Correct Tracking in FFAGs

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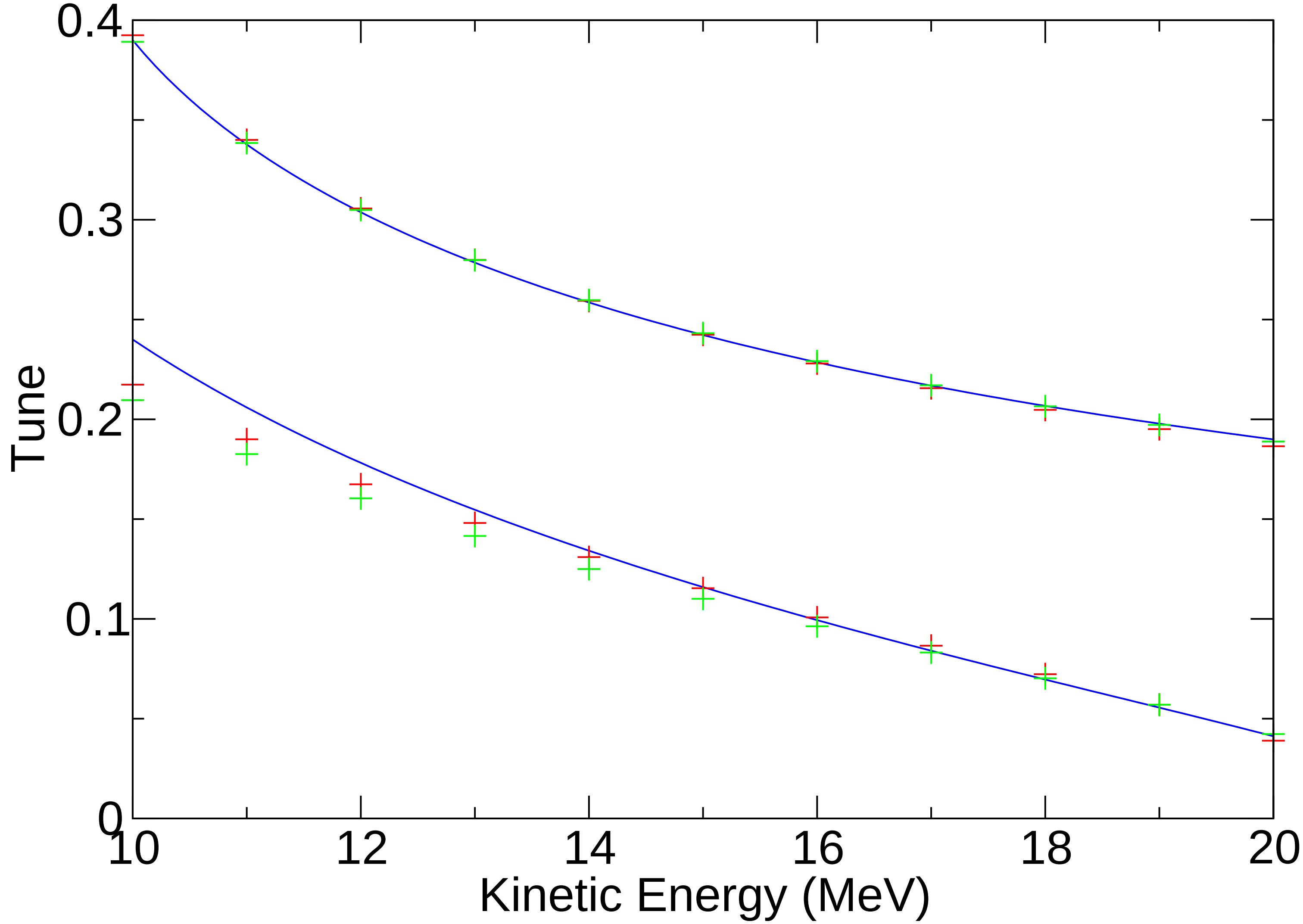
Outline

- Introduction
- Truncated power series
- Geometry
- Magnet end fields
- Distribution matching
- Conclusion

Introduction

- FFAGs break many of the approximations that are common in many accelerator design and simulation codes
- Much of this is realated to the large energy range they must cover
- Thus, many accelerator simulation codes will get incorrect answers
- Discuss some approximations used in codes, and how to handle things correctly

Introduction
Error in Tune Computation



Truncated Power Series (TPS)

