Theoretical and Simulation Treatment of Phase Rotation Channel

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Basic Premises

- Start with a distribution of pions in energy, $\rho(E_\pi)$
- Assume zero transverse momentum
- Assume equal probability of decay forward and backward in COM frame
- Look at distribution in energy-time of muons at distance L

Basic Formulas

- Note $c = 1$
- Compute pion energy/momentum from muon energy/momentum

$$E_{\pi,\pm} = \frac{E_\mu(m_\pi^2 + m_\mu^2) \mp p_\mu(m_\pi^2 - m_\mu^2)}{2m_\mu^2}$$

$$p_{\pi,\pm} = \frac{p_\mu(m_\pi^2 + m_\mu^2) \mp E_\mu(m_\pi^2 - m_\mu^2)}{2m_\mu^2}$$

  - Upper sign is for forward COM decay, lower is for backward

- Muon arrival time in terms of pion decay time

$$t_{\mu,\pm} = \frac{LE_\mu}{p_\mu} \pm t_\pi \frac{m_\pi^2 - m_\mu^2}{2E_{\pi,\pm} p_\mu}$$

Constructing a Distribution

- Decay time t_π distributed according to

$$(m_\pi/\tau_\pi E_\pi)e^{-t_\pi m_\pi/\tau_\pi E_\pi}$$

- Thus distribution in t_μ and E_μ is ($H(z) = 1$ for $z > 0$, 0 otherwise)

$$\sum_{\pm} \rho(E_{\pi\pm})\frac{dE_{\pi\pm}}{dE_\mu} H\left(\pm\left[t_\mu - \frac{LE_\mu}{p_\mu}\right]\right)\frac{1}{2\tau_\mu}e^{\mp(t_\mu - LE_\mu/p_\mu)/\tau_\mu}$$

$$\tau_\mu = \tau_\pi \frac{m_\pi^2 - m_\mu^2}{2m_\pi p_\mu}$$

Symmetriy of distribution

- Distribution would be symmetric about $t_\mu = LE_\mu/p_\mu$ were it not for the first two factors
- $\rho(E_{\pi\pm})$ weighted by $dE_{\pi\pm}/dE_\mu$

$$\frac{dE_\pi \pm}{dE_\mu} = \frac{p_\mu(m_\pi^2 + m_\mu^2) \mp E_\mu(m_\pi^2 - m_\mu^2)}{2m_\mu^2 p_\mu}$$

- Forward decays weighted below backward
- Lower energy πs contribute less for given $\rho(E_\pi)$

Computing Distribution Moments

- If $\rho(E_{\pi\pm})(dE_{\pi\pm}/dE_\mu)$ independent of $\pm$,

$$\sigma_T(E_\mu) = \tau_\pi \frac{m_\pi^2 - m_\mu^2}{\sqrt{2} m_\pi p_\mu}$$

- If instead one sign is missing,

$$\sigma_T(E_\mu) = \tau_\pi \frac{m_\pi^2 - m_\mu^2}{2 m_\pi p_\mu}$$

- Gives range of possible σ_T, upper and lower differ by a factor of $\sqrt{2}$

Computing Phase Space Area

- Assume uniform distribution in E_μ
- Assume we capture particles in a fixed energy range $[E_0, E_1]$
- Area computed as integral of σ_τ over E_μ:
- Upper bound:

$$\frac{\tau_\pi(m_\pi^2 - m_\mu^2)}{\sqrt{2}m_\pi}\int_{E_0}^{E_1}\frac{dE_\mu}{p_\mu} = \frac{\tau_\pi(m_\pi^2 - m_\mu^2)}{\sqrt{2}m_\pi}\ln\frac{E_1+p_1}{E_0+p_0}$$

