Acceleration: Goals and Constraints

- Accelerate to 20–50 GeV
- Cost: acceleration is one of the most expensive parts of the machine
- Transverse acceptance
  - Normalized acceptance of 30 mm
  - Maybe lower with more cooling, but cooling is expensive
- Bunches
  - US bunching/cooling scheme: arriving in 200 MHz RF buckets, 150 mm normalized longitudinal acceptance
  - May come as one larger bunch
- Minimize losses: requires high average gradient (> 1 MV/m)

Acceleration Rate

- Equation for decay

$$\frac{dN}{dt} = -\frac{mc^2}{E}\frac{1}{\tau}N$$

- With constant gradient $dE/ds = v$

$$\ln\frac{N}{N_0} = -\frac{mc^2}{c\tau v}\ln\frac{E+pc}{E_0+p_0c}$$

  - Example: accelerate 4–20 GeV, $\tau = 2.2\ \mu$s, $mc^2 = 105.658$ MeV, tolerate 20% decays, get $v = 1.2$ MV/m

- v is average gradient
  - Divide total voltage by total length
  - Total length includes drift spaces, focusing magnets, etc.

RF Basics

- Electromagnetic fields in cavity sinusoidal function of time: proportional to $\cos(\omega t + \phi)$
- Thus, energy gain is $V \cos(\omega t + \phi)$.
- Energy stored in the cavity is porportional to V^2:

$$U = \frac{V^2}{\omega R_S/Q}$$

  - R_S/Q depends only on geometry
  - For given shape, R_S/Q is independent of size (and thus frequency)
  - Additional weak velocity dependence: transit time factor

RF Basics: Wall Losses

- Wall losses also proportional to V^2, or equivalently U

$$P = \frac{V^2}{R_S} = \frac{\omega U}{Q}$$

  - R_S and Q are properties of the cavity structure
  - Tell you how much power must be supplied to achieve a given voltage
    - Higher Q, lower power requirement
    - Power cost is largely in peak power
    - Power required increases rapidly with voltage
  - Leads to a decay (and filling) time constant
  - Q reduces with increasing resistivity
- Can add external losses to reduce Q (allow power to propagate out of the cavity)
- Superconducting RF: very high Q

Acceleration: Methods

- Linac
- Ramping Synchrotron
- Recirculating Linear Accelerator
- Fixed Field Alternating Gradient (FFAG) accelerator

Acceleration: Linac

- Low frequency required for longitudinal acceptance
- Large aperture for transverse acceptance
- Leads to large, expensive structures: over 1.3 m outer diameter!
- Approximately $1.2 billion for 20 GeV, not counting magnets
- Reduce costs by considering systems that re-use linac: multiple passes

Beam Loading

- Cavity fill times long compared to acceleration time
  - At 1 MV/m, takes 70 μs to accelerate by 20 GeV
  - Room temperature fill times around 200 μs
  - Superconducing fill times around 2 ms
- Energy used to accelerate particles must be extracted from cavity
  - Example: 3×10^{12} muons, 18 MV energy gain per cavity. Energy loss 8.7 J.
  - Multiple pass system: extract 8.7 J at each pass
  - Compare stored energy: 1 kJ
  - To restore in 10 μs: 870 kW
  - Compare 510 kW to fill superconducting cavity
  - Large increase in power requirement, let voltage drop
- Voltage drops as stored energy drops
- Energy gain becomes current dependent

Acceleration: Ramping Synchrotron

- Angle that magnetic field bends is proportional to the magnetic field, inversely proportional to the momentum.
- If fields are increased proportionally to momentum as you accelerate, transverse dynamics are exactly the same at all energies
- Must accelerate rapidly to avoid decays
- Rough example:
  - 1 T max bending field at 20 GeV, requires 420 m of bend
  - 30 cm aperture, total stored energy is 12 MJ
    - Reduced beam emittance, lower stored energy
  - Ramp up from 4 to 20 GeV in 50 μs, requires 190 GW!!!
    - Like kicker power supply, but a lot more
  - Just the average power is 180 MW (15 Hz rep rate)

Acceleration: Ramping Synchrotron

- Eddy currents
  - Changing magnetic flux, induces currents
  - Currents induce additional undesired magnetic fields
  - Currents lead to resistive losses
    - ⋆ More power required to generate fields
    - ⋆ Heating of magnets: want to dissipate pulsed power elsewhere
  - Produce magnets using very thin laminations, blocking current path
- Situations where this may be practical
  - Smaller emittance, lower aperture (less stored energy and power)
  - Higher energy:
    - ⋆ More time to ramp magnets, less power needed
    - ⋆ Adiabatic damping, smaller beam size, smaller aperture

Acceleration: Recirculating Linac

- Make racetrack shaped machine, with multiple arcs connecting two linacs
- Bending magnet directs beam into a different arc at each pass, depending on energy
- Arc costs proportional number of passes
- Linac costs inversely proportional number of passes
- Cost formula: optimum when linac cost equal to arc cost

$$\frac{C_L}{n} + C_A n$$

- Optimum often at large number of passes
  - Complex switchyard difficult to make
  - Lowest energy from one pass cannot overlap highest energy from previous pass
  - Extra energy gap needed for coils, etc.
  - Finite transverse beam size requires even more space

Acceleration: FFAGs

- Problems with previous solutions
  - Magnets ramped too quickly in synchrotron
  - Limited number of passes in recirculating linac, mainly because of switchyard
- **F**ixed **F**ield **A**lternating **G**radient accelerator addresses these problems
  - Arc has an extremely wide energy acceptance: factor of 2 or more in energy
  - Magnetic fields do not vary with time
  - No obvious reason not to go however many turns you like, constrained by
    - Beam loading
    - Decay

FFAGs: Energy-dependent parameters

- Consist of a sequence of simple, identical cells
- Cell has a “closed orbit” which depends on energy
- At a given energy, particles oscillate about this energy-dependent closed orbit
- Accelerating cavities distributed uniformly around ring
- Adiabatically follow closed orbit
- Tunes, beta functions, time-of-flight depend on energy
- Acceleration still relatively rapid
  - Accelerate rapidly through nonlinear resonances and multi-cell linear resonances (strengths are low)
  - Single-cell linear resonances still matter

FFAGs: General Orbit Properties

- Range of closed orbit positions
  - Want small to keep apertures down
  - Decreases with increasing bend radius (increased ring length)
  - Decreases with shorter cell lengths
- Range of time-of-flight
  - Cannot shift RF phase during acceleration: too much power
  - Time-of-flight depends on energy: can't remain synchronous with RF
  - Prevents going for large number of turns: walk off crest
    - More precisely: leads to minimum amount of RF needed to accelerate
  - Reduced when closed orbit range reduced