- Most analysis of accelerators is based on TPS

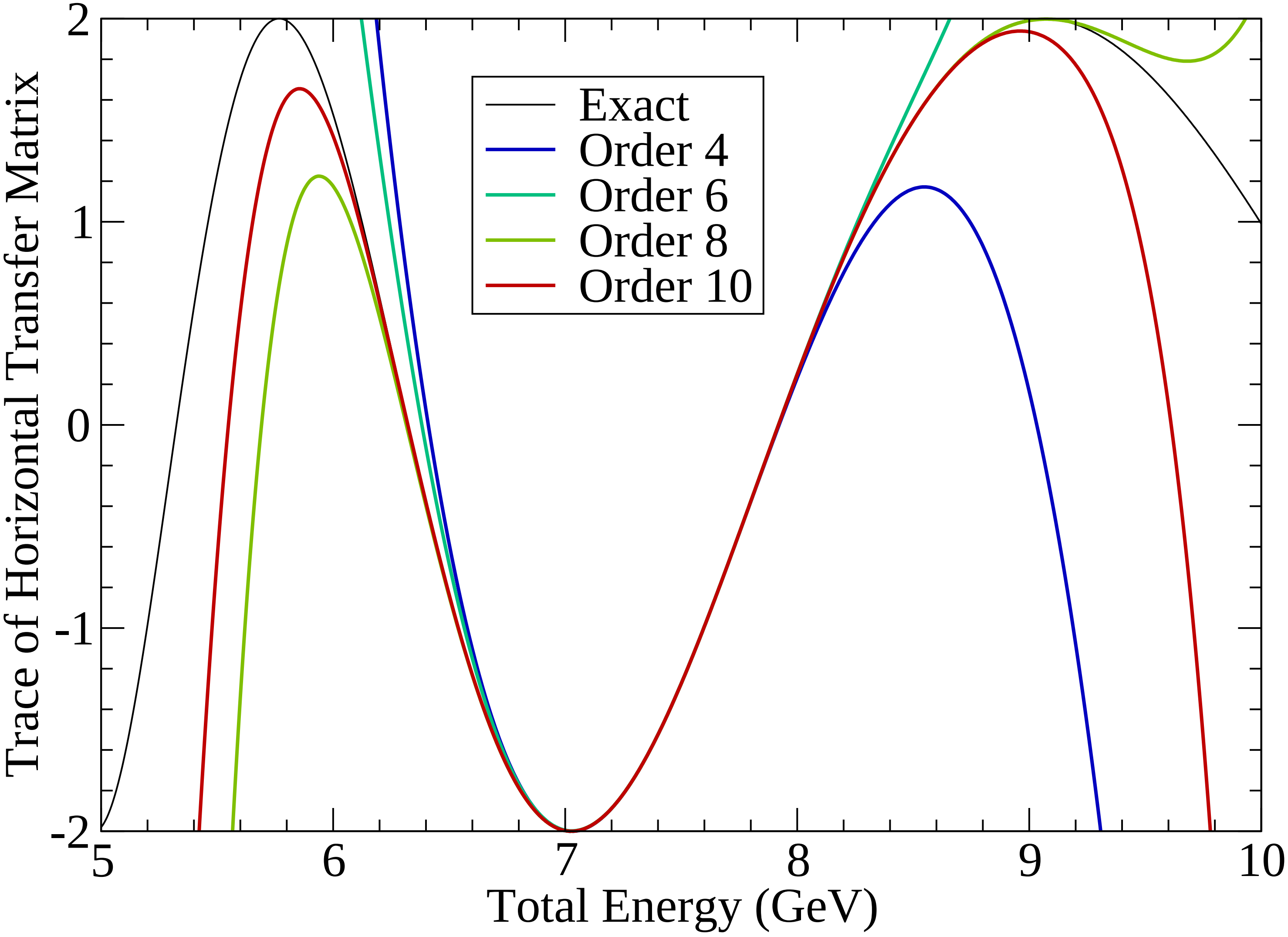
$$\sum_{|\boldsymbol{k}|\leqslant n} a_{\boldsymbol{k}} \prod_{i=1}^{6} z_i^{k_i}$$

- Linear map, needed for beta functions, tunes, dispersion, is set of first order TPS
- Magnetic field is represented (very accurately) as a TPS
  - Assuming you don't get too close to the coils/iron!
- TPS work when phase space variables (z_i) are small

Truncated Power Series (TPS)
TPS-Only Codes

- Dating back to TRANSPORT, and including modern codes like COSY, codes use a single TPS to represent the map through a section of beamline (or an entire ring)
  - The origin for that map is a so-called “reference orbit”
  - Generally, that orbit is the trajectory of an actual particle
- Problem: in FFAGs, phase space variables are not “small”
  - Primary problem: energy may vary a factor of 2 or more
  - Angles may be large (small ring, forward/backward bend)
  - Sometimes large beams (muon machines)
- Even if field well-represented by TPS, composition causes information loss
  - Composition of 2 nth order TPS, is $(2n)$th order TPS: truncation to nth order loses information