- Similar for lower bound

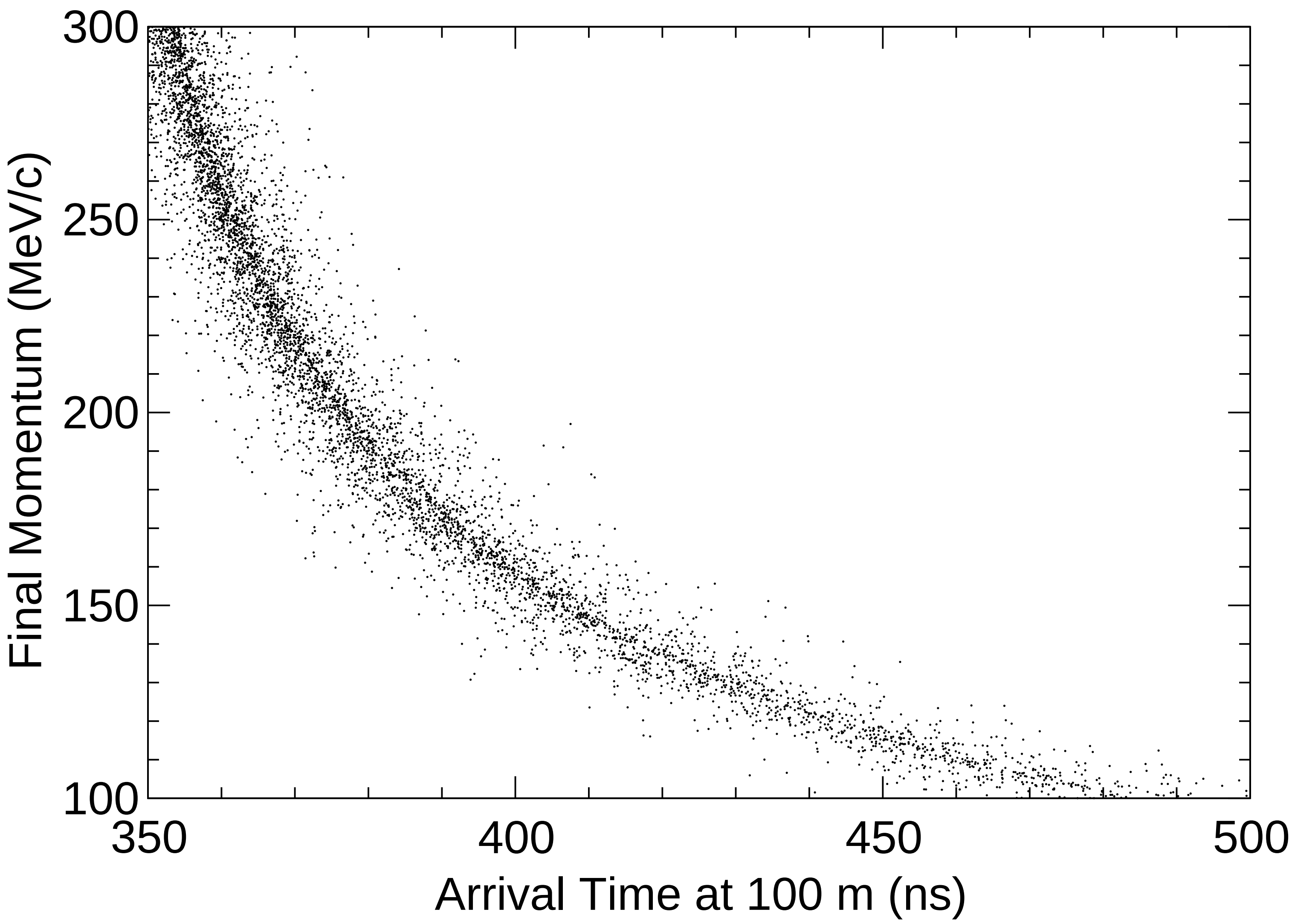
Adding a Finite Bunch Length on Target

- Arrival times get an additional Gaussian distribution added to them, RMS time σ_0
- Compute new $\sigma_\tau = \sqrt{\sigma_\tau^2(E_\mu) + \sigma_0^2}$
- Integrate over E_μ to get phase space area
- Plot inverse as a function of σ_0 to get efficiency

