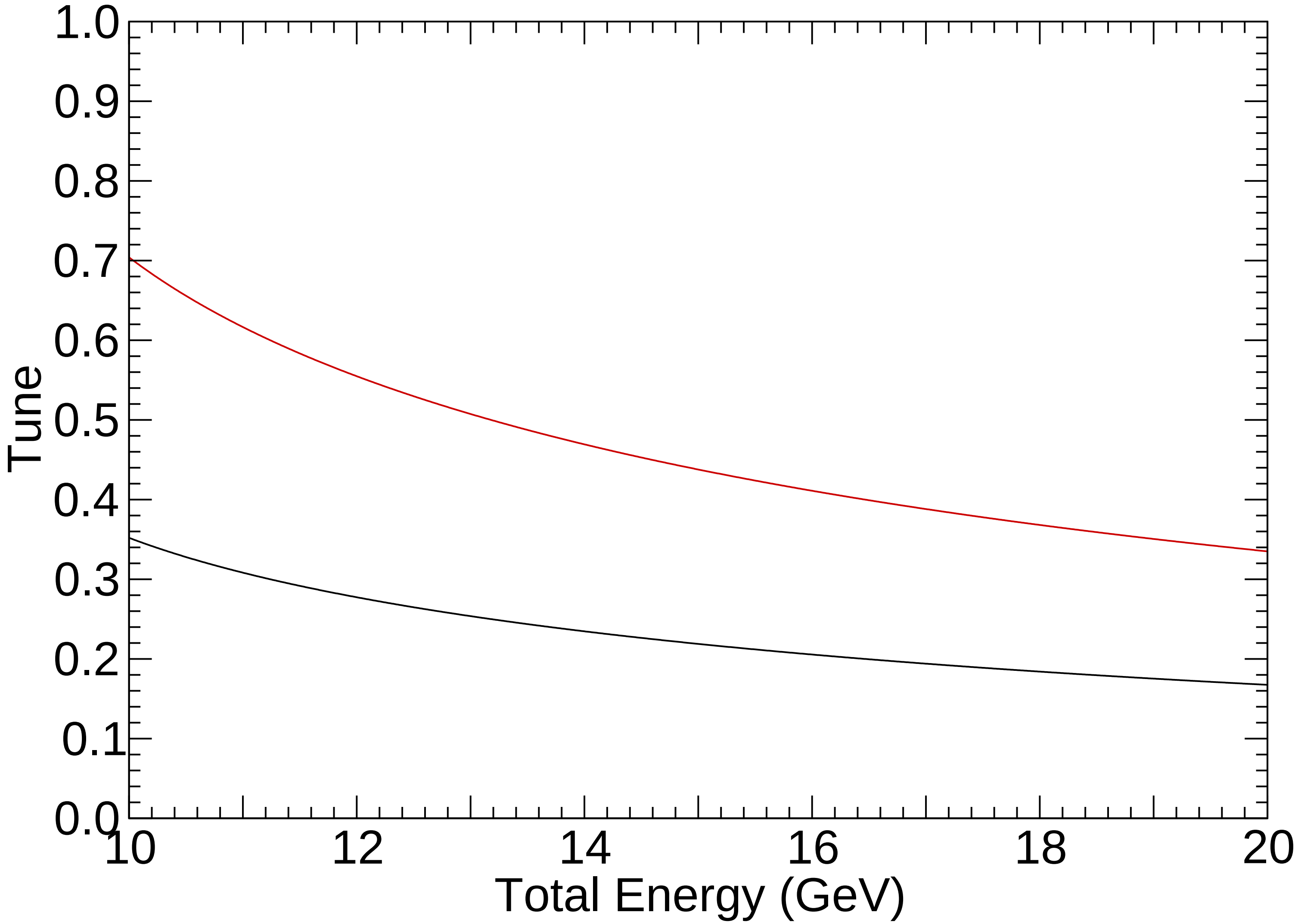
Symmetry of the Ring
Trace of the Linear Map for a Single Cell



Symmetry of the Ring
Two Cells

- Examine Dynamics of Two Cells
- Two Identical Cells
  - Dynamics determined by single cell behavior
  - Tune simply double the single cell tune
  - Trace just touches -2.0, but doesn't cross
    - Motion would be unstable if it crossed -2.0
- Two slightly different cells
  - Each close to the single cell
  - Trace now crosses -2.0
    - Motion unstable between 13.5 GeV and 15.3 GeV
- These are called linear resonances

Symmetry of the Ring
Tune for Two Identical Cells



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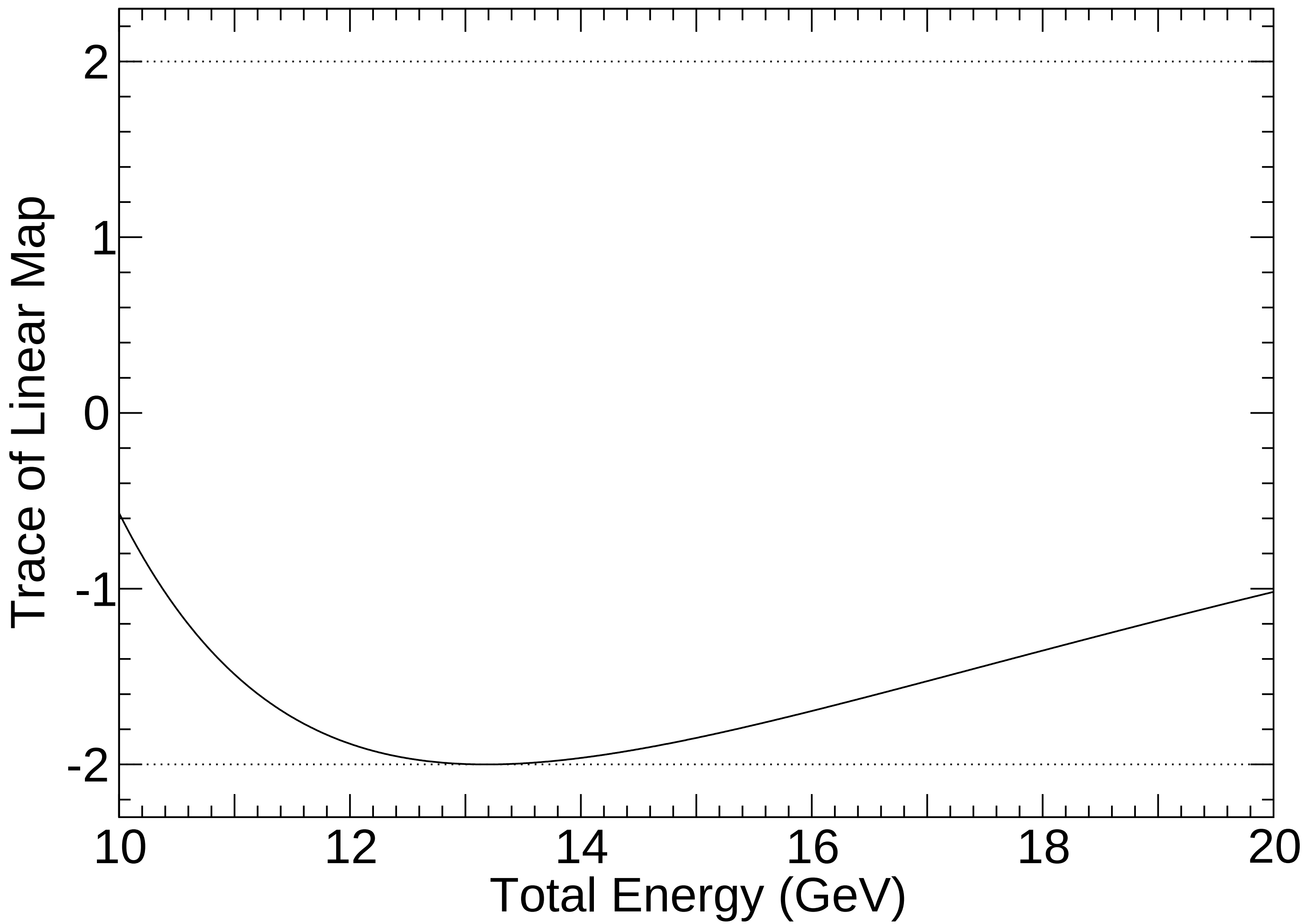
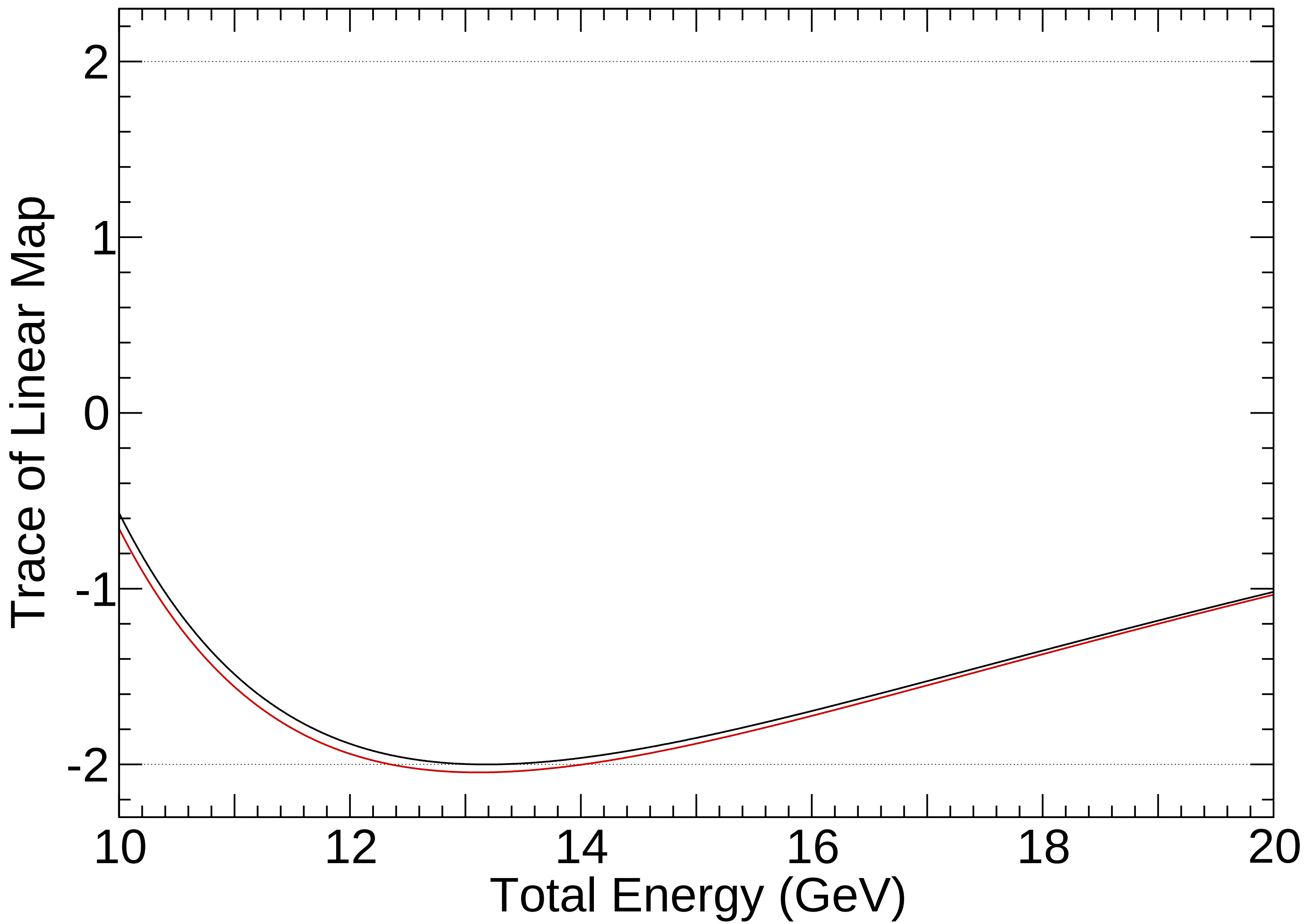
Symmetry of the Ring
Trace of Linear Map for Two Identical Cells