FFAGs: Scaling

- Avoid single-cell linear resonances by having constant tune
- Constant momentum compaction
- Use highly nonlinear magnets
  - Midplane B_y proportional to r^k
  - Larger k improves linear properties
    - Smaller dispersion: simplistically, find $x(p)$ that keeps B_y/p fixed
    - Smaller time-of-flight range, since smaller dispersion
      - Momentum compaction is $1/(1+k)$
  - Note one does not accelerate through nonlinear resonances
  - Dynamic aperture worse with increasing k

FFAGs: Scaling, Longitudinal Dynamics

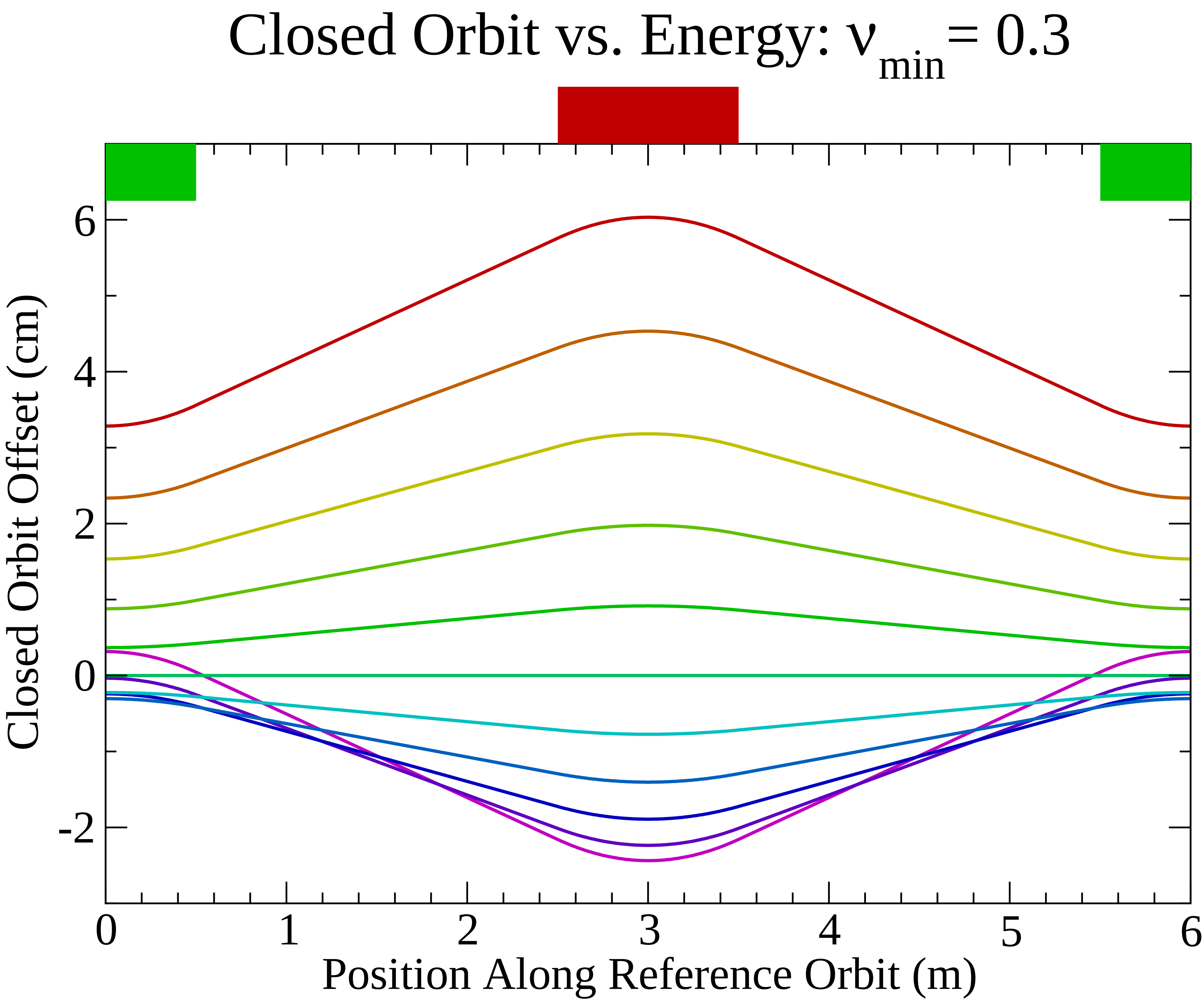
- Half oscillation in RF bucket
- Bucket height approximately

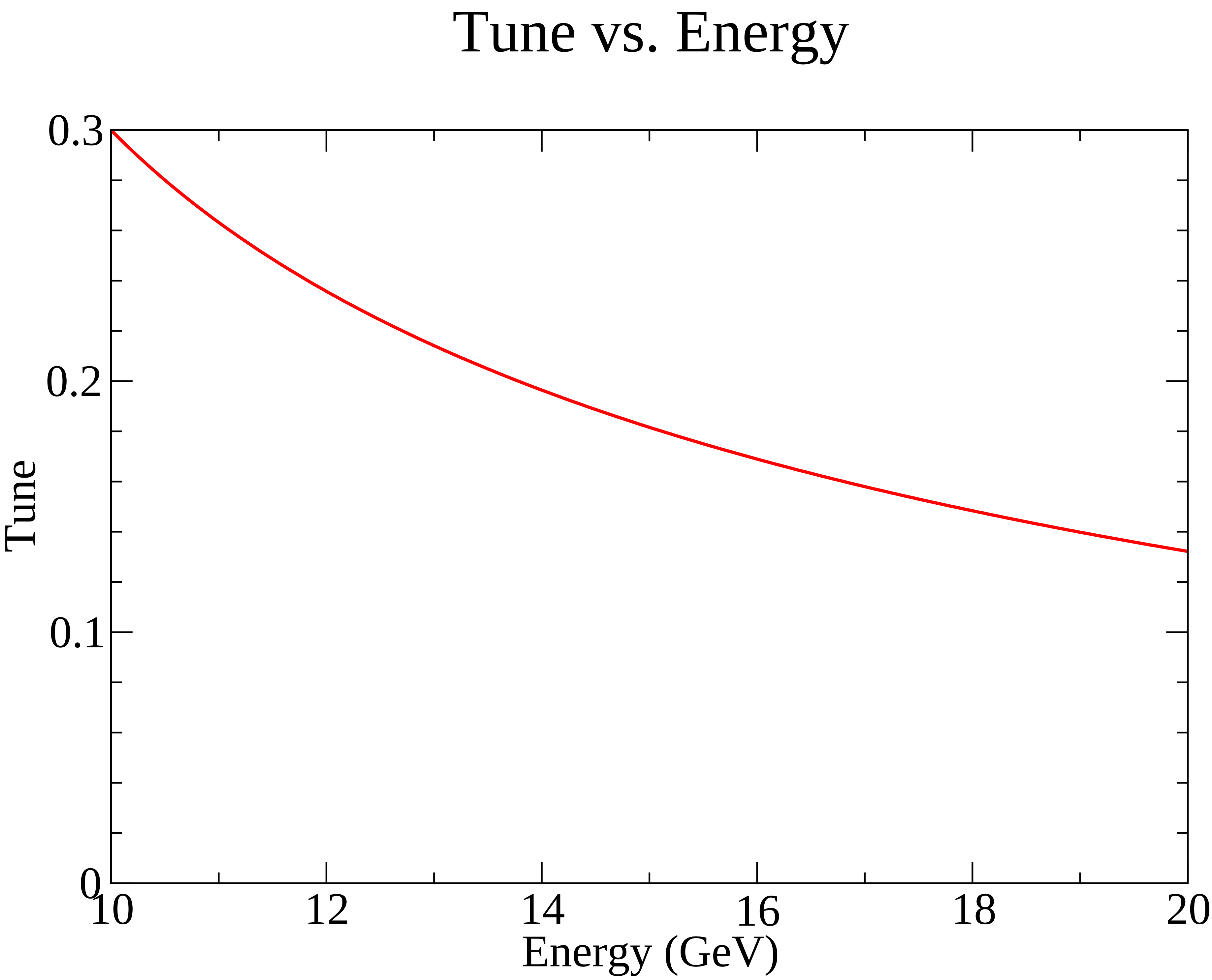
$$4\sqrt{\frac{vcE_0}{\omega\alpha_C}}$$