Simulation

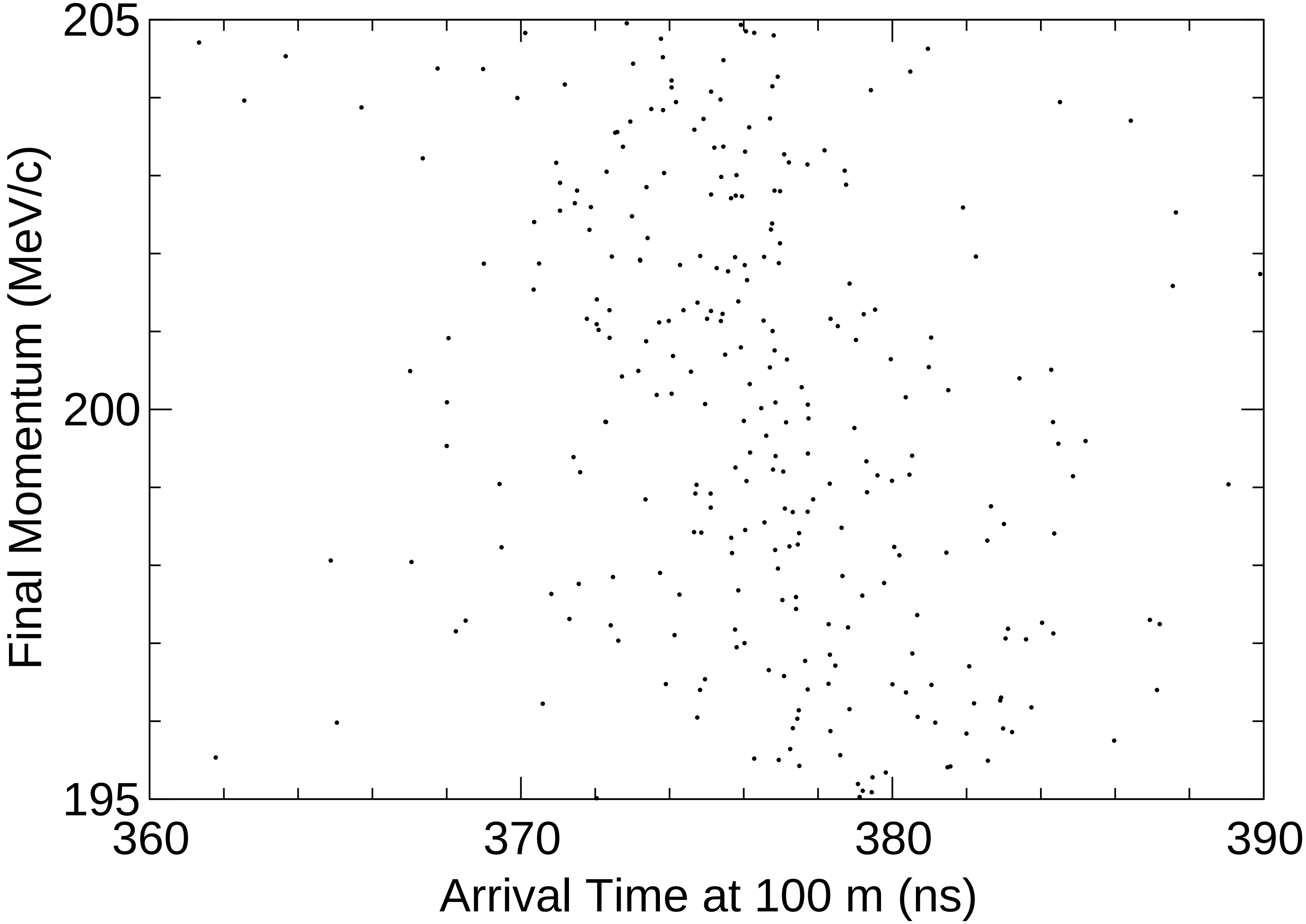
- Distribute initial times according to Gaussian
- Uniform initial energy distribution
- Do random decay times (exponential)
- Do random decay direction (forward/backward only) in COM frame
- Look only at final momentum 100 to 300 MeV/c