Symmetry of the Ring
Trace of Linear Map for Two Different Cells



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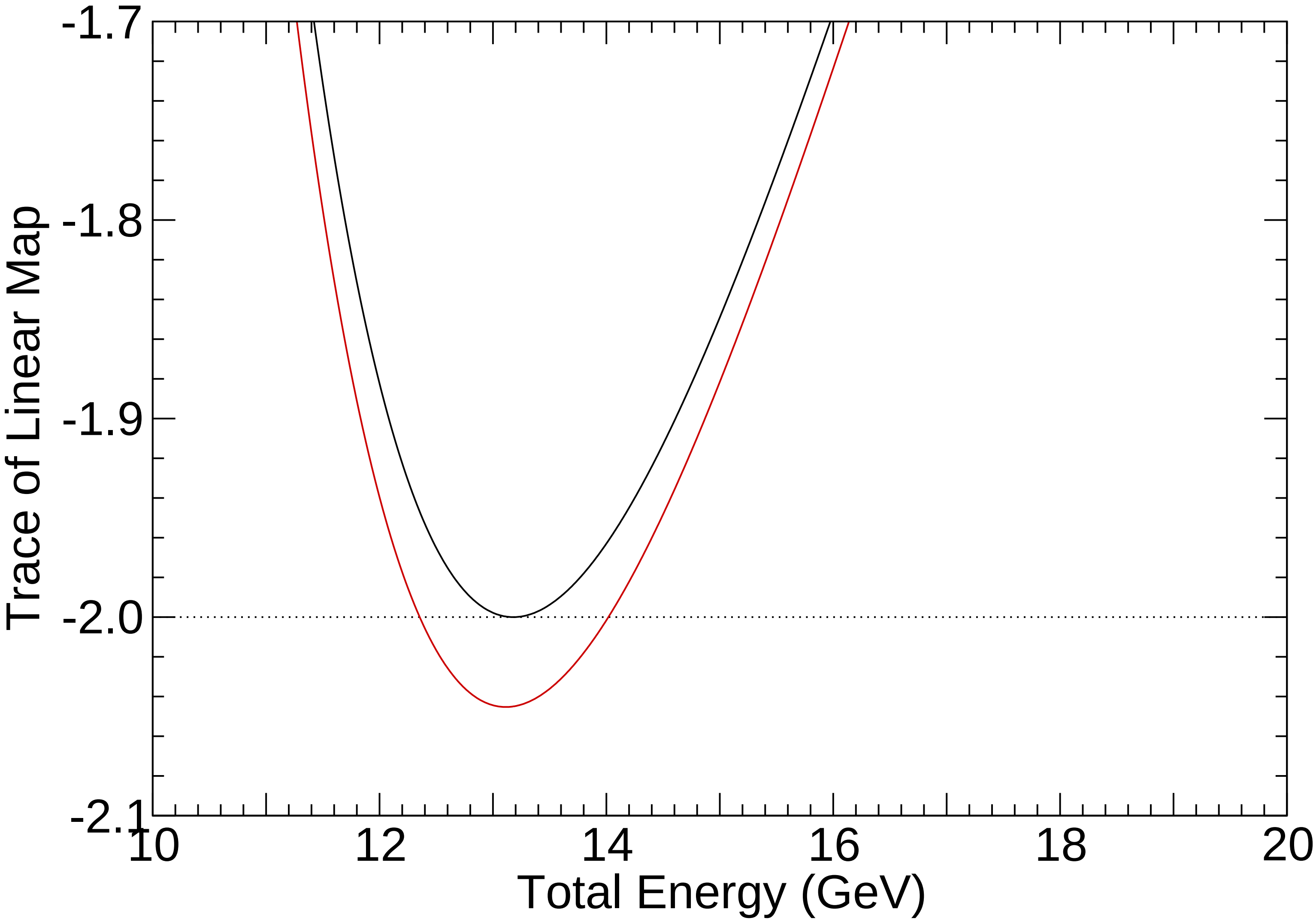
Symmetry of the Ring

Trace of Linear Map for Two Different Cells

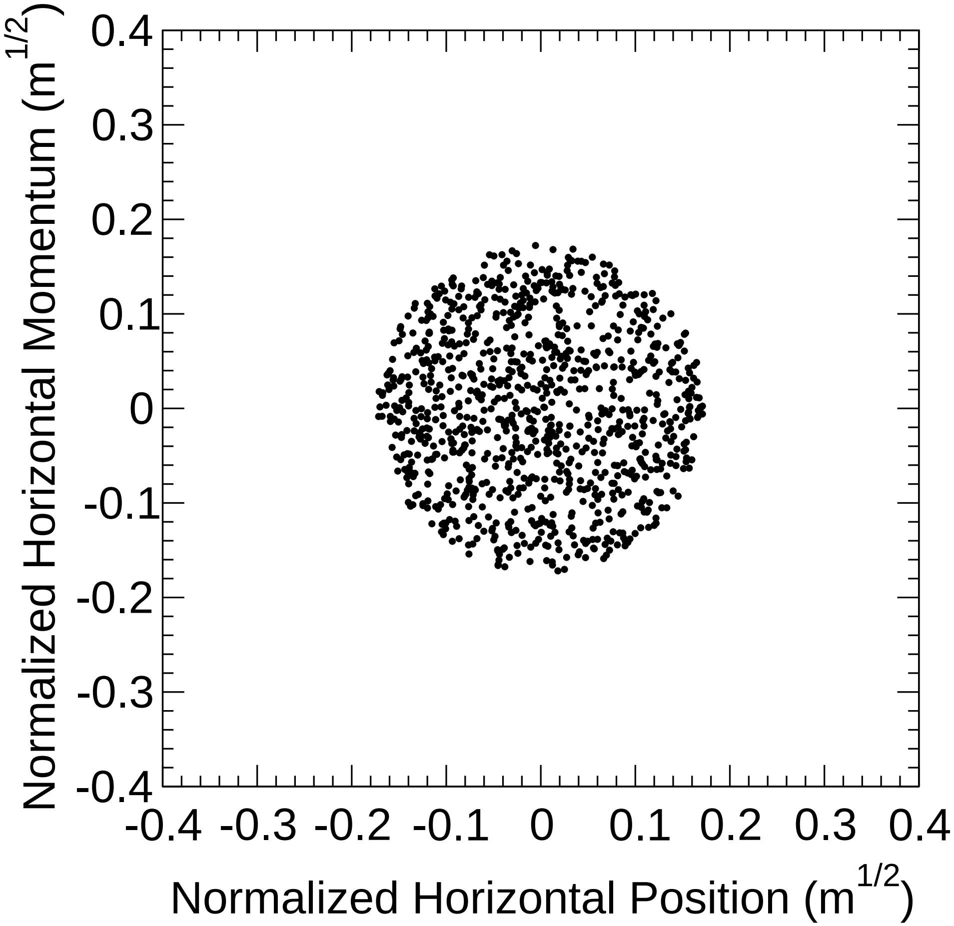


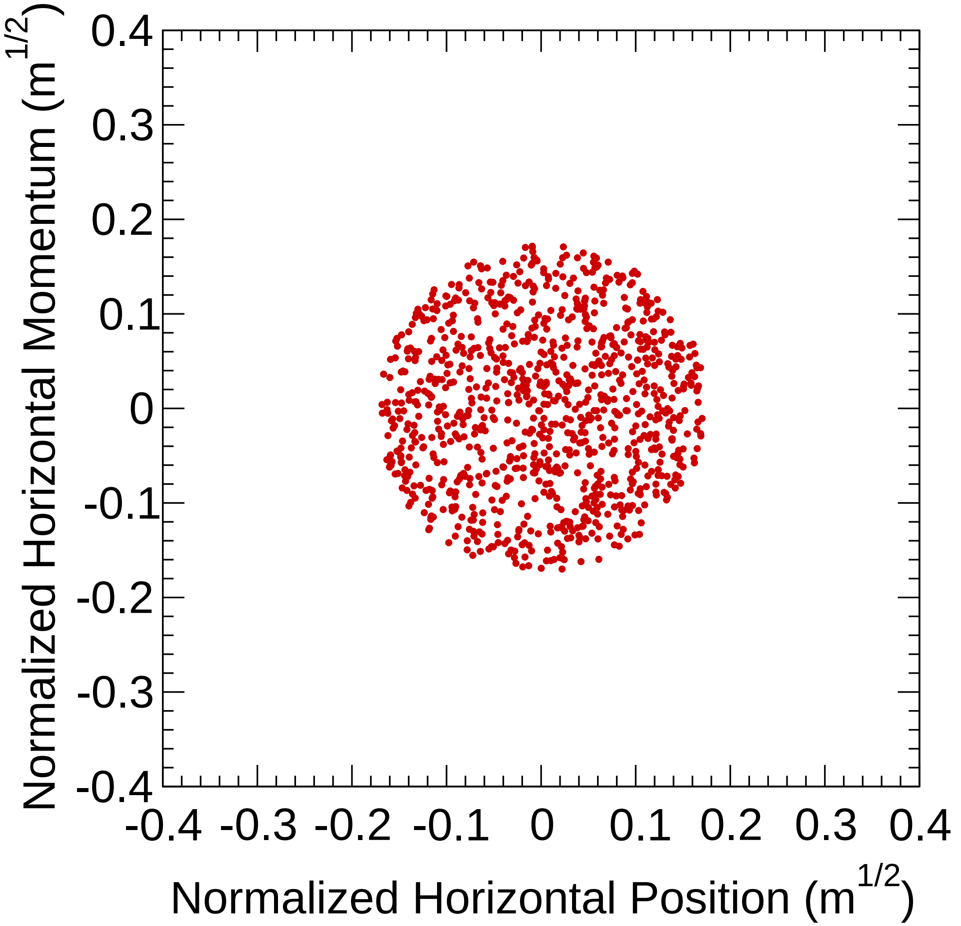
Symmetry of the Ring

Accelerating Through Resonances

- Accelerating through resonances will cause the beam to blow up.
- If cells identical, same before and after in normalized phase space
- If cells different, beam gets larger in normalized phase space
- Effect is stronger if you accelerate more slowly
  - More time spent in unstable state

Symmetry of the Ring
Accelerating in Two Identical Cells





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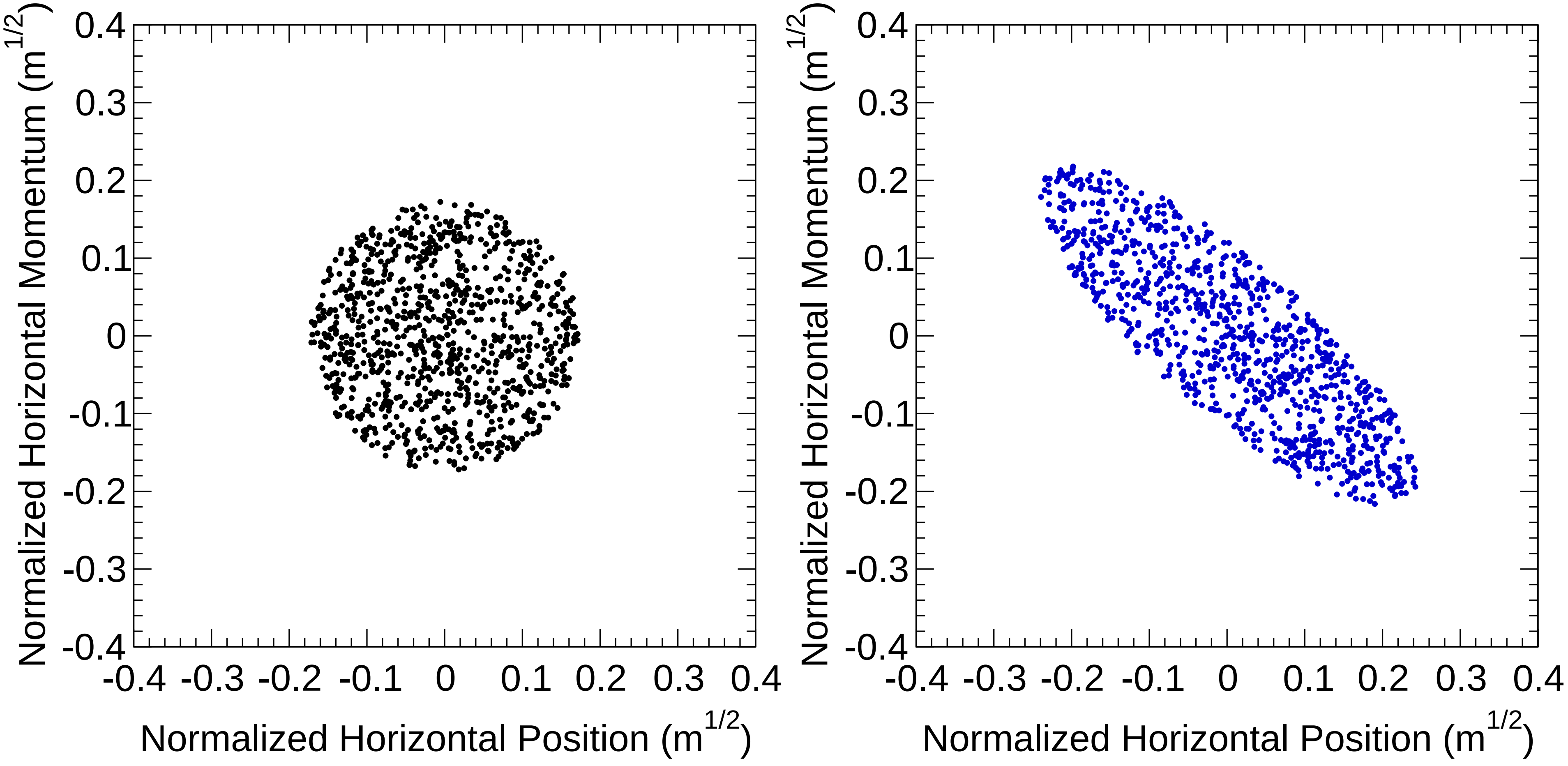
Symmetry of the Ring
Accelerating in Two Different Cells