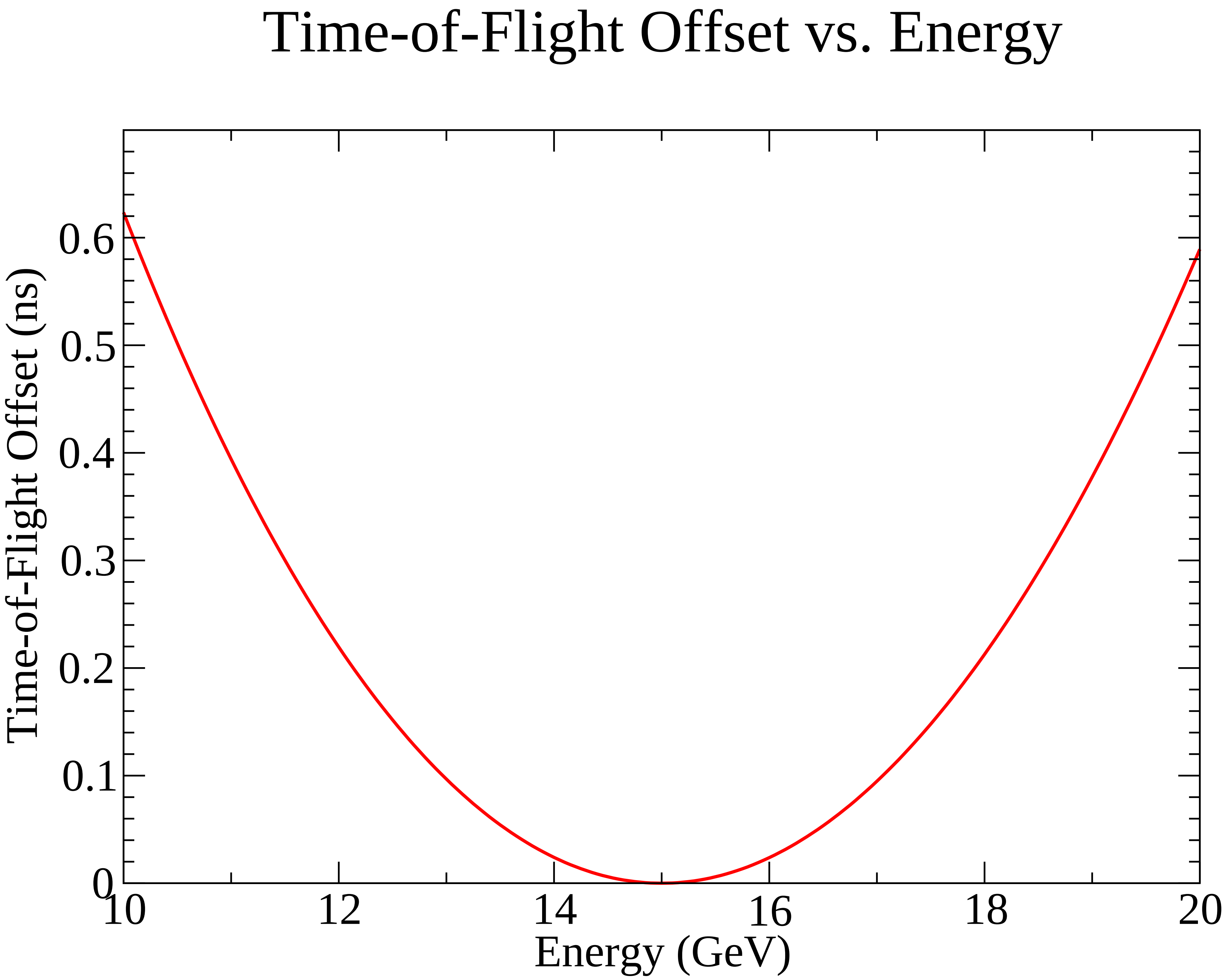
  - Lower frequency, less voltage required
  - Higher k (smaller α_C), smaller voltage required