Truncated Power Series (TPS) Computing Tunes



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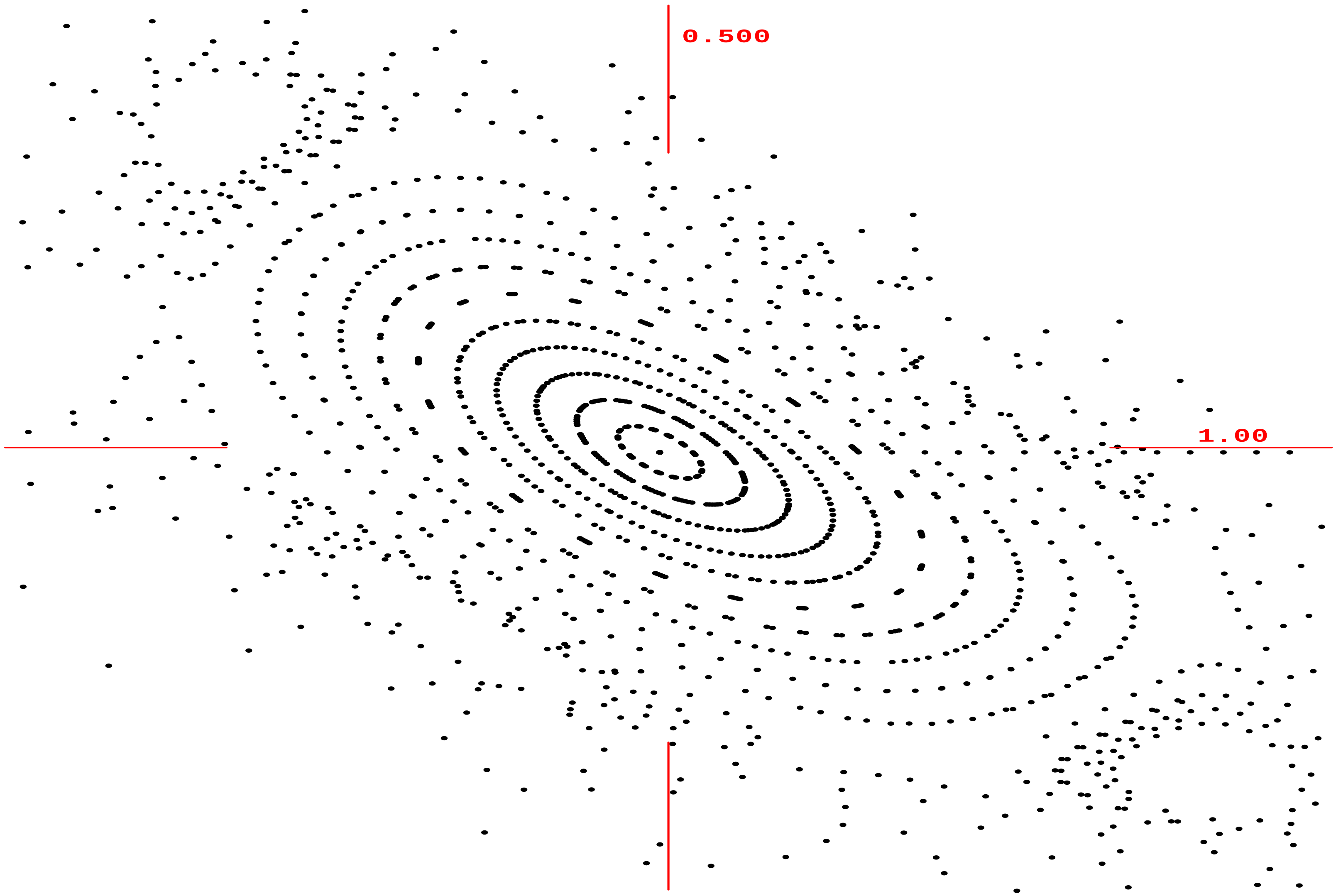
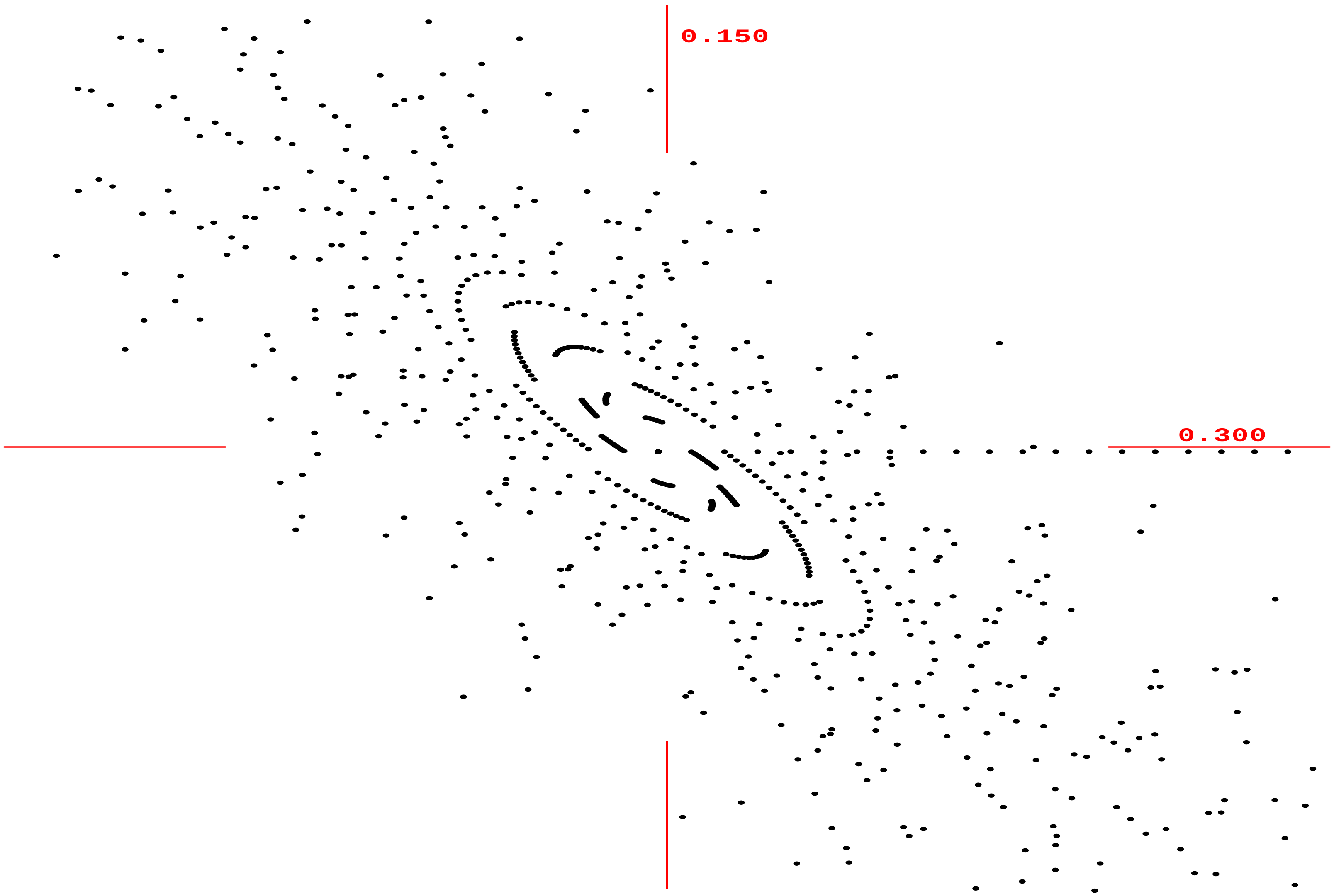
Truncated Power Series (TPS) Tracking On-Energy