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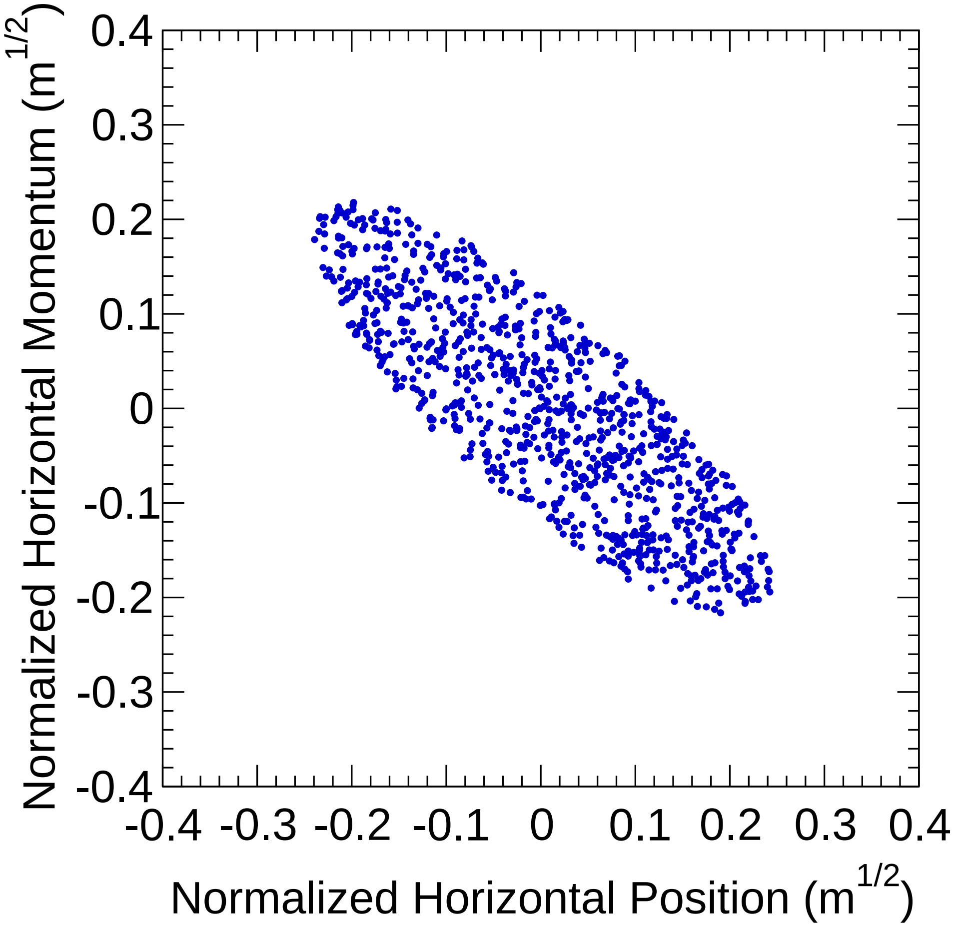
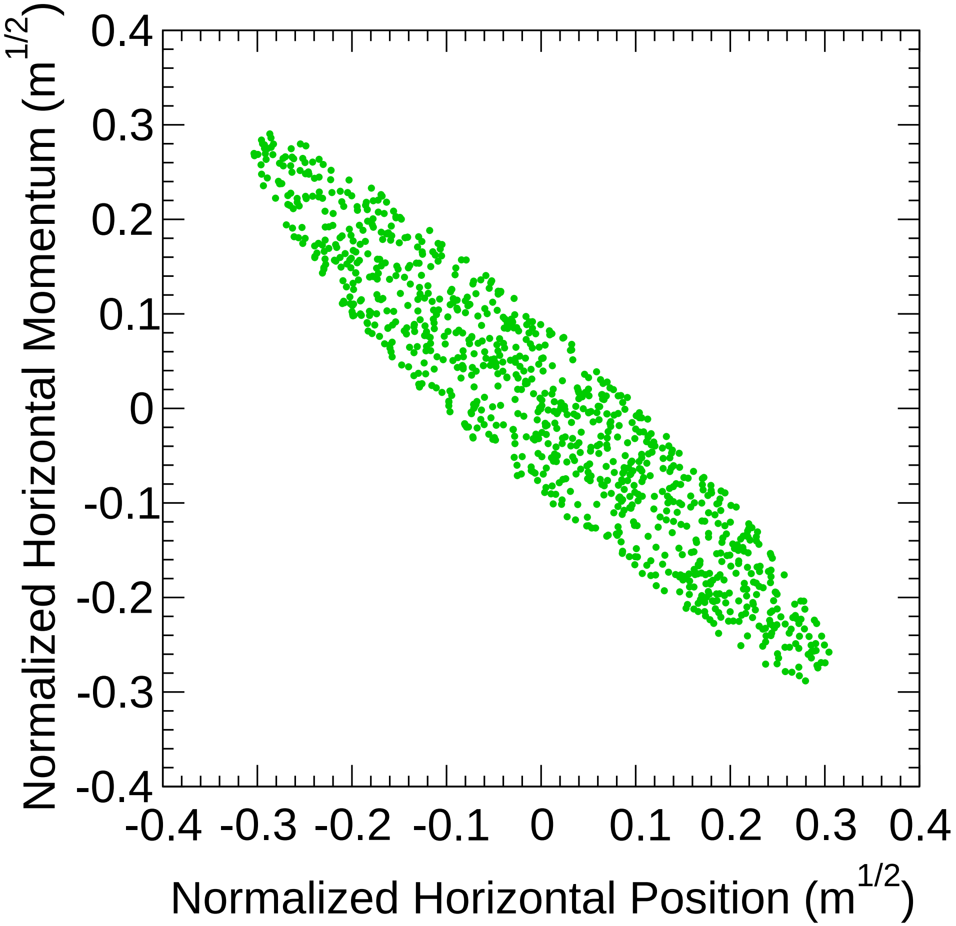
Symmetry of the Ring
Accelerating at Different Rates

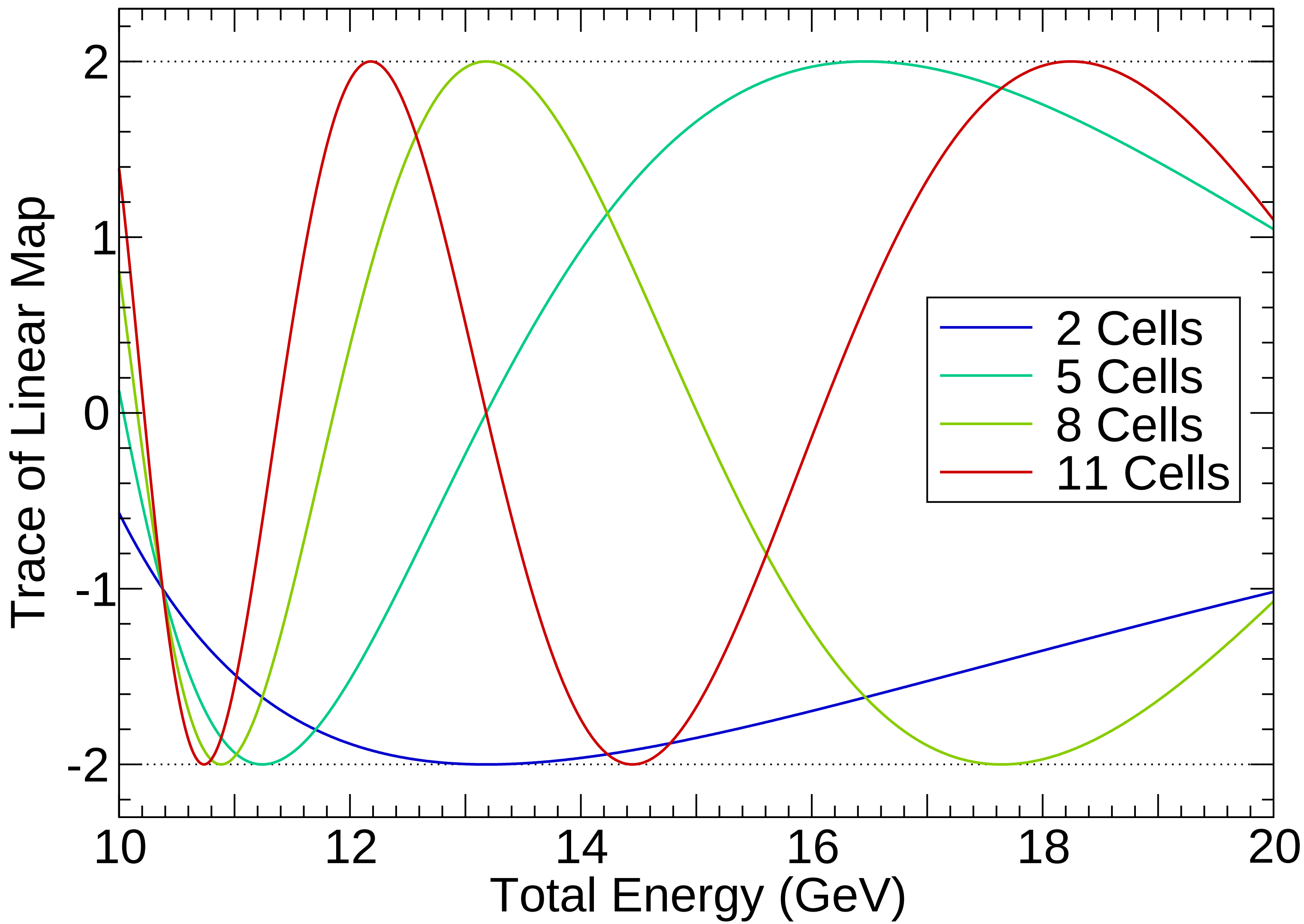




Symmetry of the Ring
More Cells

- Instead of two cells, do more cells
- If cells identical, trace touches -2 and 2 in more places for more cells
  - These are places where, when the symmetry is broken, things could go unstable
- Most rings have little symmetry
  - Detector, injection/extraction, etc.
  - Regions between trace crossing ± 2 are small
  - This is the first reason why most machines can only transmit a small momentum range with fixed fields

Symmetry of the Ring
Trace for Many Cells



Symmetry of the Ring
Results for FFAGs

- These results have an impact on FFAG design
- (Non-scaling) FFAGs should consist entirely of short, identical cells
  - This will prevent the linear resonances from causing (too much) instability
  - Magnetic fields and magnet placement must be very precise
- Accelerate rapidly to minimize the impact of the unavoidable errors

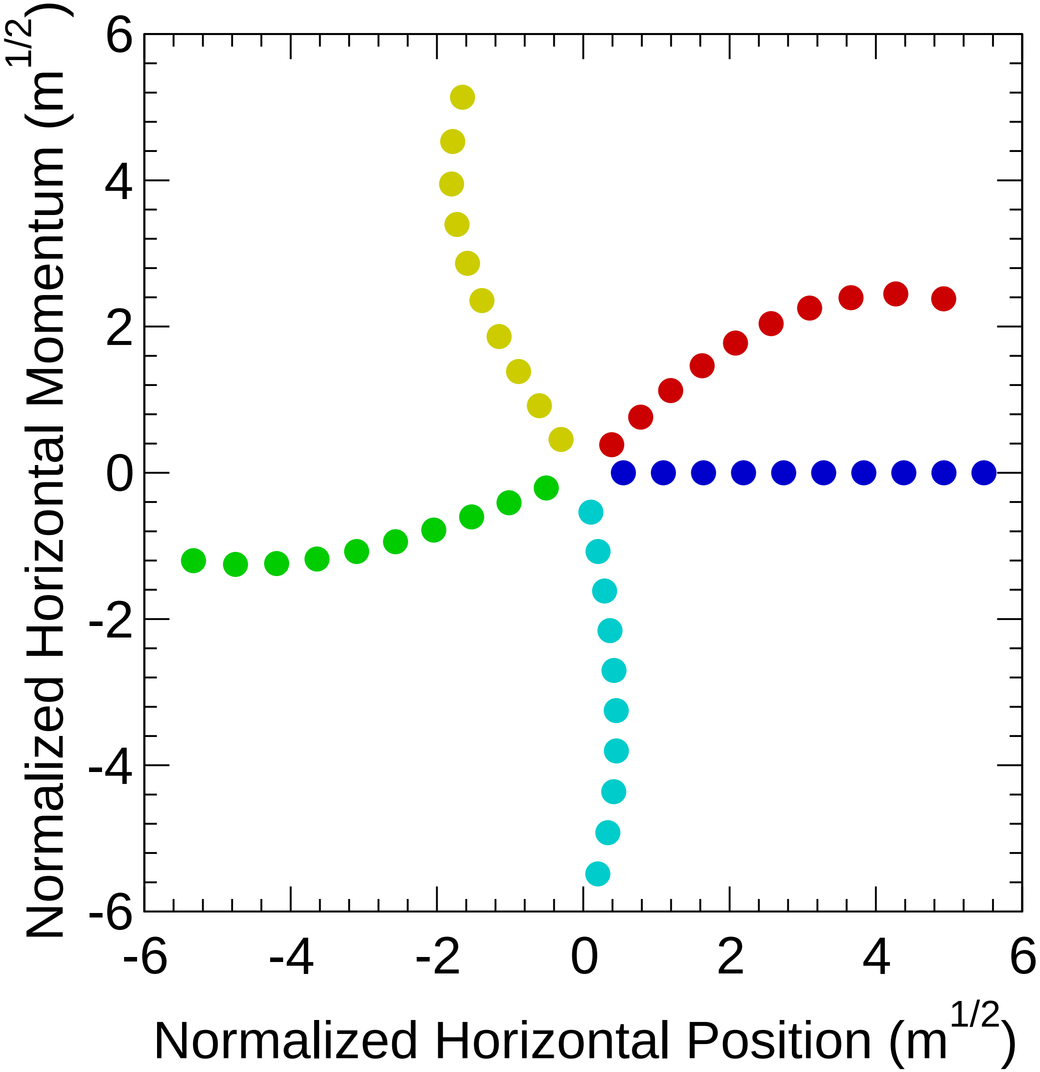
Nonlinearities

- Discussion thus far assumed that a linear map about the closed orbit was an exact representation of the motion
- Real machines have higher order terms about the closed orbit
- Some terms are unavoidable
  - Terms due to the beam not making small angles
  - Terms coming from the curvature of the ring
  - Terms coming about because fields change from their values in the magnets to zero outside the magnets (Maxwell's equations)
  - Space charge (forces of particles on each other)
- Sometimes nonlinearities are intentionally introduced
  - Reduce tune variation
  - Control time of flight (more later)
  - Control collective effects