Distribution in Final Phase Space

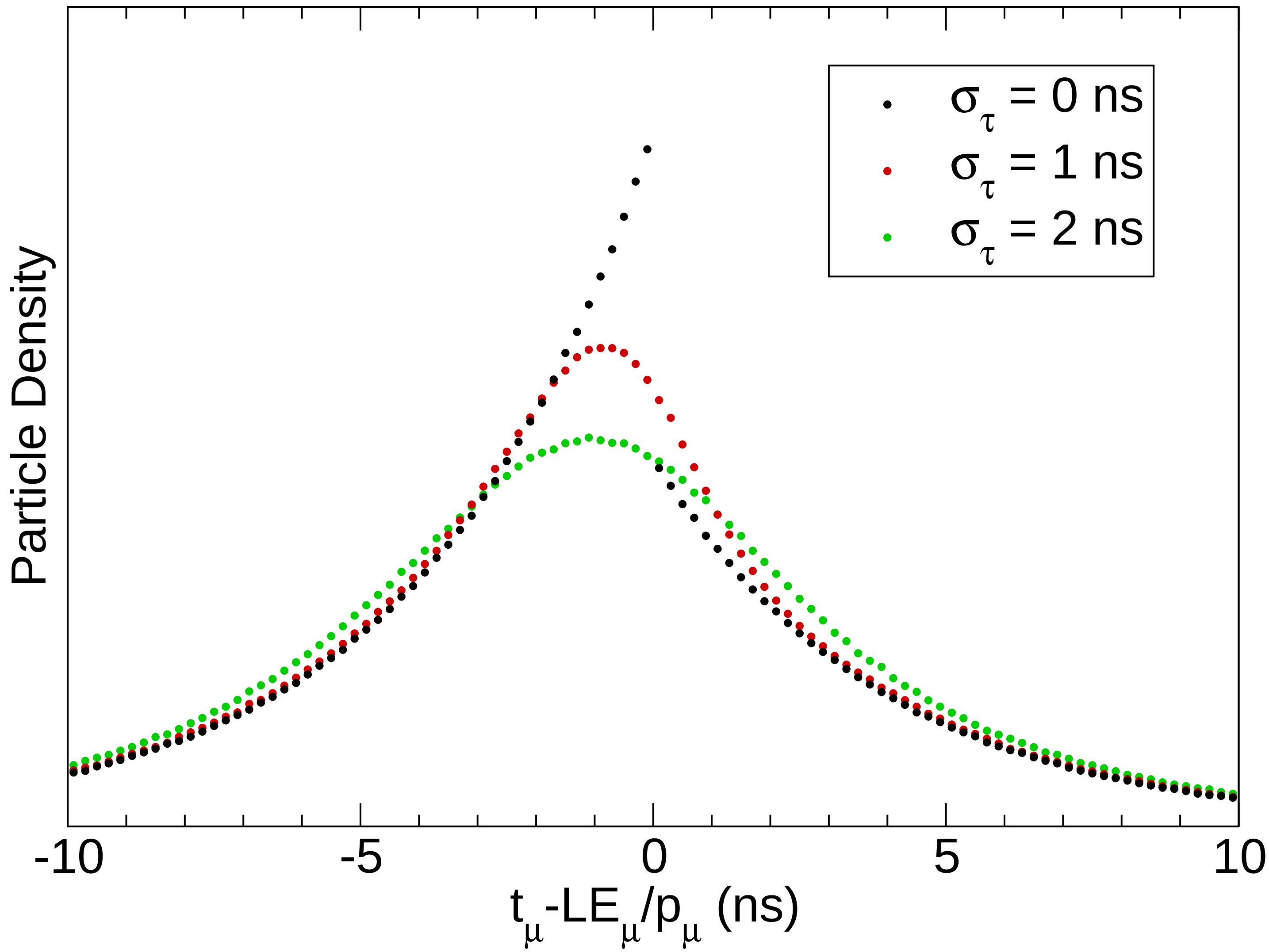


Distribution in Final Phase Space

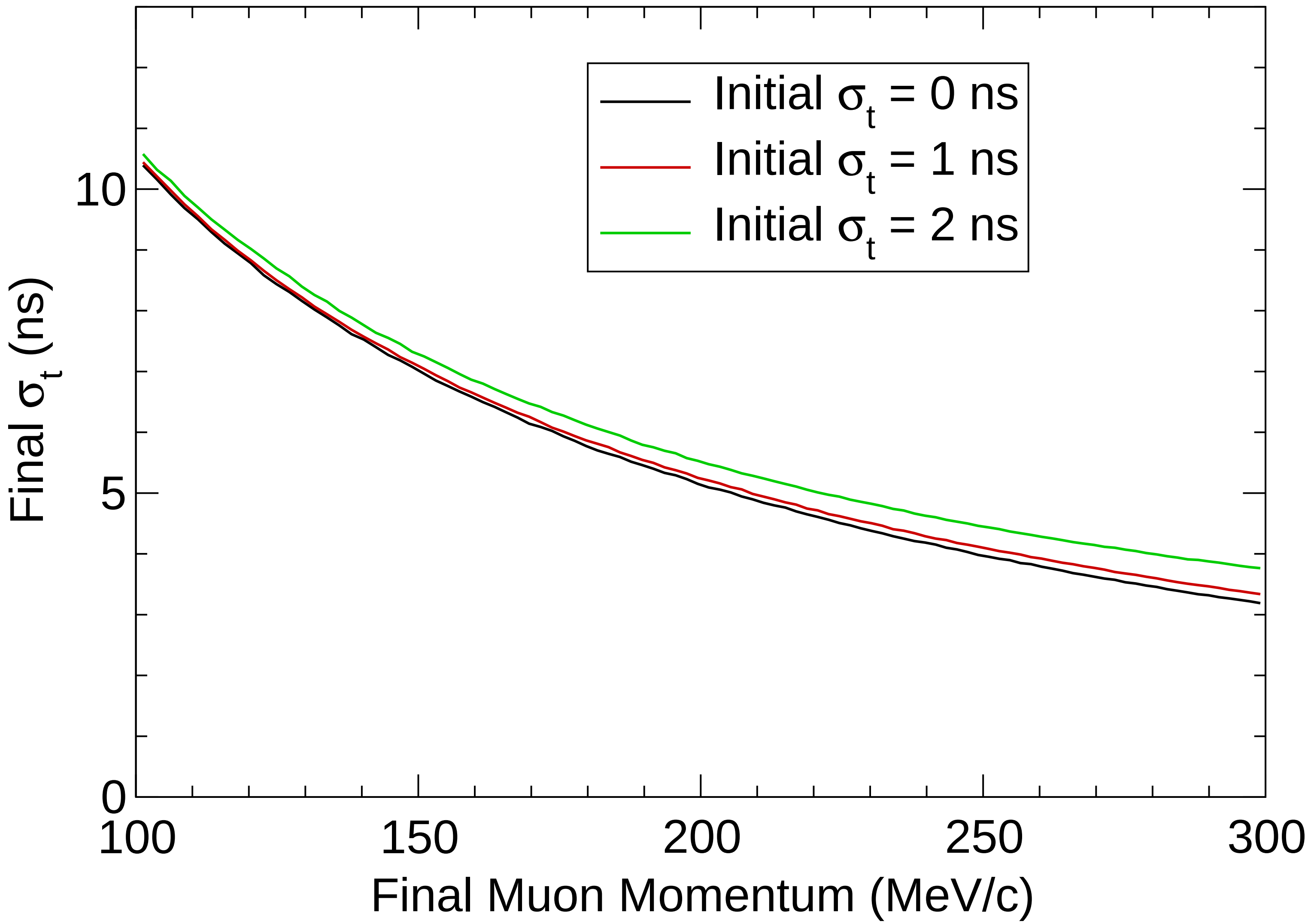
Zoom In Near 200 MeV/c



Distribution in Time



σ_T vs. Energy



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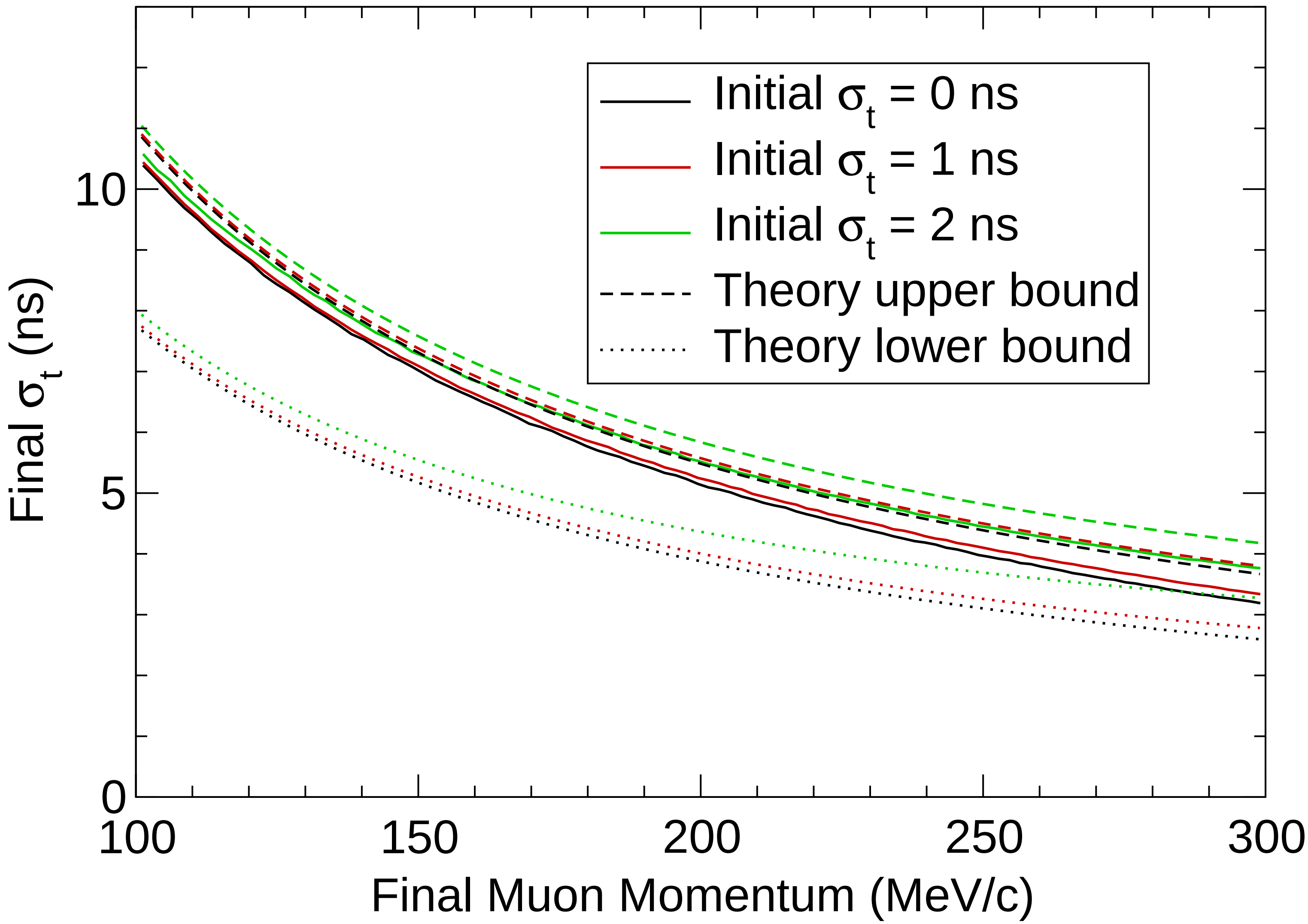