Truncated Power Series (TPS)
Tracking Off-Energy



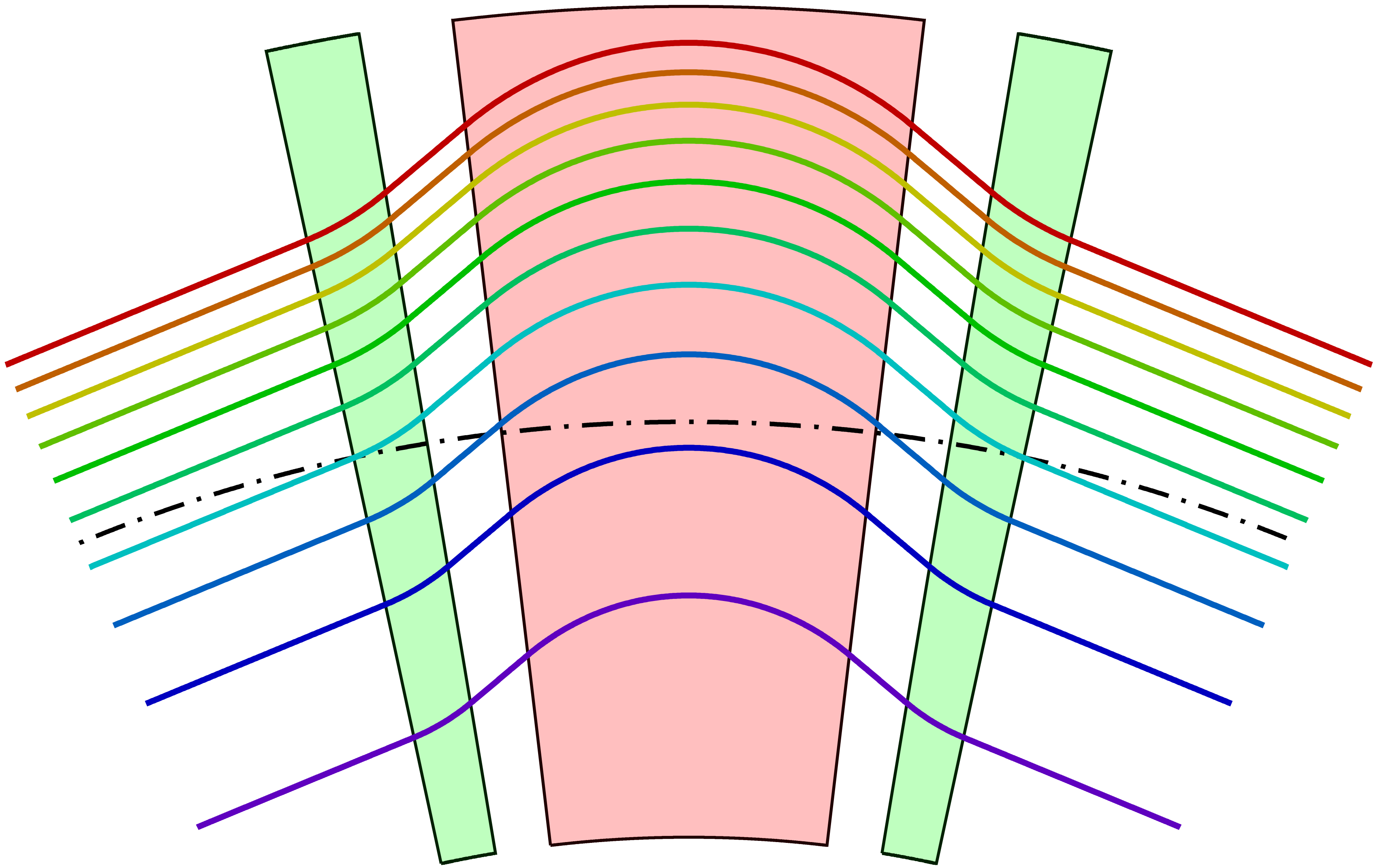
Truncated Power Series (TPS) Proper Usage in FFAGs

- Find closed orbit using exact Hamiltonian: tracking!
- Make a TPS about this closed orbit
- Use that for analysis (e.g., linear map to get tunes)
- Don't get too greedy. . .
- Some codes can't do this out of the box

Geometry

- Many codes define coordinate system by the motion of a "reference particle" following the field (MAD (sort of), COSY, ICOOL (some cases)).
- Making coordinate system follow reference particle problematic in some circumstances
  - Scaling FFAGs: fields defined in cylindrical coordinate system
  - Varying end fields changes geometry
  - Field errors will change geometry
  - Exact handling of rectangular bends
- Fix geometry, calculate closed orbit
- Étienne Forest has been telling us this for a while. . .