Accelerator Concepts
Tune Shift with Amplitude

- Nonlinearity leads to a tune that depends on the distance from the closed orbit
- Even if the tune at the closed orbit is OK, tune may be “bad” at some distance from the closed orbit

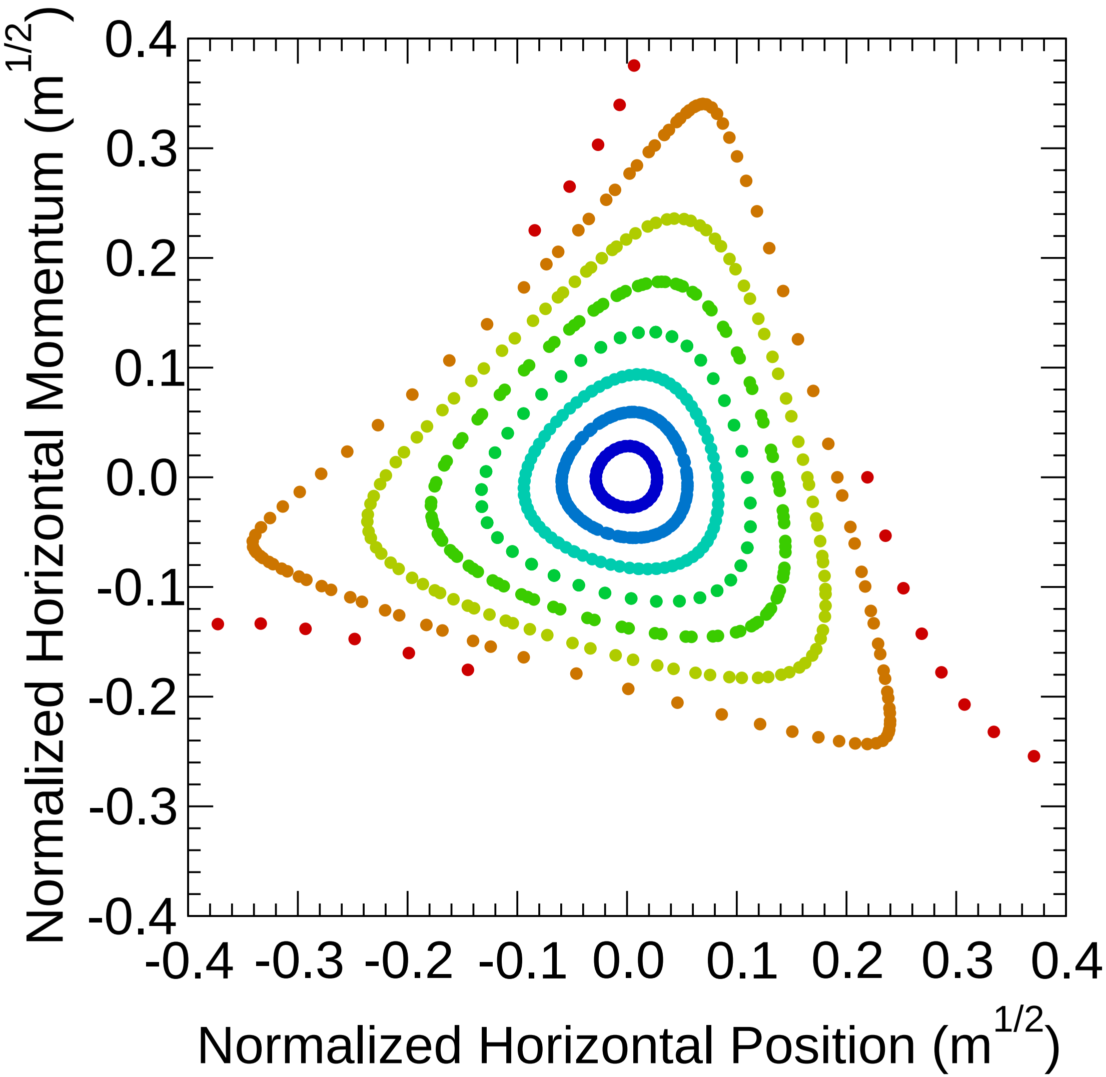
Accelerator Concepts
Tune Shift with Amplitude



Accelerator Concepts
Nonlinear Resonances

- Single cell example
- At small amplitudes, motion is like linear
- At larger amplitudes motion develops a three-fold symmetry
  - This is a sign that the tune is approaching 1/3
- At large enough amplitude
  - Tune becomes exactly 1/3
  - Motion becomes unstable
- Resonant phenomenon, similar to linear resonances above
  - Happens at nonzero amplitude, however

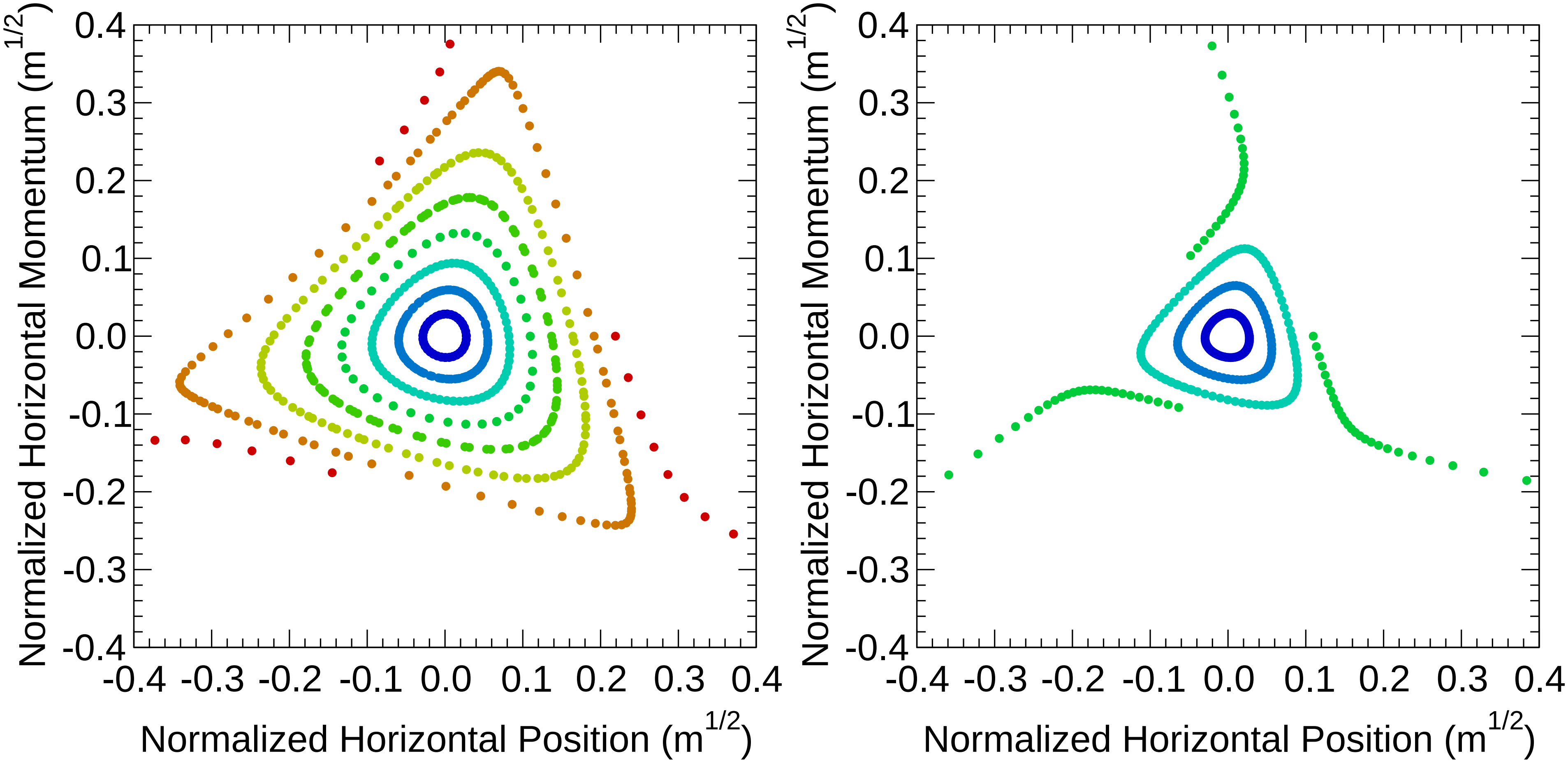
Accelerator Concepts
Nonlinear Resonance



Accelerator Concepts
Nonlinear Resonances: Energy Variation

- As energy changes, zero-amplitude (linear) tune approaches 1/3
- Amplitude where tune becomes 1/3 and particles are lost becomes lower
- Smaller phase space area is transmitted
- When linear tune reaches 1/3, transmitted area can be zero

Accelerator Concepts
Nonlinear Resonances: Energy Variation



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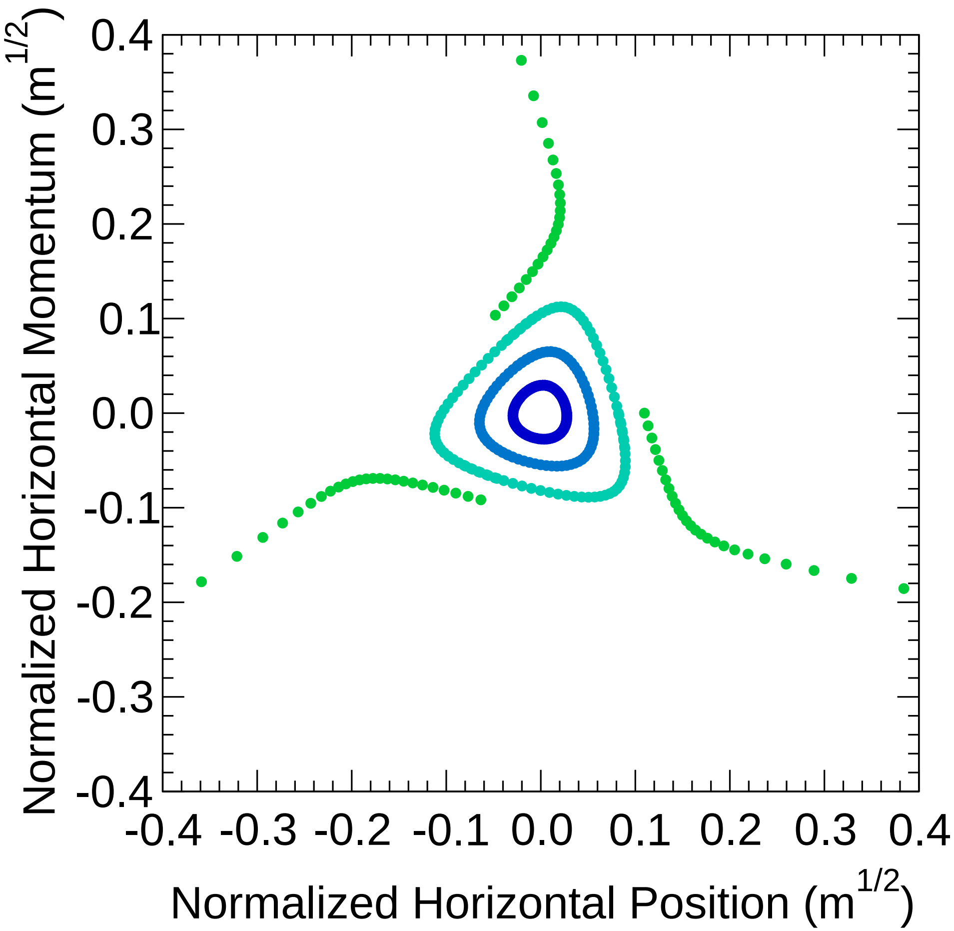
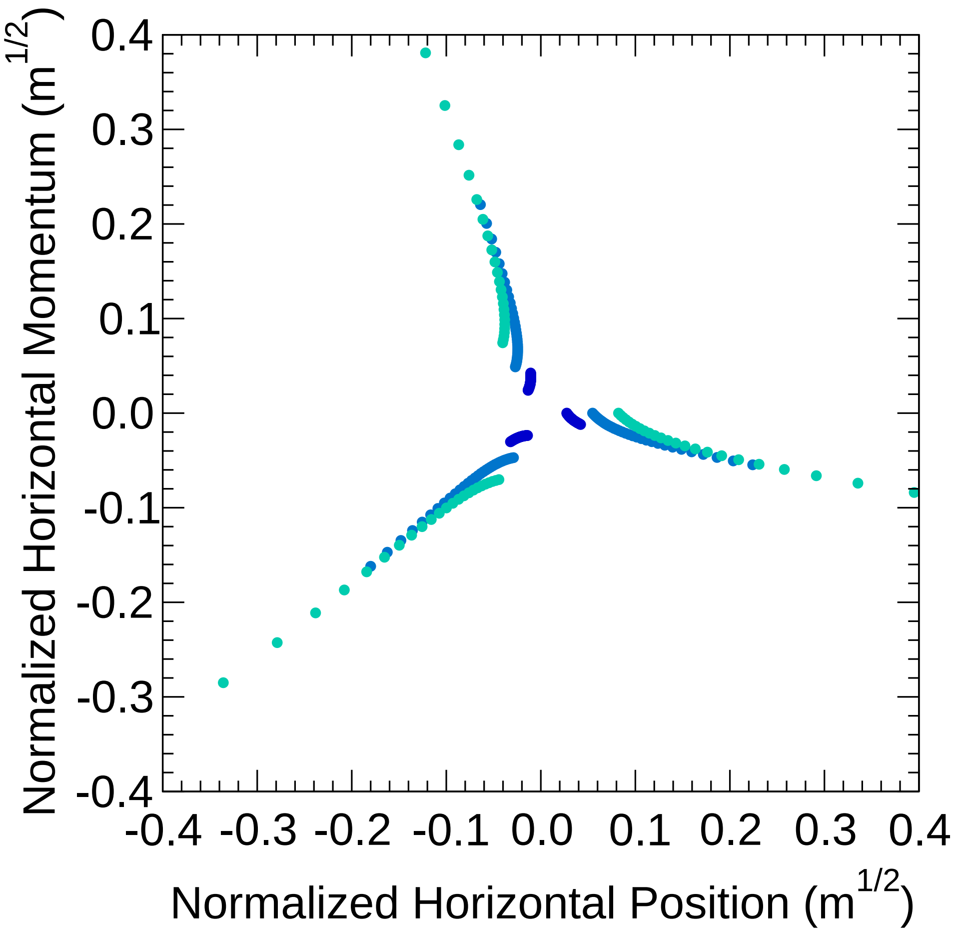
Accelerator Concepts
Nonlinear Resonances: Energy Variation