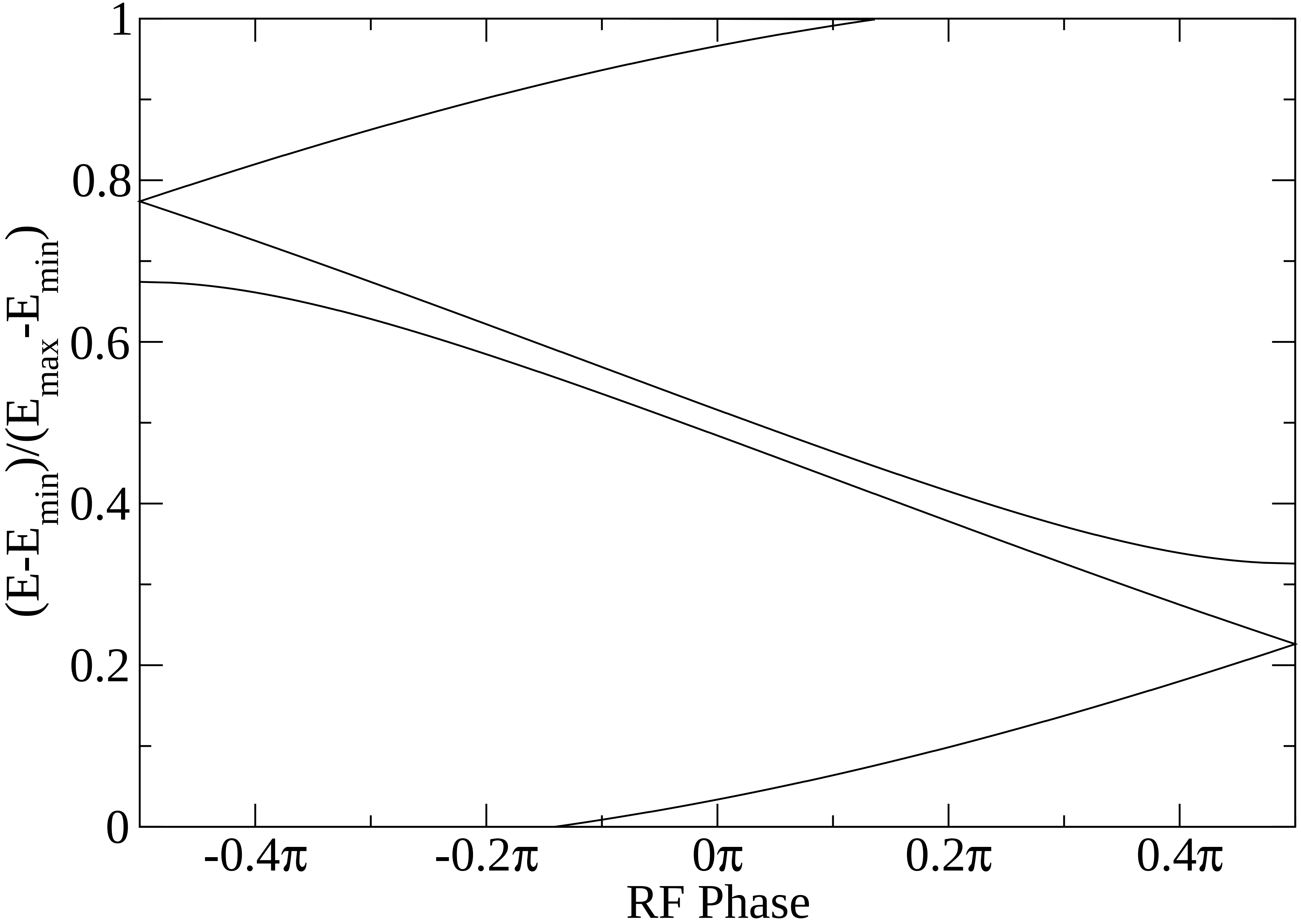
FFAGs: Non-scaling

- Use simple cells: FODO, Triplet
  - Highly linear lattice, good dynamic aperture
- Avoid single-cell linear resonance by keeping tunes below 0.5 at low energy
  - Tune decreases with increasing energy
- Parabolic time-of-flight dependence on energy
  - Longitudinal motion: crosses crest three times
  - Minimum voltage required to achieve this
    - Proportional to time-of-flight range
  - Extra voltage required to get decent phase space volume through

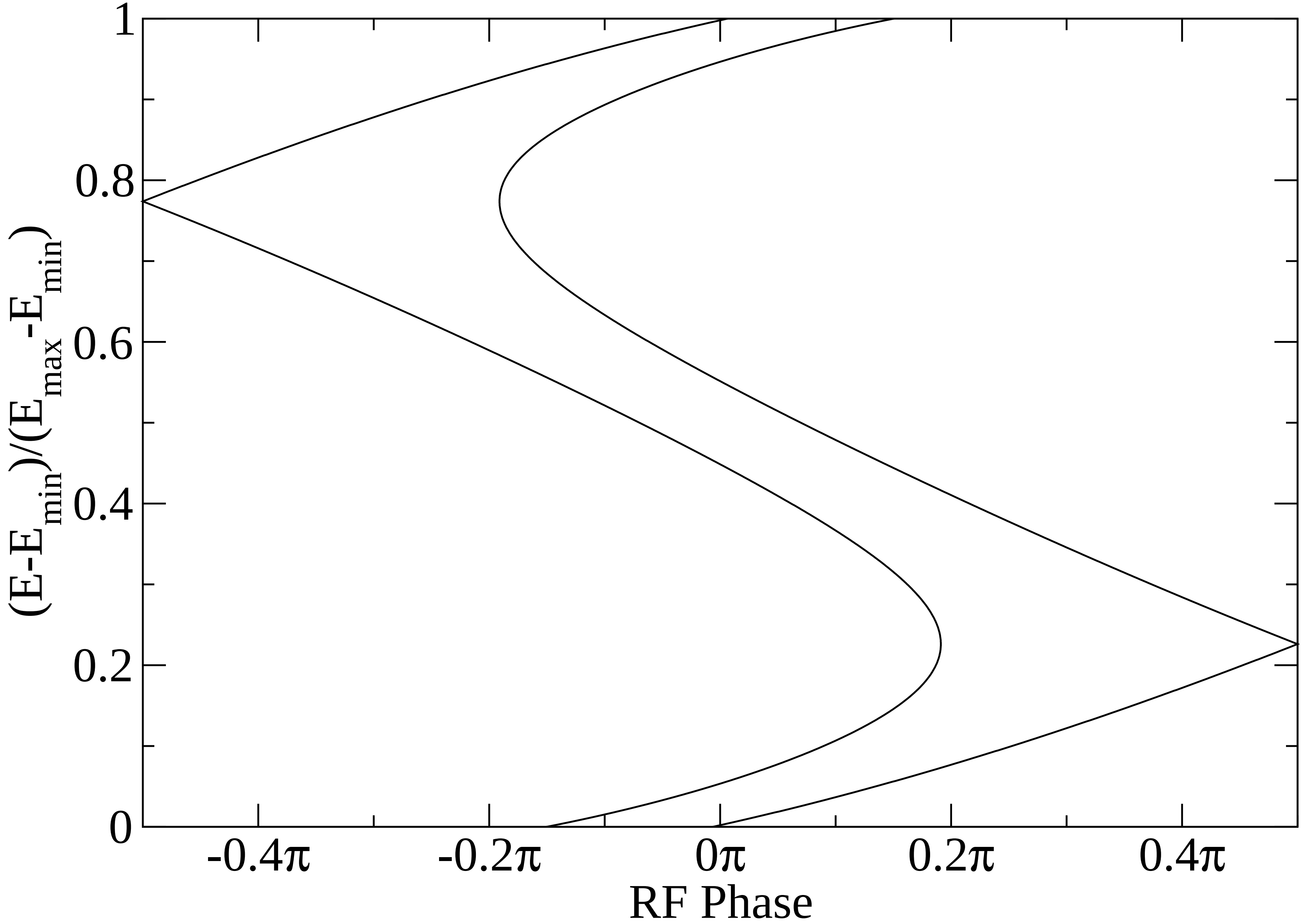

Closed Orbit vs. Energy: ν_min = 0.3
6
4
2
0
-2
Closed Orbit Offset (cm)
0
1
2
3
4
5
6
Position Along Reference Orbit (m)