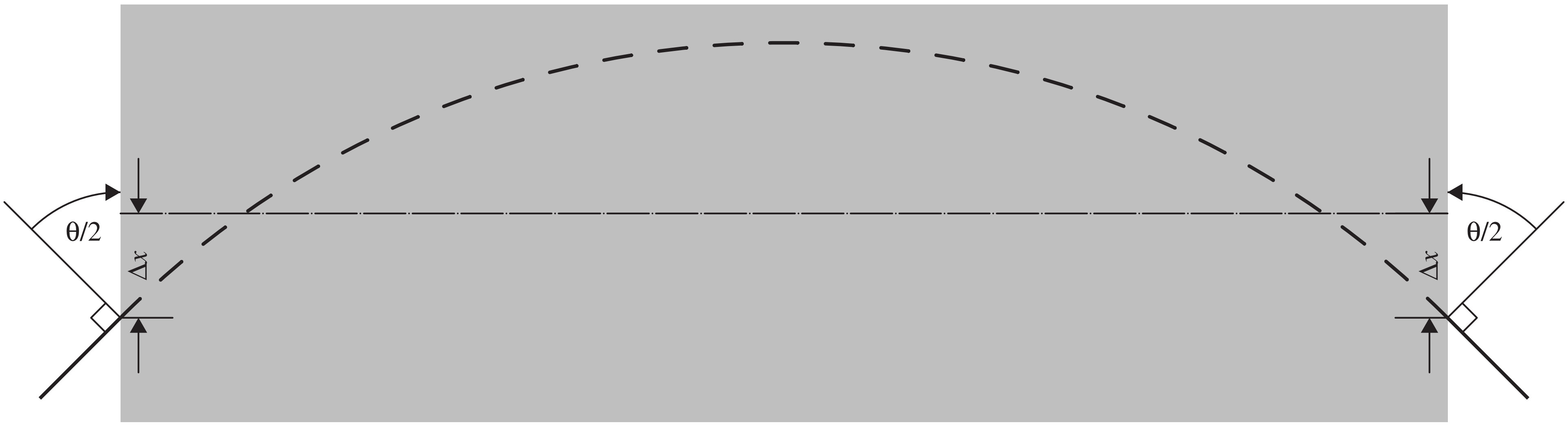
Geometry
Scaling FFAG



Geometry
Gradient Rectangular Bends

- Lines of constant field are straight
- No particle follows the arc of a circle (gradient nonzero)
  - Closed orbit at reference energy not at zero
  - Get close: integral of x along symmetric arc is zero
- Many codes represent as sector bend with rotated pole faces
- Different nonlinearities in sector and rectangular bend: like x/ρ, which can be large in FFAGs
- Want to integrate in coordinate system of magnet
  - Change to magnet coordinates as enter magnet

Geometry
Rectangular Bend



Geometry
Time of Flight

- If there is no "reference particle," how do you do time of flight?
- Simplest way to handle time of flight: use absolute time, not relative to "reference particle"
- If you want to use relative time anyhow:
  - Use constant velocity along zero axis of coordinate system
  - Add time jumps as necessary (e.g., rectangular bend)
    - Find closed orbit to compute what is needed
  - Note: this will not increase your precision over absolute time unless done extremely carefully
  - Alternative: compute closed orbit, use time along that (tricky)
- Adjusting frequency (zero of parabola)
  - Absolute time: easy, just do it
  - Relative time: advance cavity phase by a certain amount for each cavity; alternatively, change velocity

Magnet End Fields

- Real magnet fields don't suddenly go to zero: they fall off slowly
- Maxwell's eqauations lead to higher order fields: nonlinearities
- In linear non-scaling FFAGs, this is a main source of nonlinearity
- Approaches
  - Field map
  - Specify functional form, compute Maxwell's equations
  - Hard edge

Magnet End Fields
Field Map

- Best thing you can do when you have real magnets
- Requires running magnet design code: not ready at design stage
- Huge amount of data
- Functional form not smooth
- Getting interpolated form to satisfy Maxwell's equations
- Constructing vector potential: symplecticity

Magnet End Fields
Functional Form

- Instead, specify shape of field at end, either in midplane or of multipole components
- Use Maxwell's equations to get fields everywhere
- How fast does it fall off
- Symmetry: e.g., quadrupole
  - Could say field is $B_y(x, 0, z) = B_2(z)x$ in midplane
  - Or, say that to lowest order in transverse coordinates, field is $B_y(x, y, z) = B_2(z)x$, $B_x(x, y, z) = B_2(z)y$, with A_z having terms only of $\cos 2\theta$ angular symmetry
  - These are not equivalent!
  - Look to magnet symmetry (e.g., scaling FFAG, will try to maintain r^k in midplane)

Magnet End Fields
Hard Edge

- Hard edge approximation is a good method for early simulations
  - Fast to evaluate
  - Don't need to know anything about magnets
- We've always done it for bend magnets

$$\Delta p_y = \frac{q\Delta B_y y p_x}{p_s}$$

- To get fully correct (B_y specified in midplane), generate transform from Lie generator

$$\frac{q\Delta B_y(x) y^2 p_x}{2p_s}$$

Magnet End Fields
Hard Edge

- There are higher order terms also
- Note: this transforms coordinates, not just momentum!
- Similar symmetry issues to functional form end fields
- Simple example of where this is important: get tunes right
  - Use p_x on closed orbit, not magnet pole face angle only
  - Use B_y where particle enters magnet, not at origin