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Accelerator Concepts

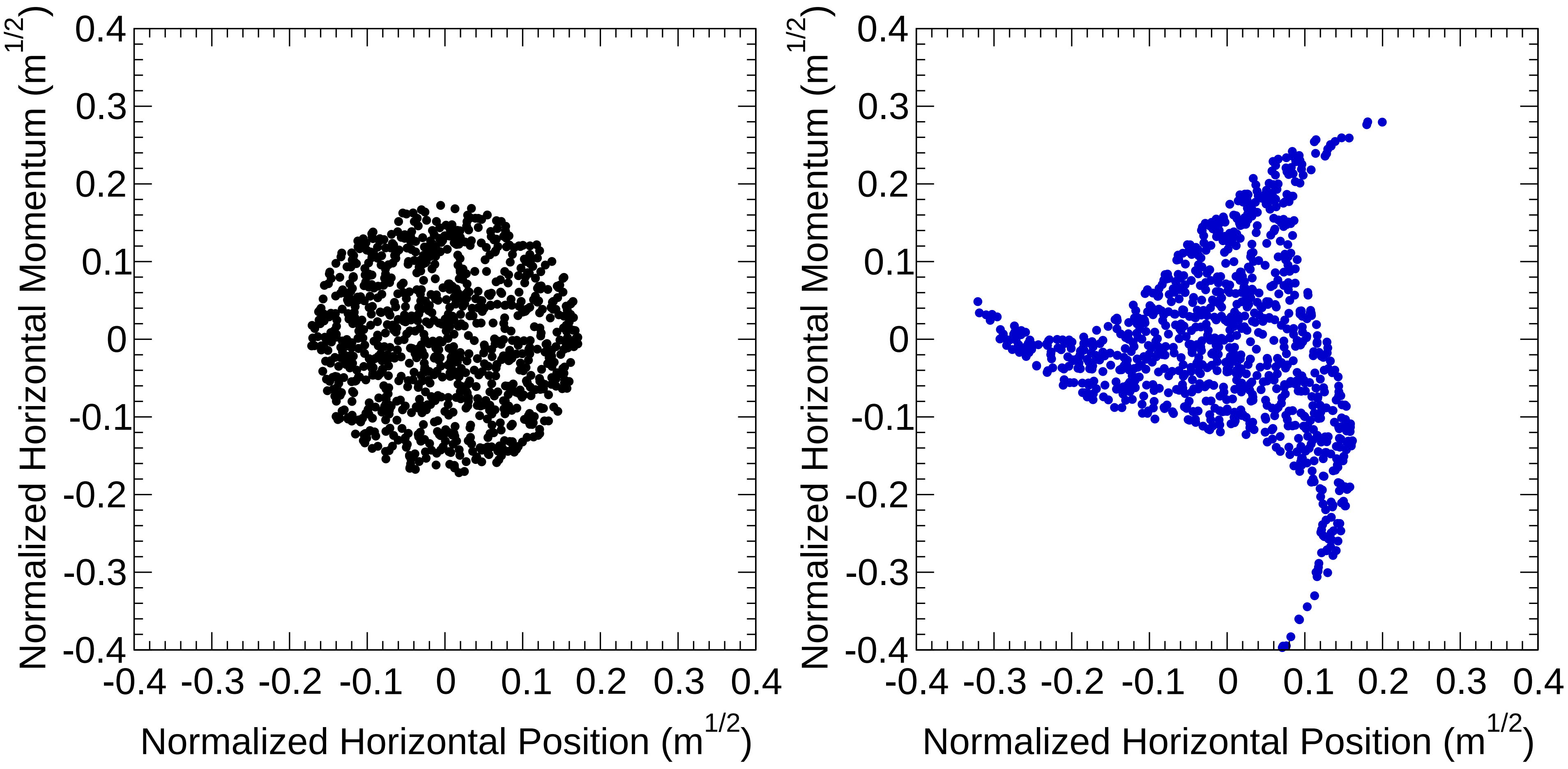
Nonlinear Resonances: Energy Variation

- With linear resonances, needed to avoid $2\nu_x = k$, for k integer.
- Including nonlinear resonances means we avoid $m\nu_x = k$ for any m and k
  - Infinite number of these
    - Driving terms may be zero or weak for many of these
    - Lower-order (m) resonances tend to be stronger
    - Must face these even with perfect single-cell symmetry
  - Energy space even more restricted than when considering linear resonances
  - This further restricts energy acceptance of real machines
- Because of two transverse planes, really deal with $m_x\nu_x + m_y\nu_y = k$

Accelerator Concepts
Accelerating Through Nonlinear Resonances

- Accelerate through nonlinear resonance
- Increase in beam size if we accelerate through rapidly
- Accelerate through more slowly
  - Further increase in beam size
  - Particle loss (12%, this example)
- Increase the resonance strength
  - Beam looks smaller
  - That's because large amplitude particles were lost (58%!)

Accelerator Concepts
Accelerating Through Nonlinear Resonance



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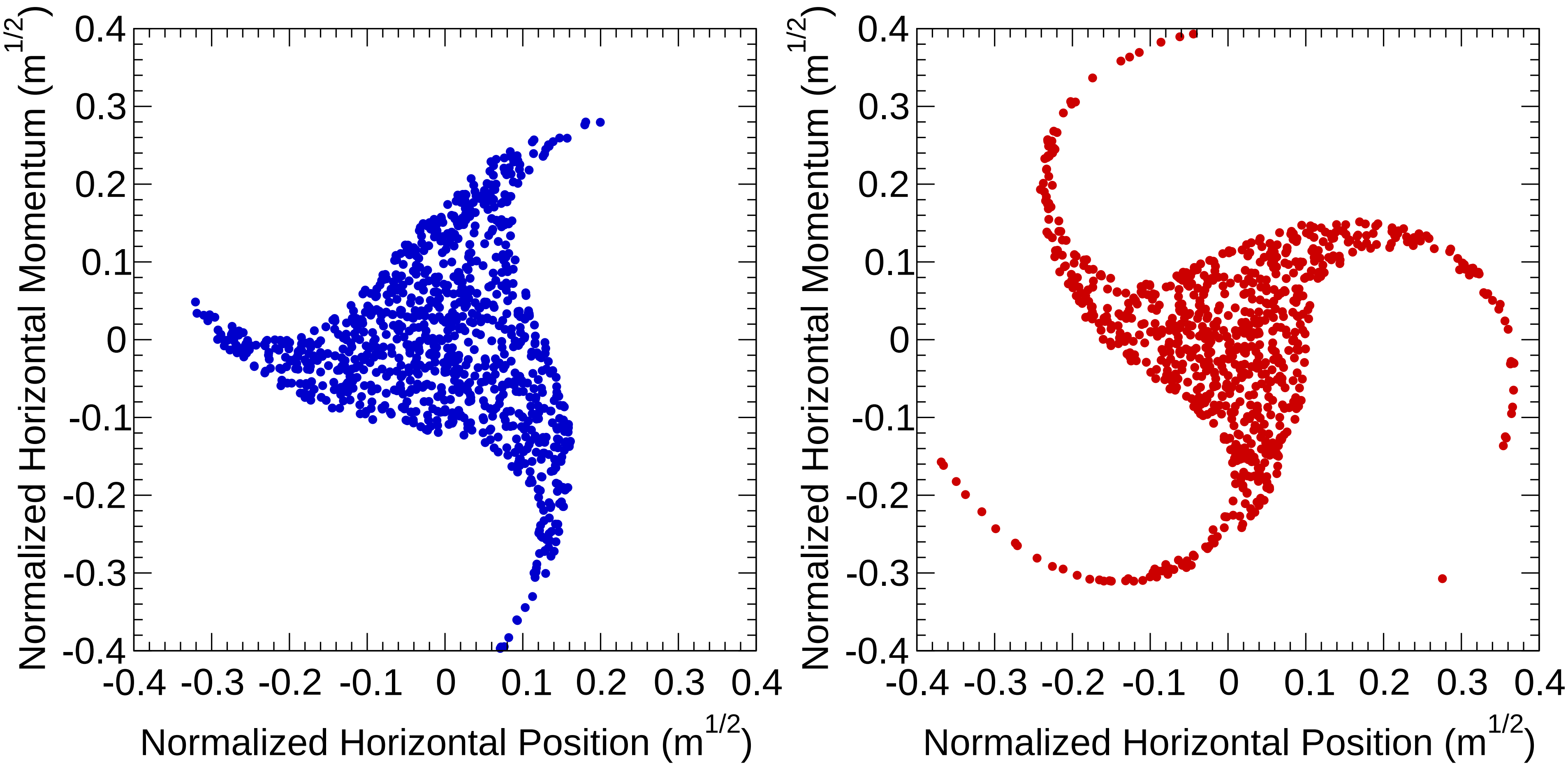
Accelerator Concepts
Accelerating Through Resonance More Slowly



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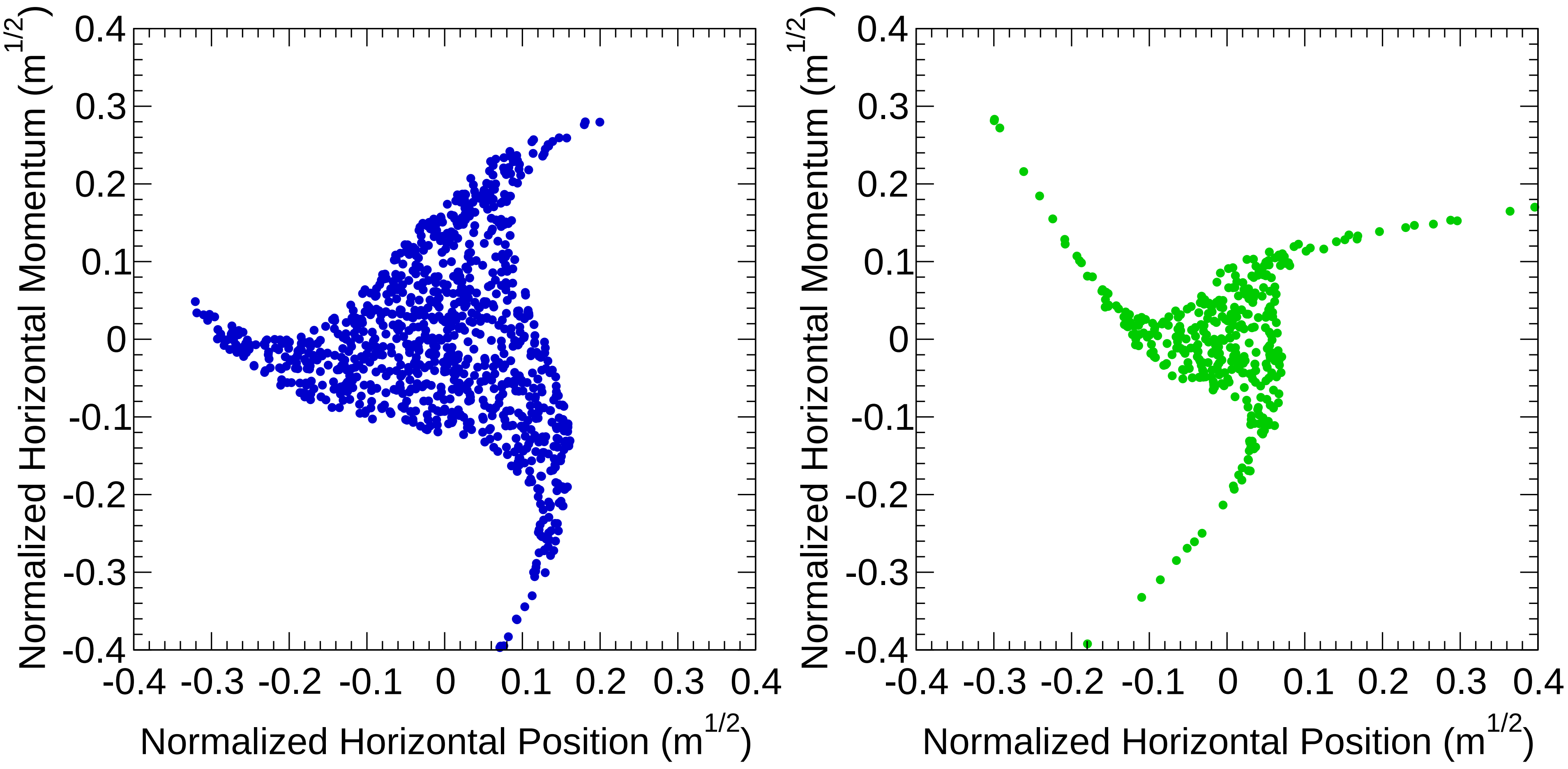
Accelerator Concepts
Stronger Resonance



