Integration Accuracy in ICOOL

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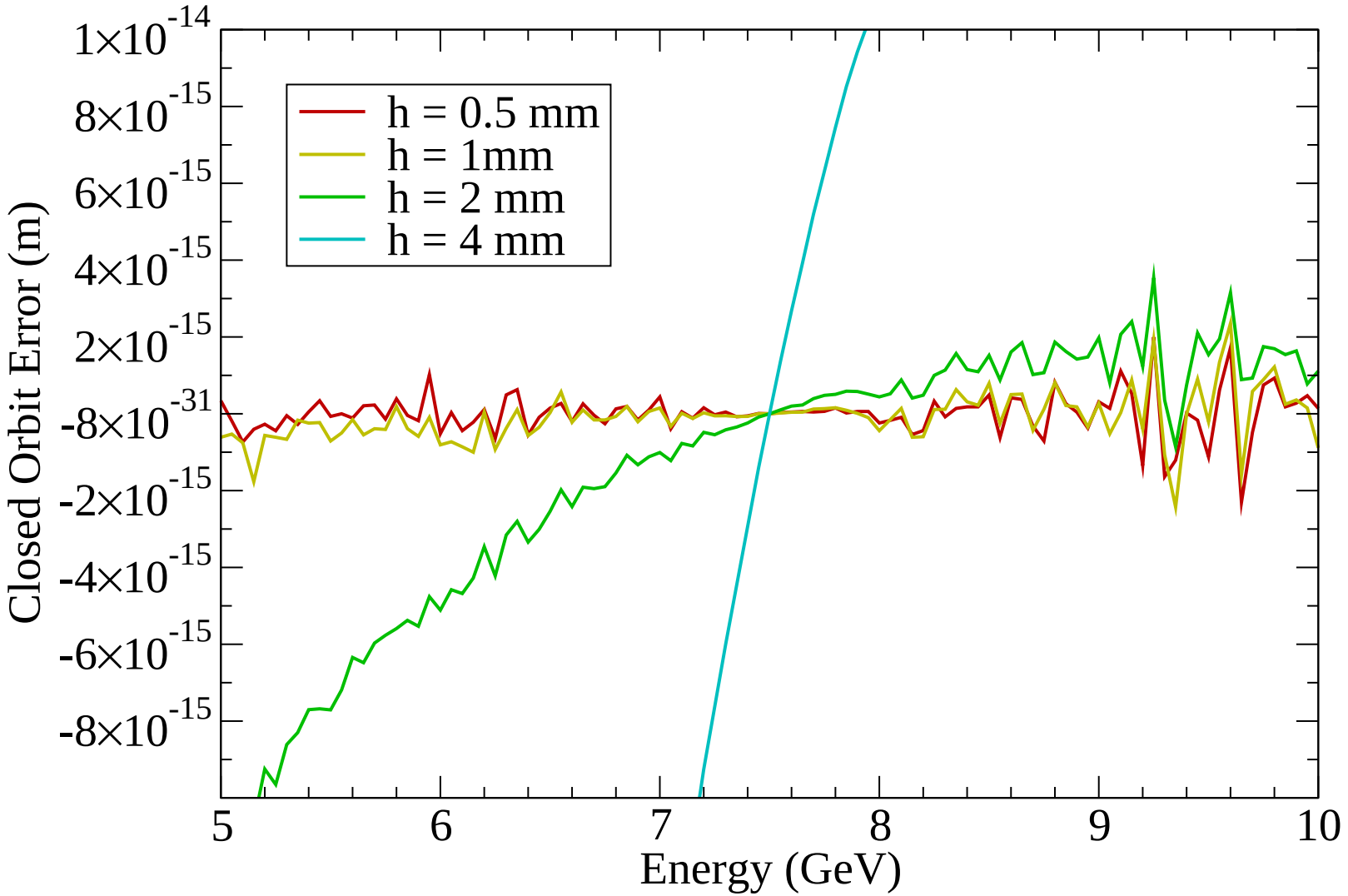
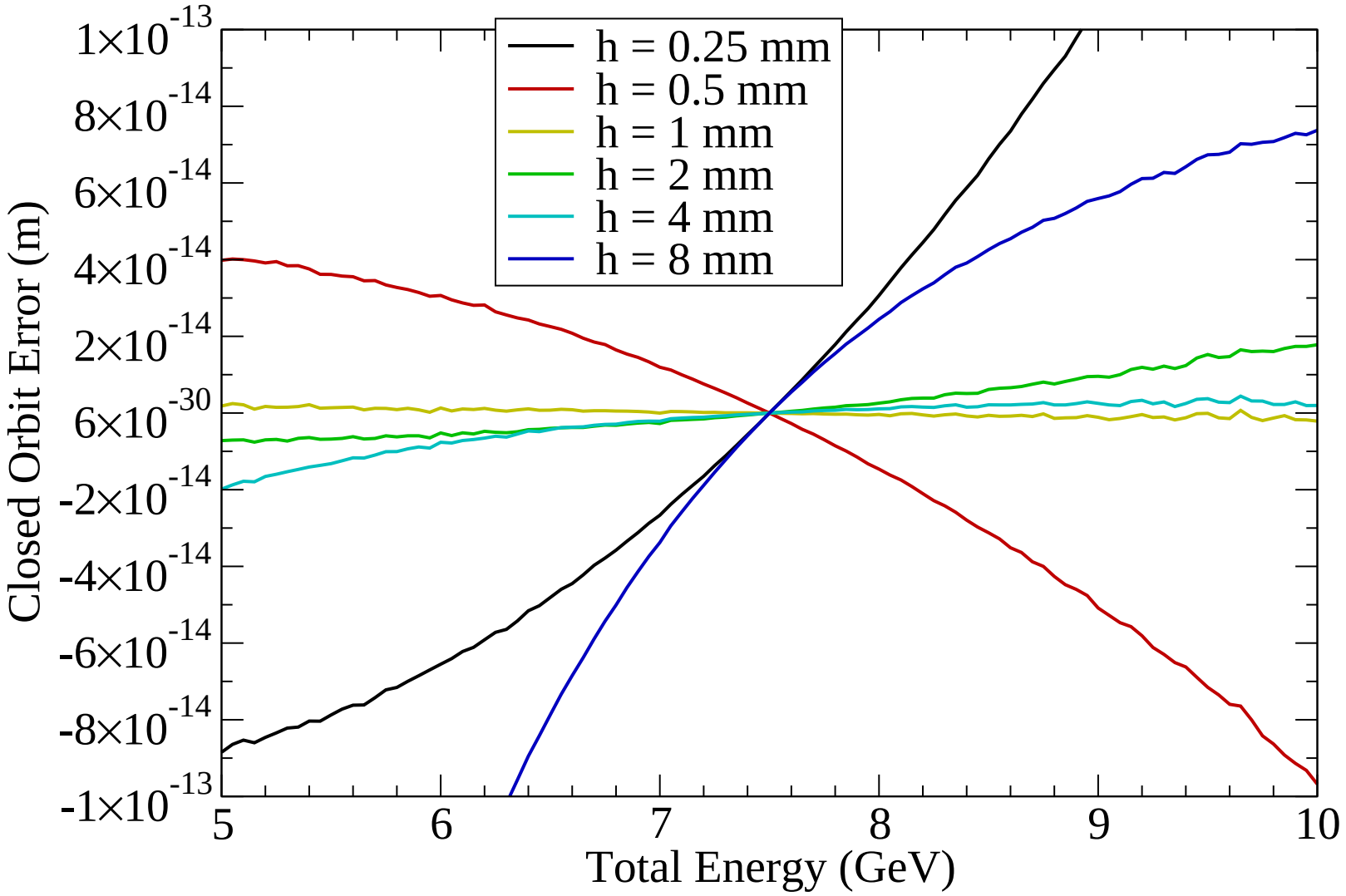
Introduction