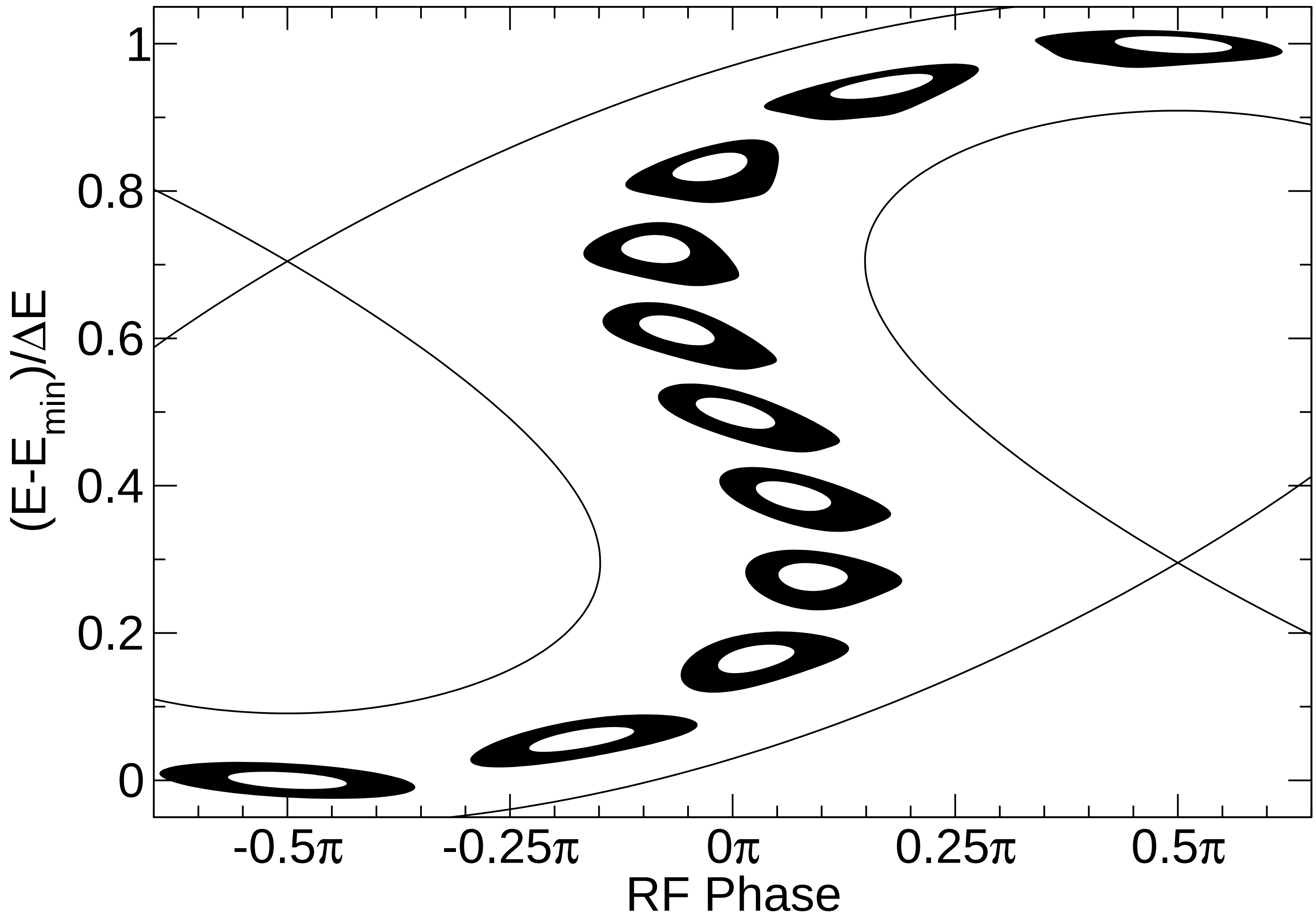
Distribution Matching

- Distribution injected into the machine should be matched to the machine
- Thus, your initial conditions when tracking must be matched as well
- Two issues that come up (especially for high-frequency gutter acceleration)
  - Orienting longitudinal ellipse properly
  - Dispersion and time-of-flight