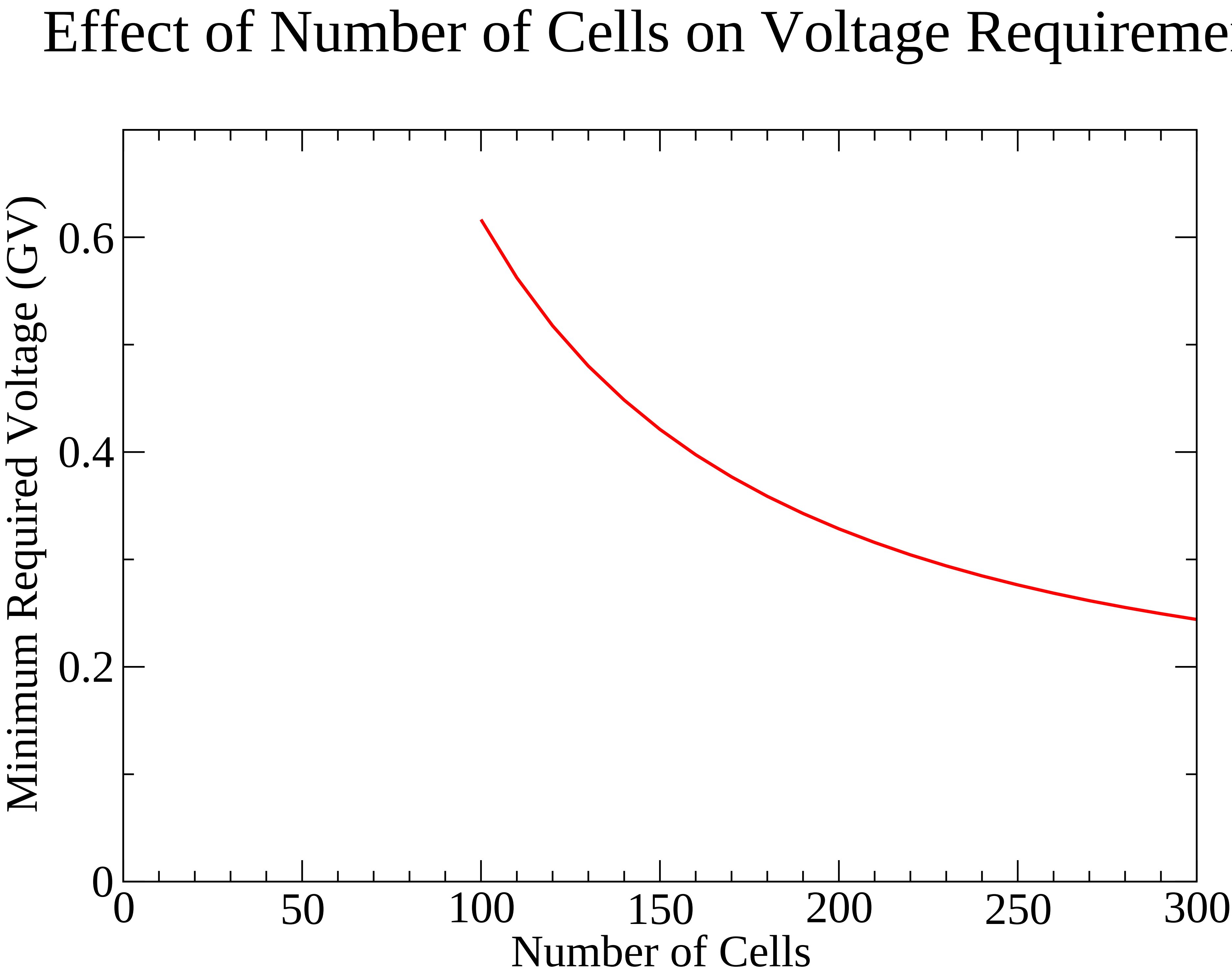

Tune vs. Energy
0.3
0.2
0.1
0
Tune
10
12
14
16
18
20
Energy (GeV)