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Summary: Constraints Imposed on FFAG Design by Resonances

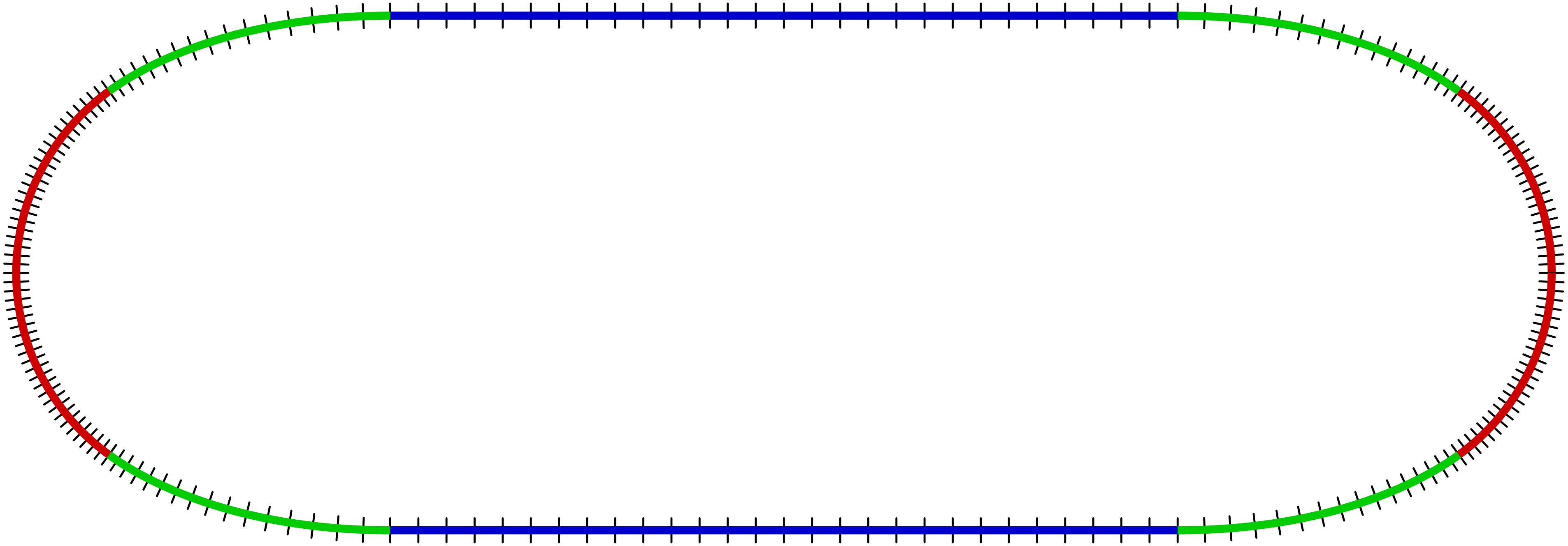
- To achieve wide energy range, FFAGs must avoid resonances
- To avoid resonances, FFAGs should be constructed as
    - Simple cells
    - All identical
    - Minimize differences due to errors
- Nonlinearities should be minimized to avoid driving nonlinear resonances
    - Must accelerate rapidly through any residual nonlinear resonances

FFAG Research Areas

- Much FFAG research involves trying to push on these constraints
- Breaking symmetry
  - Short cells keep apertures small, have other benefits
  - Long cells needed to hold RF cavities (accelerate particles)
  - Would like to combine both in one machine
  - Must transition between the two gently. . .
- Adding nonlinearities
  - Nonlinearities are desirable for various reasons
    - Control time of flight (more later)
    - Decrease tune variation with energy
      - Avoid space charge driven resonances
      - Reduce time of flight variation with transverse amplitude
  - Try to add nonlinearity while transmitting large enough beam

Breaking FFAG Symmetry

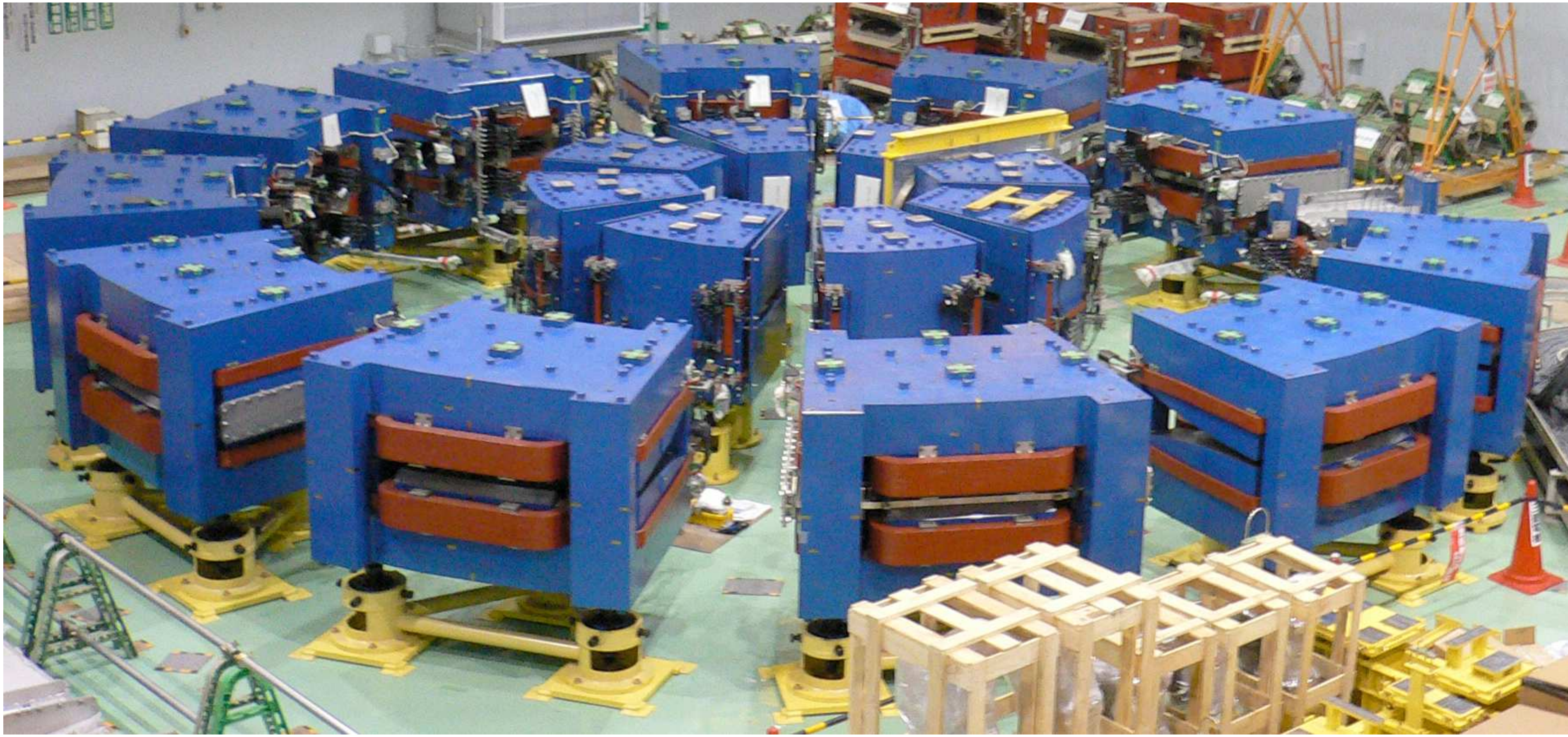
Racetrack-shaped FFAG



The Scaling FFAG

- If one could avoid tune variation, resonances might not be a problem
- The first FFAG designers (1950s!) realized this, and had a solution
- If field in cylindrical coordinates is $B(r, \theta, 0) = B_0(\theta) r^k$, tune is independent of energy
- These are currently being built (Japan)
- Why not just use these
  - Apertures larger than non-scaling FFAGs (cost)
    - Bend is in the wrong magnet
  - Time of flight (coming up)

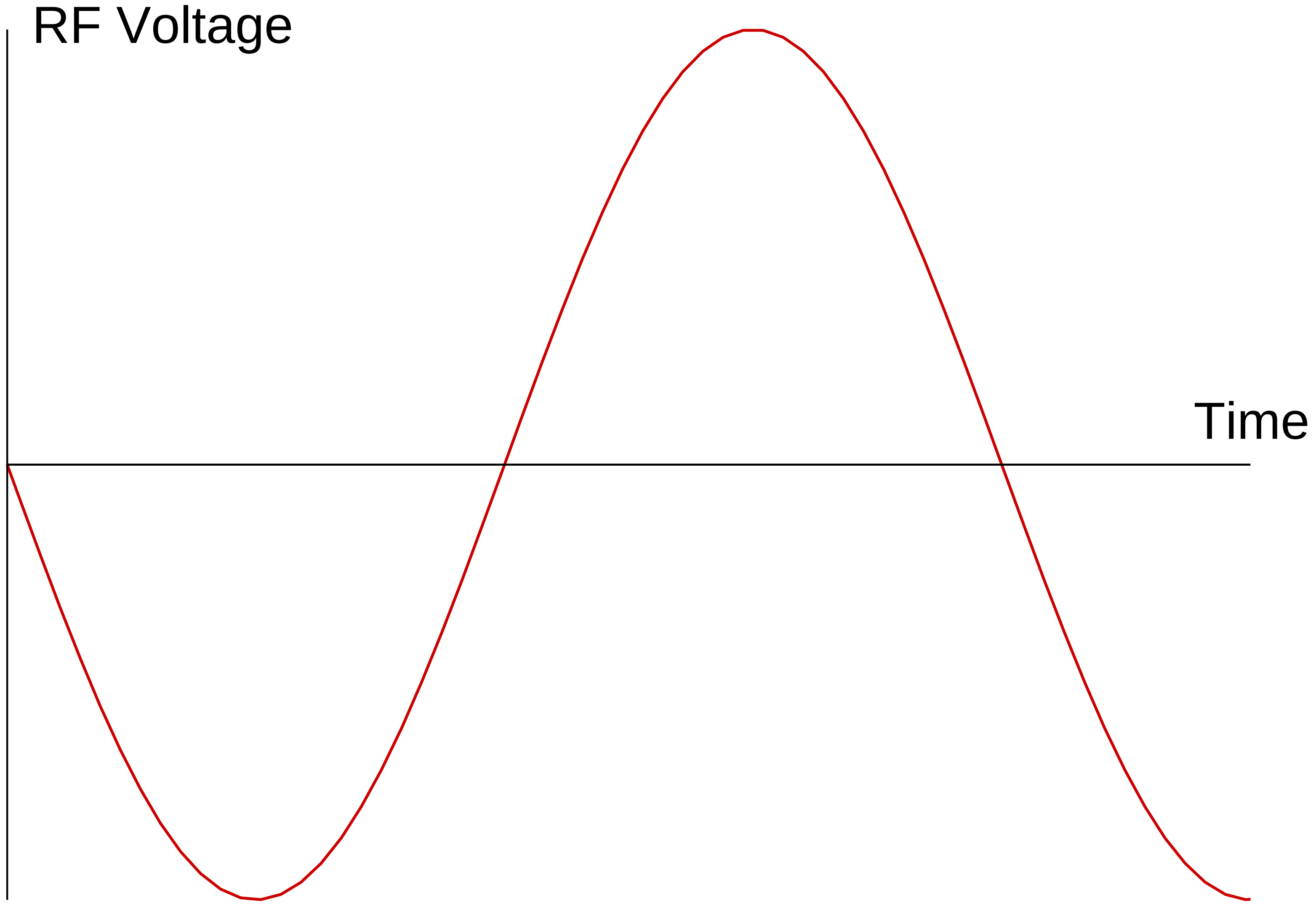
Scaling FFAGs
FFAG at KURRI



Accelerator Concepts
RF Acceleration

- Acceleration occurs using RF cavities
- Accelerating field varies sinusoidally with time
- To be accelerated, particles must arrive at the cavity at the right time (positive field)

Accelerator Concepts
Sinusoidal RF



Time of Flight Depends on Energy

- Time of flight depends on energy
  - Especially important for large energy range in FFAGs
- Particle velocity varies with energy
  - Important when particles aren’t too relativistic
- Length of closed orbit varies with energy
  - Becomes important for highly relativistic particles
- If keep RF frequency constant, eventually will no longer accelerate
  - Particle returns to same RF phase for one energy only