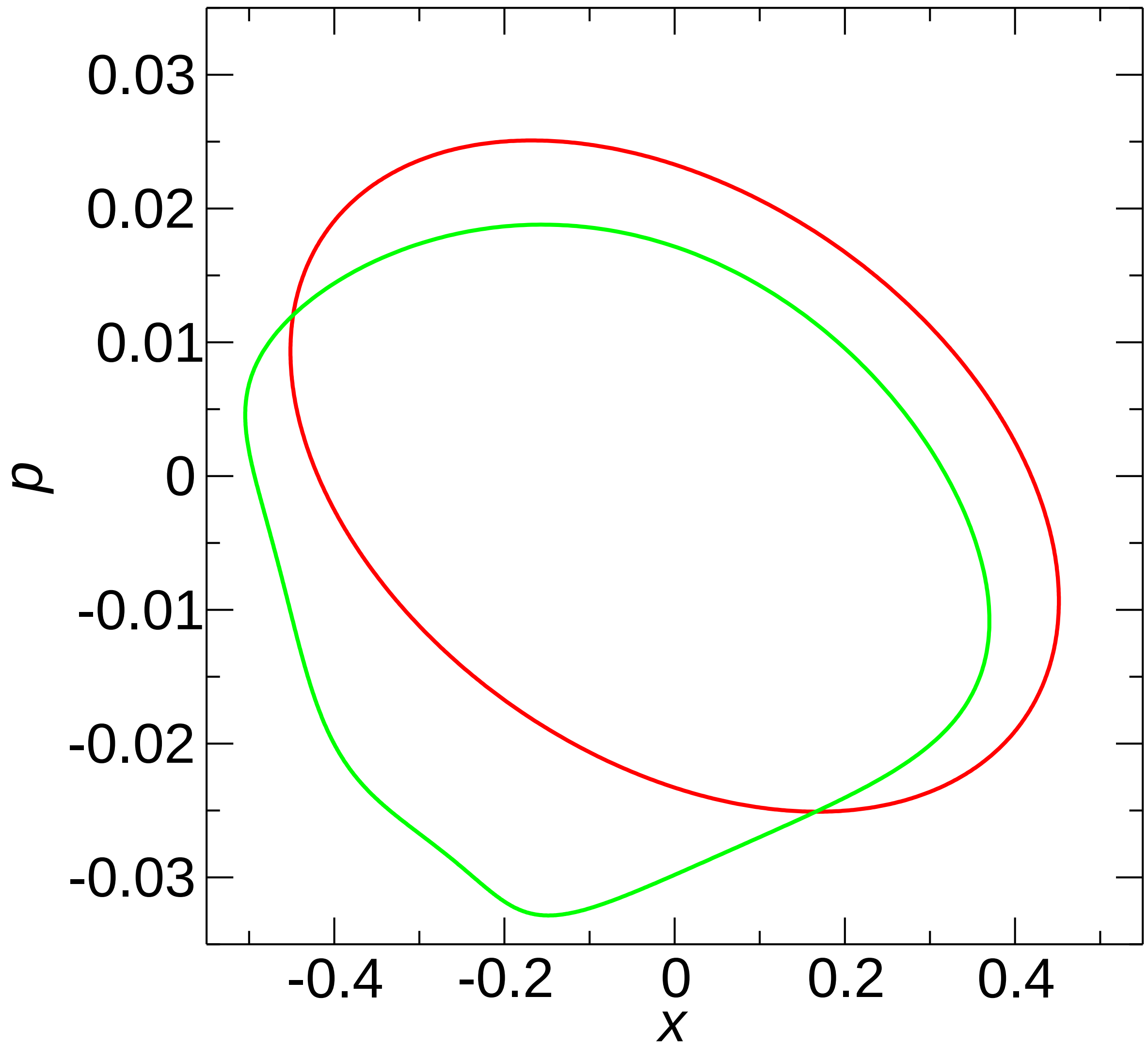
Distribution Matching
Orienting Longitudinal Ellipse

- As bunch accelerates, distribution is distorted
- Want ellipse to remain as elliptical as possible
- There is an orientation and aspect ratio that minimizes this ellipse distortion
- The ellipse orientation is NOT parallel to the phase space trajectories

Distribution Matching
Longitudinal Ellipse Evolution



Distribution Matching
Longitudinal Ellipse Distortion



Distribution Matching
Dispersion and Time of Flight

- Dispersion matching: put a correlation between position and energy, momentum and energy

$$x = \bar{x} + D_x\delta \qquad\qquad p = \bar{p} + D_p\delta$$

- Dispersion also affects the time of flight

$$\tau = \bar{\tau} + D_p\bar{x} - D_x\bar{p}$$

- In dispersion matched machine, this will get taken care of automatically
- In simulations, must put these correlations into initial conditions
- Becomes important in high-frequency FFAGs: RF period small
- Dispersion at cavities will add another twist. . .
- Important to simulate mismatch!

Conclusioon

- Truncated power series should be used with caution. Avoid a global TPS map
- Geometry should not be determined by a “reference particle” that follows the actual magnetic field
- Treating magnet ends correctly is important
- Your initial conditions should be a (nearly) matched distribution; this is slightly more complicated than normal