- In checking ICOOL computations, I was finding small but slightly excessive errors in closed orbits, times of flight
  - Errors are larger than my code
- General behavior of integrator (4th order Runge-Kutta)
  - Large step sizes: proportional to h^4, systematic
    - Nearby system
  - Small step sizes: proportional to $h^{1/2}$, random
    - Roundoff, random walk

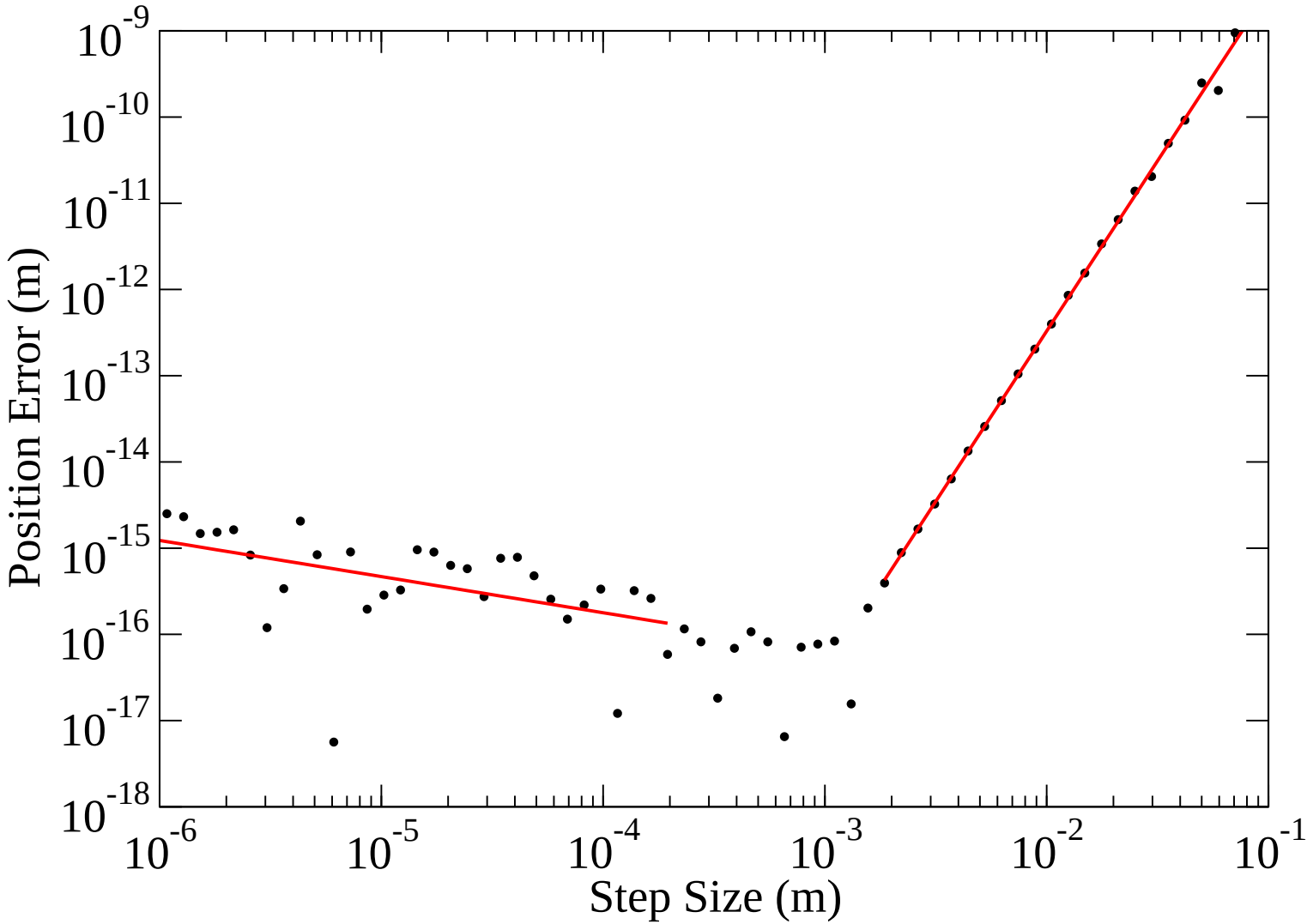
Closed Orbit Error vs. Step Size: Berg



Closed Orbit Error vs. Step Size: ICOOL



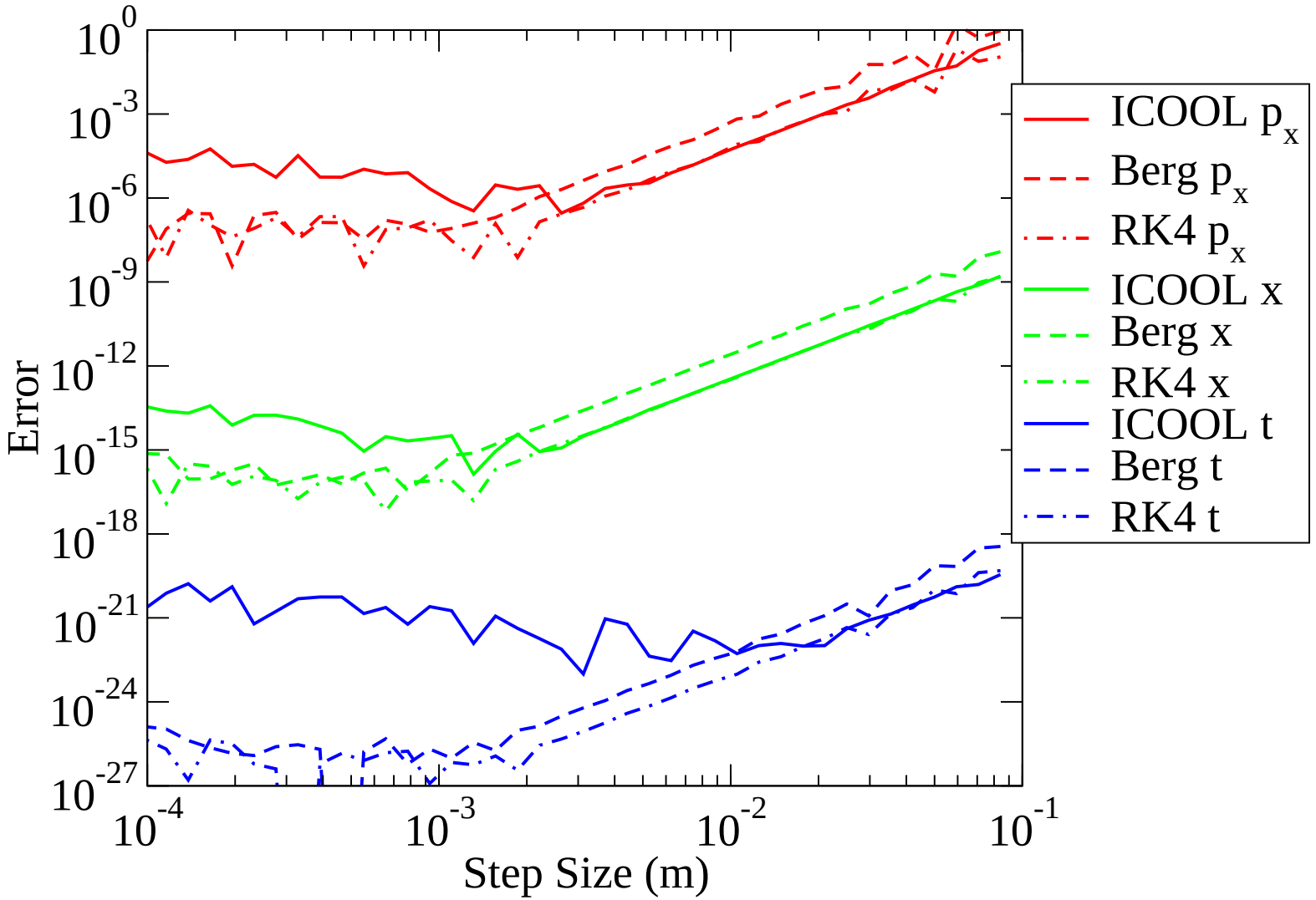
Integration Error vs. Step Size: Ideal



Analysis

- Comparing error vs. step size
  - My code worse than ICOOL for large step sizes (symplectic integrator)
  - Small step sizes, I do much better
  - If I do Runge-Kutta, I match ICOOL at larger step sizes, do better at small step sizes
- ICOOL power law dependence at small step sizes is wrong: $h^{-1.41\pm0.22}$
- Error is systematic vs. closed orbit at small step sizes: should be random
- Why worry? These are small!
  - I'm only doing one cell here
  - Sometimes you do really tiny step sizes without analysis
  - Should be understood for confidence in code
- ICOOL's time-of-flight error is larger: not computing relative to reference particle (trick to do right)

Integration Error vs. Step Size: All



Integration Error vs. Step Size: ICOOL