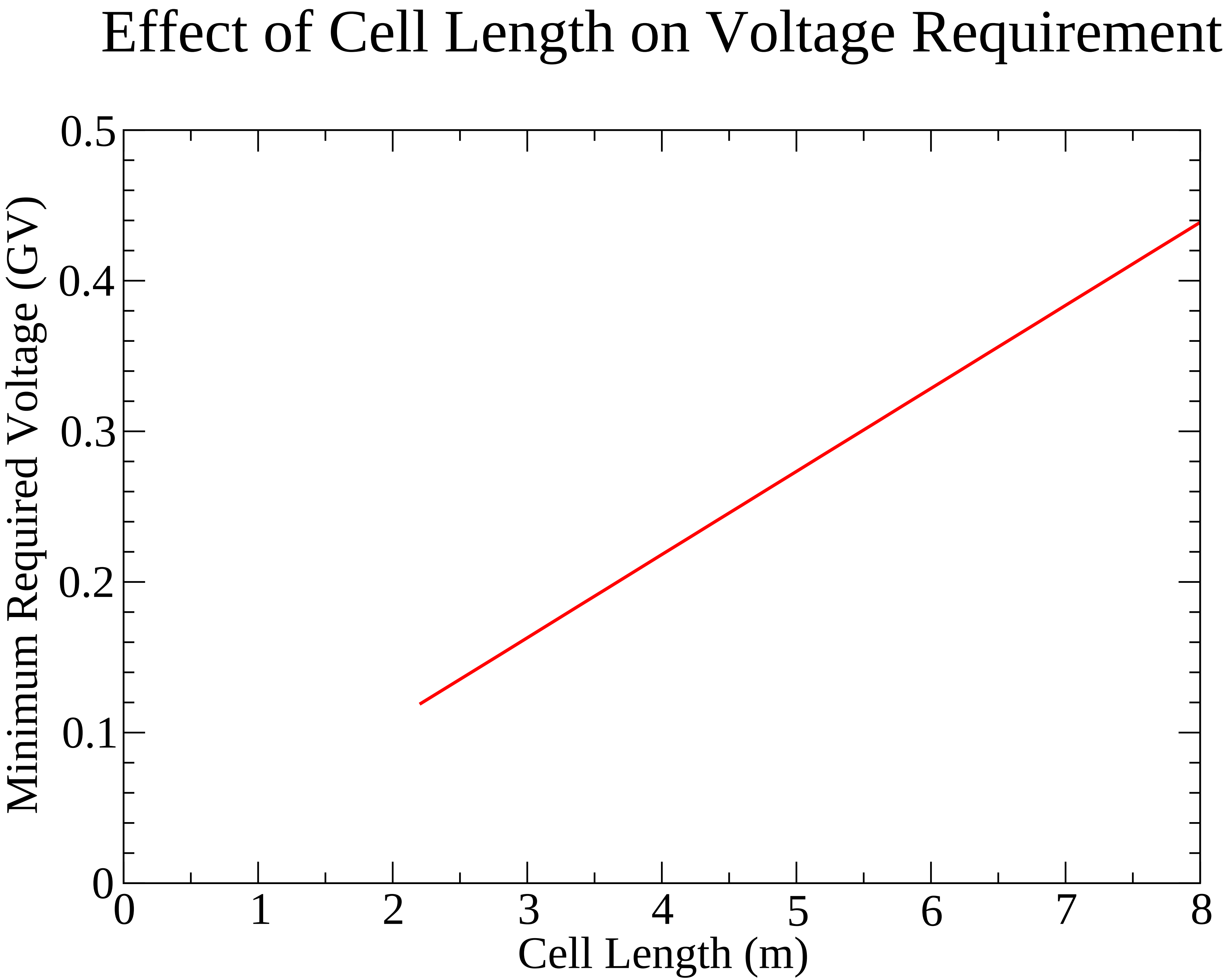

Time-of-Flight Offset vs. Energy
Time-of-Flight Offset (ns)
0
0.1
0.2
0.3
0.4
0.5
0.6
10
12
14
16
18
20
Energy (GeV)



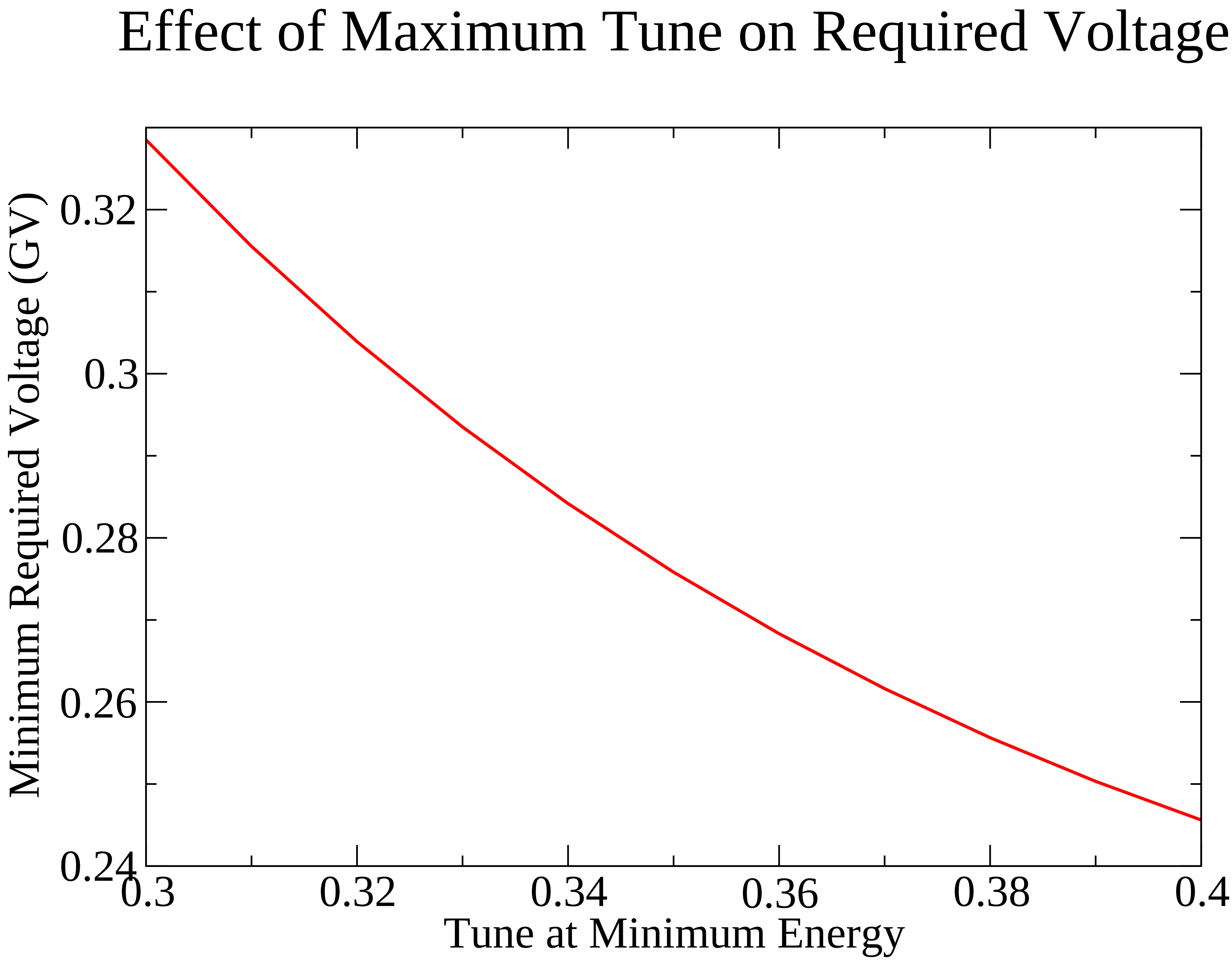
1
0.8
0.6
0.4
0.2
0
-0.4π
-0.2π
0π
0.2π
0.4π
RF Phase
(E-E_min)/(E_max-E_min)


1
0.8
0.6
0.4
0.2
0
$(E-E_{min})/(E_{max}-E_{min})$
-0.4π
-0.2π
0π
0.2π
0.4π
RF Phase