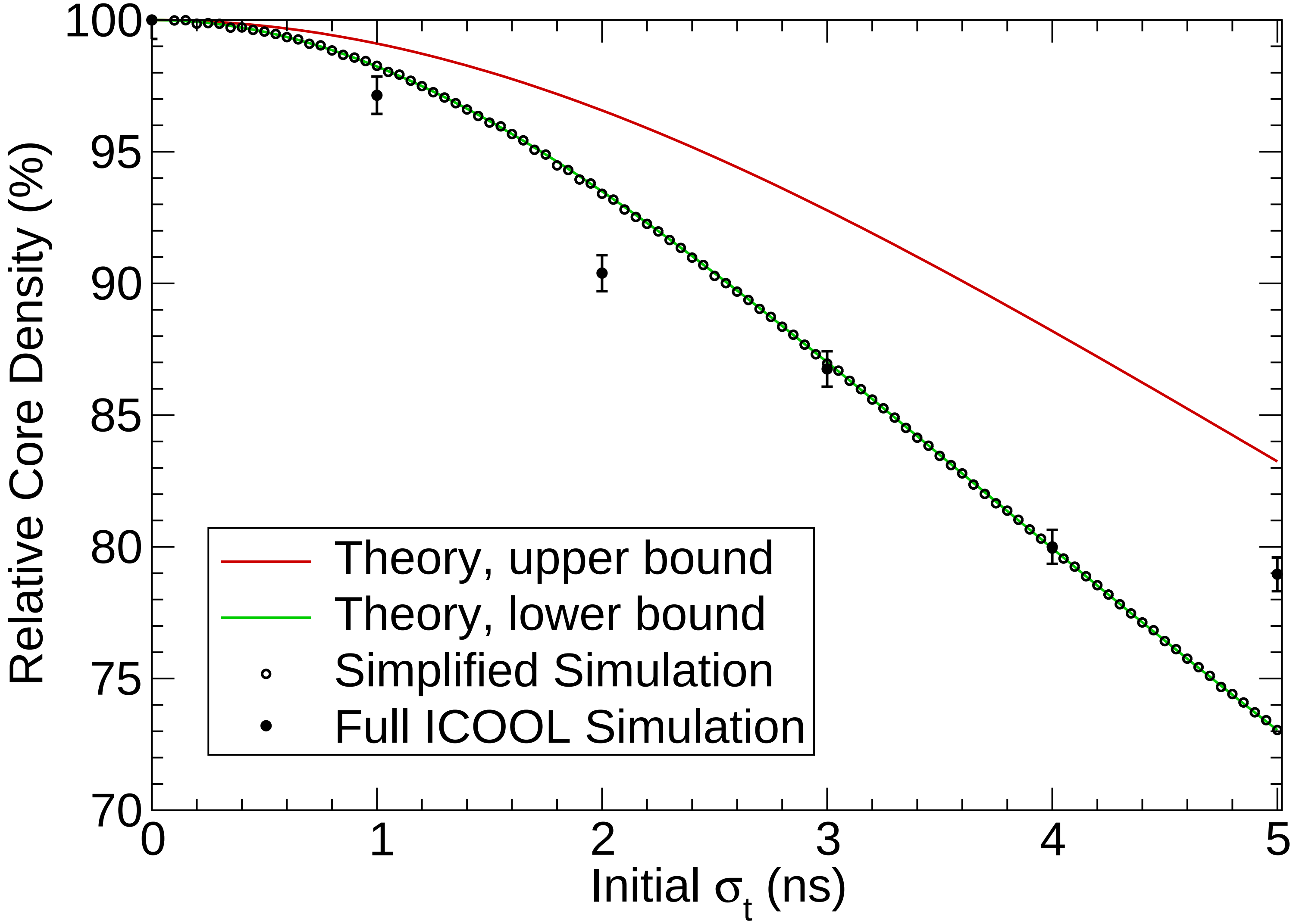
σ_T vs. Energy Theory Included



Estimated Capture Efficiency vs. σ_0



Discussion

- Significant difference, but agrees in middle of range
- Simulation close to lower bound
  - Asymmetry of $dE_{\pi\pm}/dE_\mu$
  - Adding RMS in quadrature not quite right
- Can potentially get different effect from phase rotation
  - Phase rotation gives largest energy spread on early times and high initial energy
  - Energy cut at capture will affect different pieces differently
- Could also try using real initial distribution of pions
  - However, since simulation matches theory lower bound, don't expect much

Perfect Phase Rotation

- Reduce E_μ by

$$\frac{m_\mu}{\sqrt{1-(L/t_\mu)^2}}$$

- Not really "perfect"
  - Should first leave high energy part with spread
  - Allow to stretch out more in time
  - Then rotate it down
  - Makes energy spread uniform in time

Phase Space After Perfect Rotation