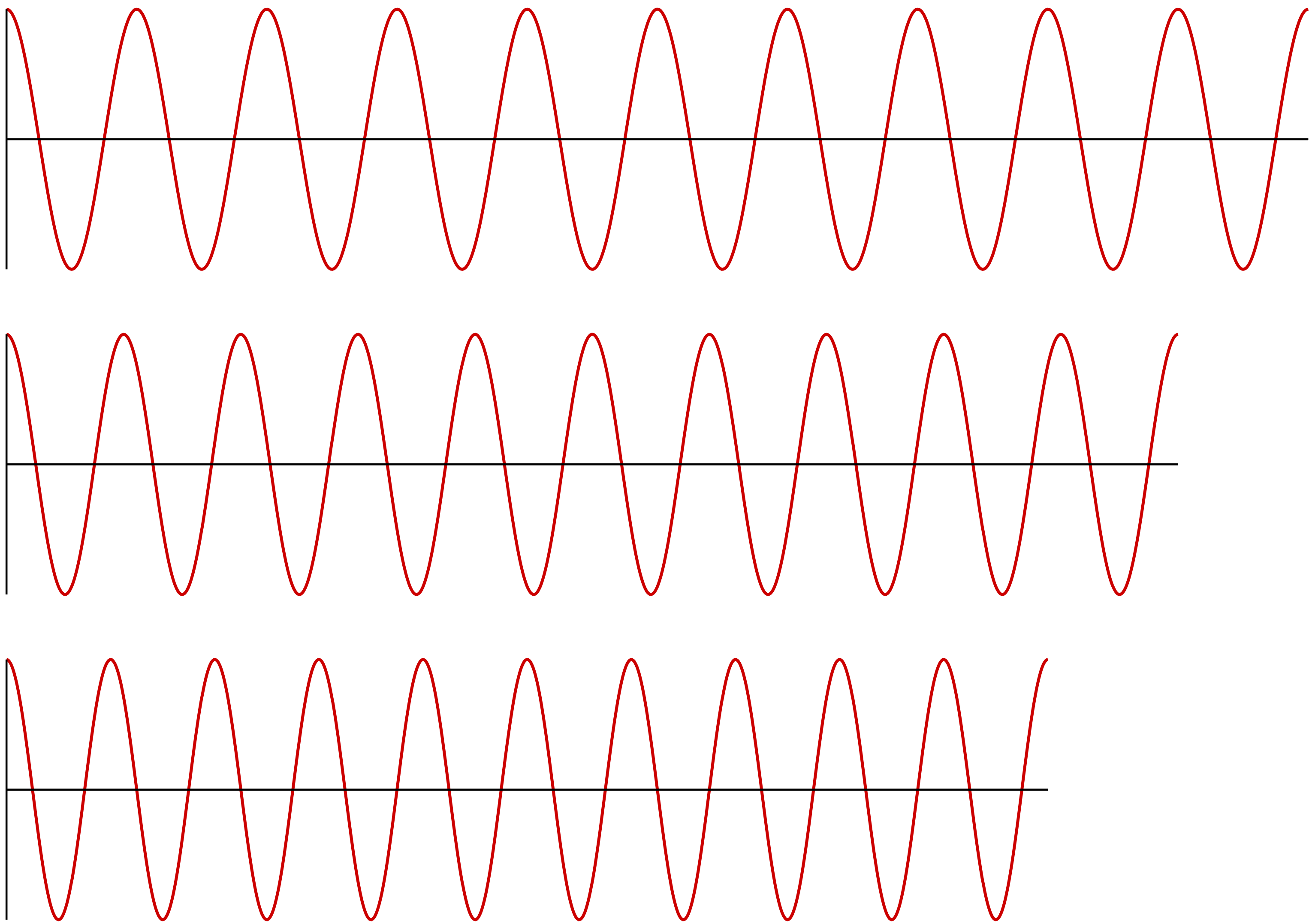
Time of Flight Depends on Energy
Methods for Handling

- Vary RF frequency
- Fix RF frequency, accelerate rapidly
- Harmonic number jump

Time of Flight Depends on Energy
Vary RF Frequency

- Change RF frequency as you accelerate
- Time of flight at each energy is a given number of RF periods
- Problem: making a cavity vary its frequency fast enough
  - High-frequency, high gradient cavities cannot vary their frequency fast enough
  - Low-frequency cavities have lower gradients
    - Limits acceleration rate
  - Higher gradient and faster frequency variation both increase loss in cavity
    - Cooling to extract heat ends up making losses worse
  - Important R&D area for FFAGs
- This is what most current FFAGs do

Time of Flight Depends on Energy
Vary RF Frequency



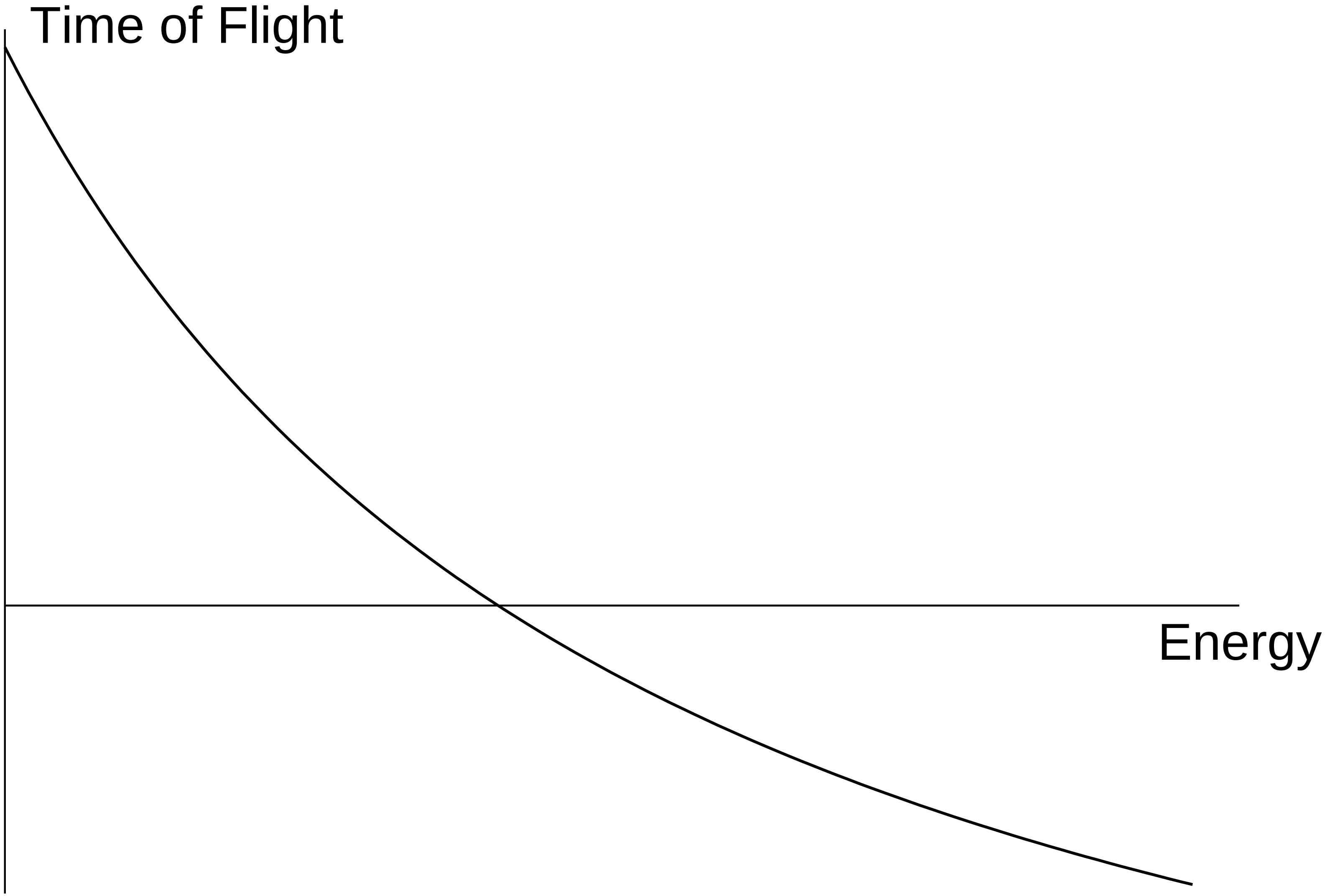
Time of Flight Depends on Energy
Fix RF Frequency, Accelerate Rapidly

- If we accelerate rapidly enough, we won't get too far off the RF crest
- Requires design of an FFAG which where the time of flight range is small compared to the RF period
  - Use low-frequency RF
    - Difficult to get high gradients for rapid acceleration
  - Keep time-of-flight range in FFAG small
    - Impacts overall FFAG design
    - Only works for fairly relativistic FFAGs: otherwise, velocity variation with energy dominates

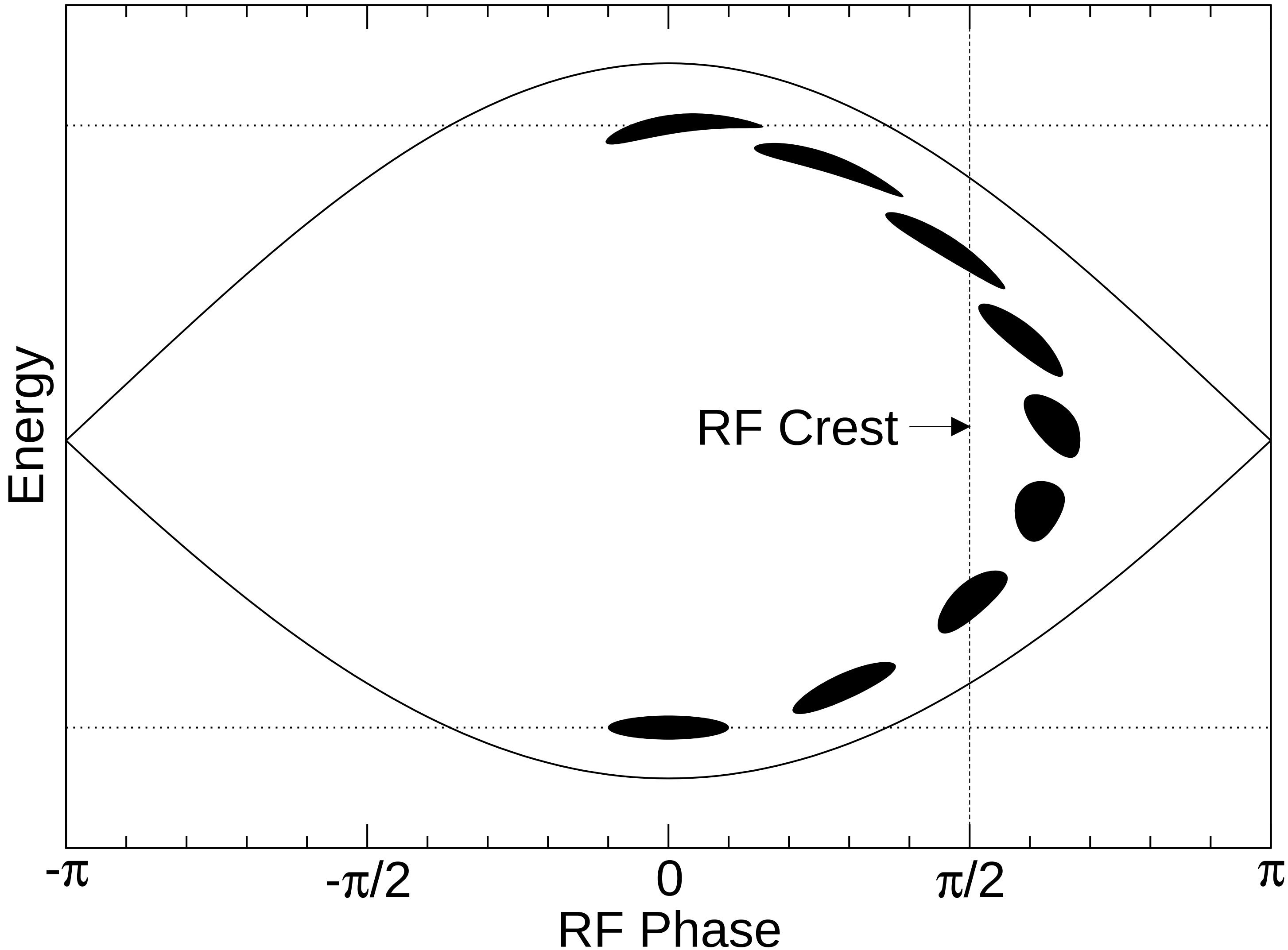
Fix RF Frequency, Accelerate Rapidly
Scaling and Non-Too-Relativistic FFAGs

- Time of flight varies monotonically with energy
- Synchronize RF to time of flight near the central energy
- Acceleration process
  - Start on one side of crest
  - Time of flight too long, move toward crest and cross
  - Reach energy where time synchronized
  - Time of flight gets too short, move across crest in other direction
- Applications
  - Frequency required too low for not-too-relativistic situations
  - Scaling FFAGs: interesting for muon acceleration (very rapid)
    - Requires rather low frequency RF: low gradient