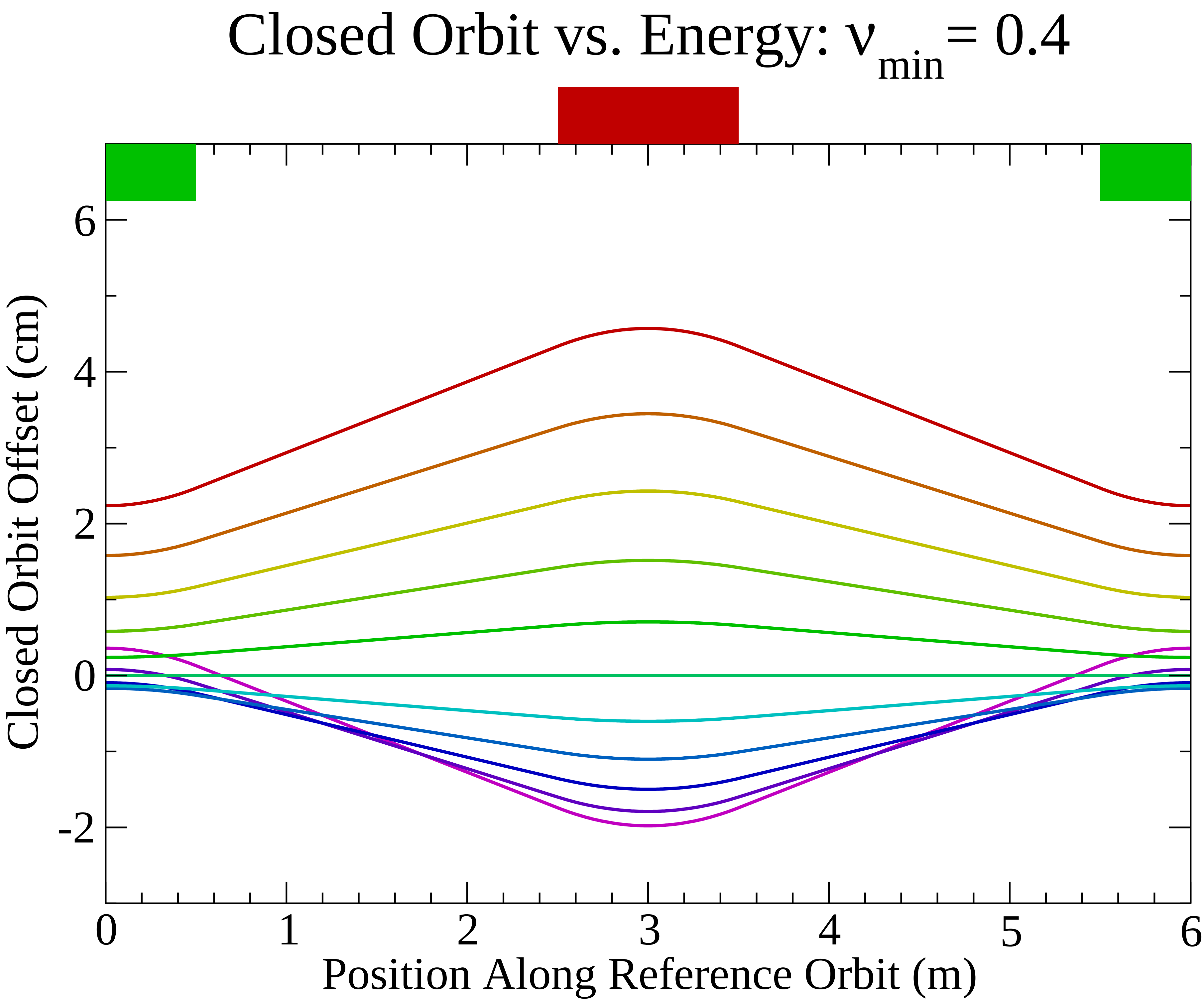

Effect of Number of Cells on Voltage Requirement
Minimum Required Voltage (GV)
0.6
0.4
0.2
0
0
50
100
150
200
250
300
Number of Cells



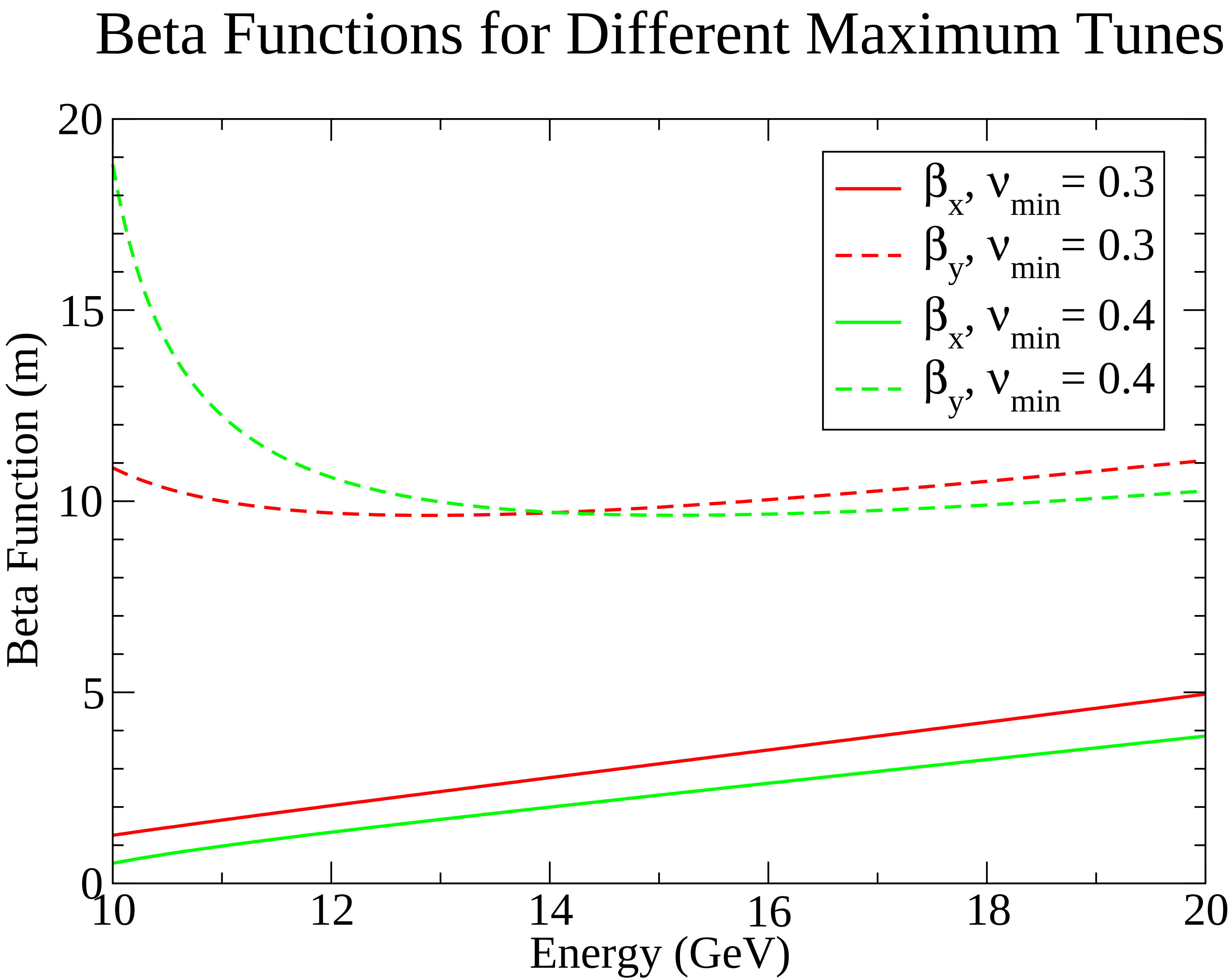
Effect of Cell Length on Voltage Requirement
Minimum Required Voltage (GV)
0
0.1
0.2
0.3
0.4
0.5
0
1
2
3
4
5
6
7
8
Cell Length (m)



Effect of Maximum Tune on Required Voltage
Minimum Required Voltage (GV)
0.32
0.3
0.28
0.26
0.24
0.3
0.32
0.34
0.36
0.38
0.4
Tune at Minimum Energy