Fix RF Frequency, Accelerate Rapidly
Monotonic Time of Flight



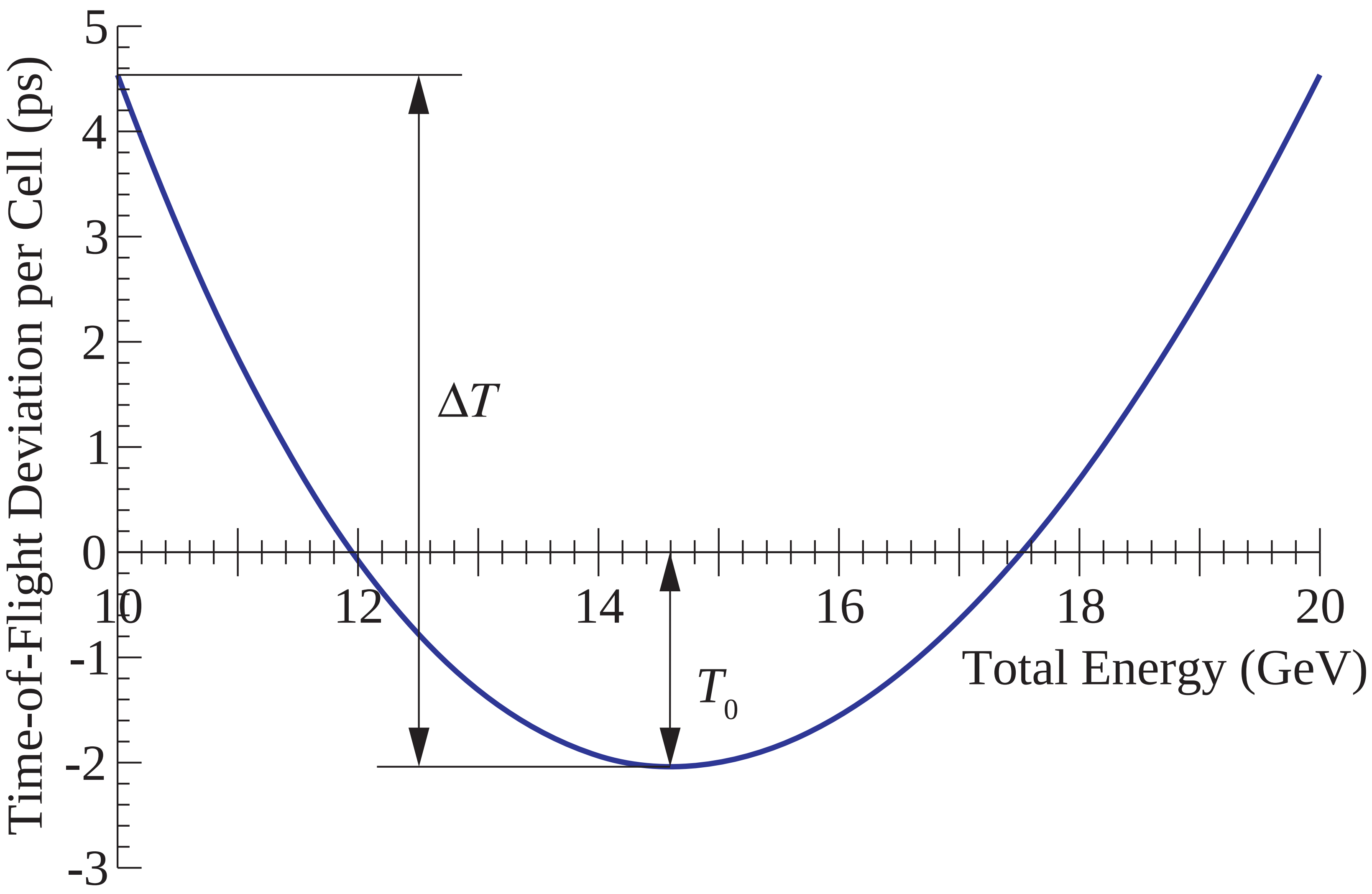
Fix RF Frequency, Accelerate Rapidly
Acceleration with Monotonic Time of Flight



Fix RF Frequency, Accelerate Rapidly Non-Scaling FFAGs

- Advantage of non-scaling FFAGs: can make derivative of time of flight zero at some energy for relativistic energies
- This minimizes the drift off of the RF crest
- Allows for the use of higher frequency RF
- Don't need as much RF as for scaling FFAGs
- Plus: cross crest three times instead of twice
- Some non-scaling FFAG designs make time of flight nearly constant
    - Requires large nonlinearities
    - Leads to problems with beam size transmitted, at least for muon applications

Non-Scaling FFAGs
Time of Flight



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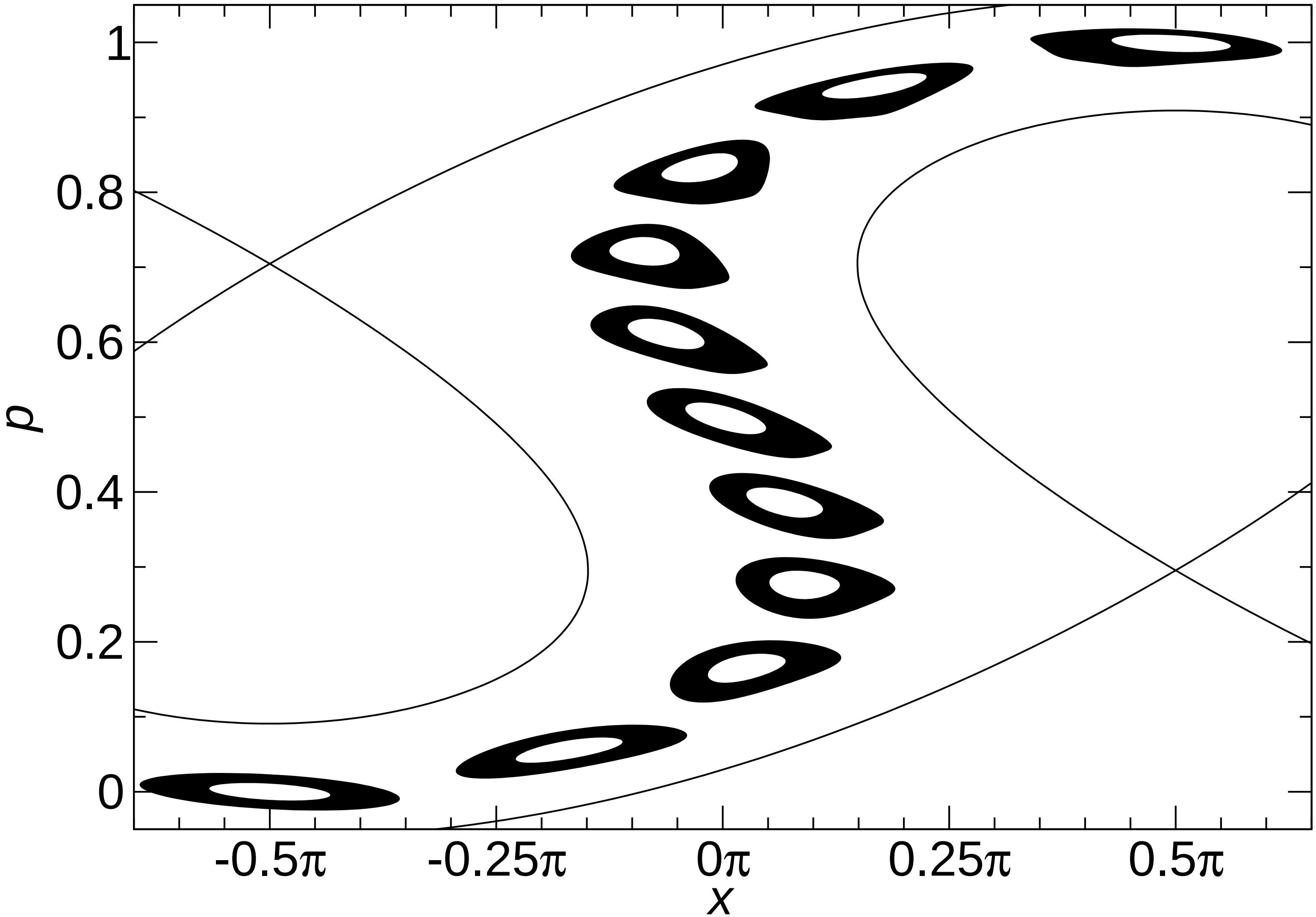
Non-Scaling FFAGs
Dynamics in Phase Space



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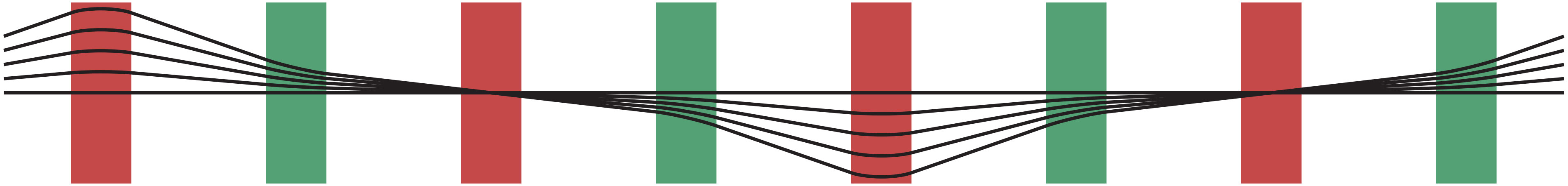
Fix RF Frequency, Accelerate Rapidly
Time of Flight Dependence on Transverse Amplitude

- Time of flight depends on transverse amplitude
- Reason: finite angles give longer geometrical path

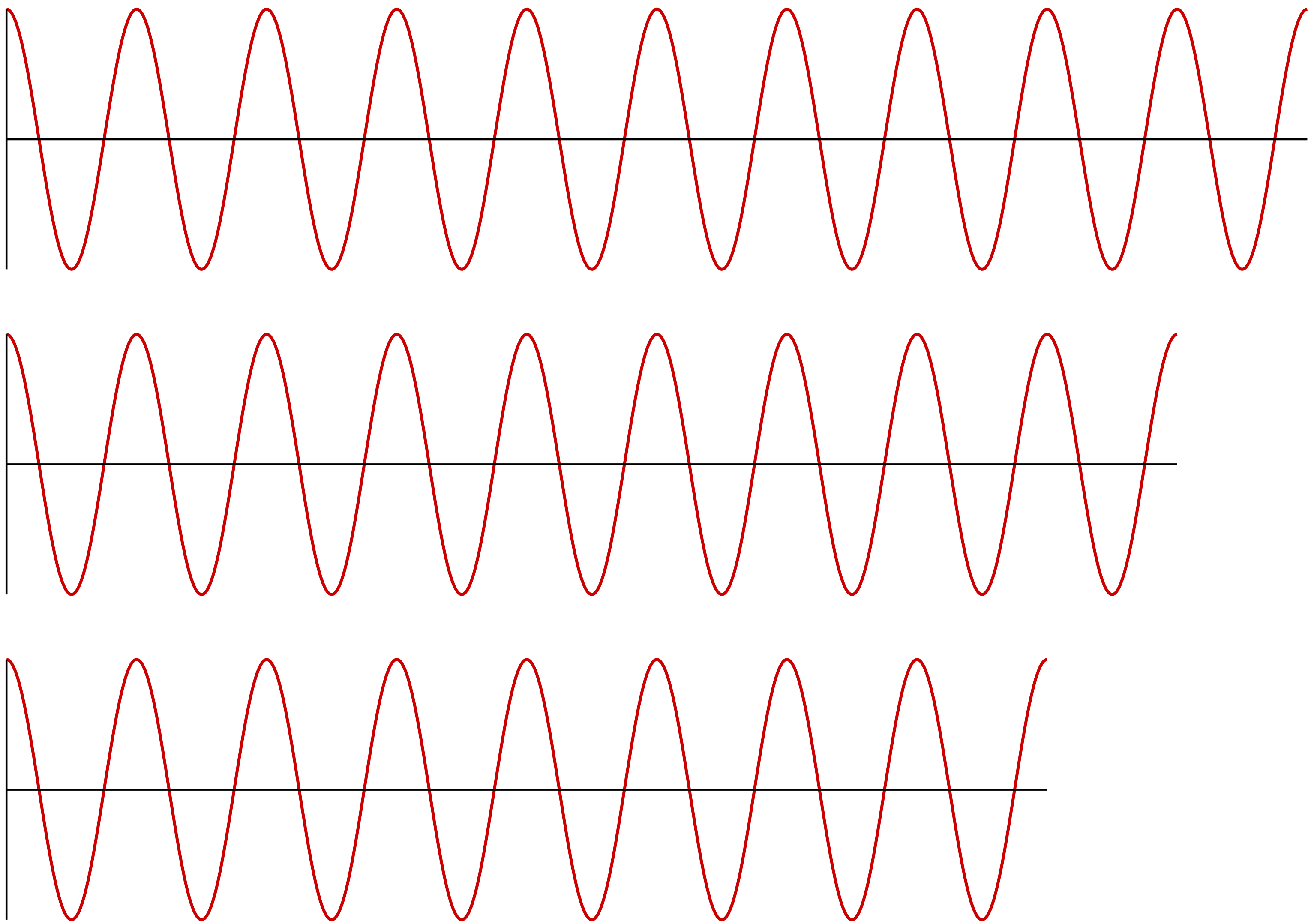


- Low amplitude particles synchronized to RF, high amplitude aren't
- Problem especially for muon machines with large transverse beam sizes
- Potential fixes (important research area!)
  - Add nonlinearities to correct this
    - Potentially reduces beam size transmitted!
  - Modified RF (higher harmonic)
  - Add RF voltage, increasing cost

Harmonic Number Jump

- RF frequency stays constant
- On each turn, the number of RF periods changes by 1 as the energy changes
- Must carefully control the energy gain on each turn
  - If energy gain per turn were linear, no problem; but it isn't
  - Need to vary energy gain with energy
    - Can't change RF: too fast
    - Cavity's energy gain could vary with position
    - Various RF manipulation tricks can be tried
    - Research area

Time of Flight Depends on Energy
Harmonic Number Jump



Relativistic Cyclotrons

- Cyclotrons have an energy-independent circulation period
  - Avoids RF synchronization problems!
- At low (non-relativistic) energies field is constant (B_0)
- To achieve relativistic energies, the field looks like

$$B(r) = \frac{B_0}{\sqrt{1 - \left(\frac{qB_0cr}{mc^2}\right)^2}}$$

  - $B(r) \to \infty$ at finite r
- Gradients get very large as energies become higher
  - Particles lose focusing vertically
- For applications suggesting cyclotrons but needing relativistic energies: consider FFAGs

Final Summary

- FFAGs are useful when rapid acceleration is needed
- The have advantages over cyclotrons because
  - They can have smaller apertures
  - They can more easily reach relativistic energies
- FFAGs must be carefully designed to avoid problems with resonances
  - These resonances prevent ordinary accelerators from having large energy spreads
- FFAGs must somehow deal with the fact that circulation time of the beam depends on energy
  - We have several methods for dealing with this