Closed Orbit vs. Energy: ν_min = 0.4
6
4
2
0
-2
Closed Orbit Offset (cm)
0
1
2
3
4
5
6
Position Along Reference Orbit (m)



Beta Functions for Different Maximum Tunes
20
15
10
5
0
10
12
14
16
18
20
Energy (GeV)
Beta Function (m)
βx, νmin= 0.3
βy, νmin= 0.3
βx, νmin= 0.4
βy, νmin= 0.4

RF Cavity Placement

- Can't have significant magnet fields as superconducting cavities
  - Generally must be below 0.1 Gauss
  - Can be as high as 0.1 T once cavity has cooled down
- Gap required between superconducting cavities and magnets
- Want to minimize gap: smaller cell length gives smaller closed orbit range and smaller time-of-flight range
- Could use room-temperature RF
  - Power requirements significantly larger, thus costs higher
  - Can't reduce power by using lower voltage: need stored energy

Storage Ring

- Goal: cause as many muons as possible to decay toward detector
- Elongated racetrack shape, with one straight pointed toward detector
  - Maximum 50% of muons decay toward detector
  - Maximize ratio of straight length to arc length
  - Increasing straight length increases depth/height of ring
  - Keep arcs short!
- Keep angular spread of beam small compared to angular spread from decays
  - Large beta function, and thus large beam size

Flux Uncertainty

- Neutrino flux is convolution of muon angular distribution with decay angular distribution

$$\rho(\theta_x, \theta_y) = \int \rho_{\text{Muon}}(\theta_x - \theta'_x, \theta_y - \theta'_y)\rho_{\text{Decay}}(\theta'_x, \theta'_y)\, d\theta'_x\, d\theta'_y$$

- $N\, d\Omega_{\text{Detector}}\rho(0,0)$ is flux on detector
- One-dimensional model, Gaussian distributions: σ_0 is decay anglular spread, σ_x is beam angular spread

$$\frac{\phi(\sigma_x, \sigma_0)}{\phi(0, \sigma_0)} = \frac{\sigma_0}{\sqrt{\sigma_0^2 + \sigma_x^2}}$$

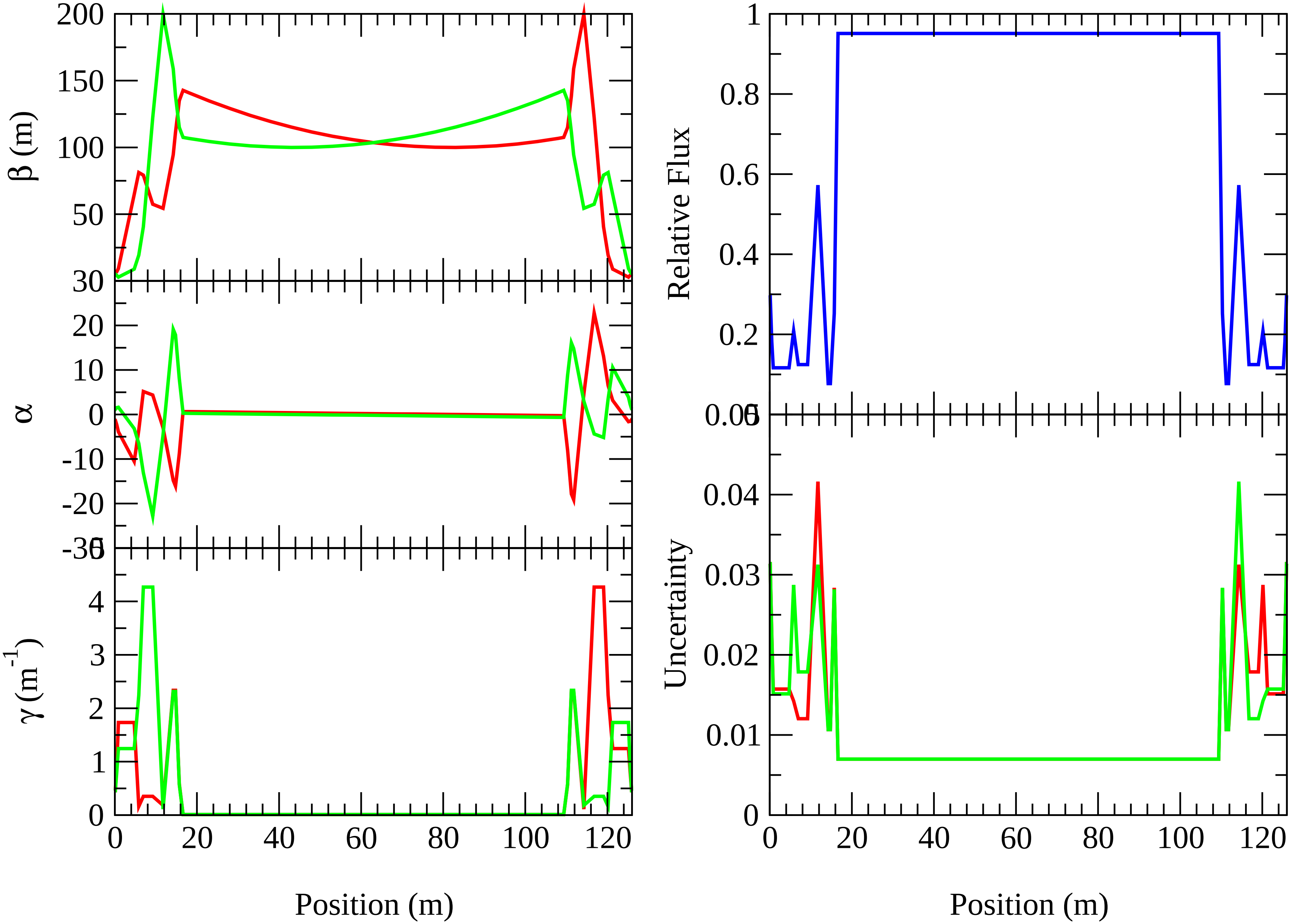
- Large angular spreads reduce flux

Flux Uncertainty

- Uncertainty in flux

$$\delta\phi = \frac{\sigma_x \sigma_0}{(\sigma_0^2 + \sigma_x^2)^{3/2}} \delta\sigma_x$$

  - Uncertainty lower when σ_x (angular spread) lower
  - Difficult to get σ_x to better than 15% or so
- Real storage ring design
  - Usually want 2π phase advance in arcs
  - Can't have beta functions as high in arcs as in straight and keep arc short
  - Matching sections (arc to straight) contribute little to flux, but much to uncertainty
  - Point matching sections away
- Can make uncertainty arbitrarily low, but costs you in ring length


β (m)
α
γ (m-1)
Position (m)
Relative Flux
Uncertainty
Position (m)