Charge Recombination in the Muon Collider Cooling Channel

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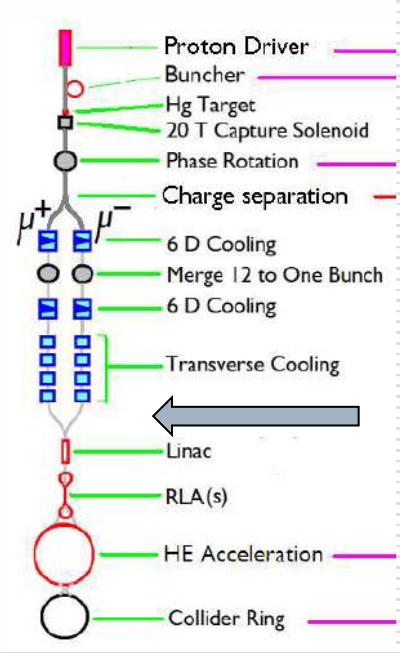
Introduction

- the cooling channel is a crucial part of a muon collider
  ~10^{34} luminosity requires reduction in muon ε_6 by factor ~10^6
- in the present design the last stage of the cooling channel must recombine
  positive and negative muon bunches before acceleration to high energies
- essential to minimize emittance growth in this system
  since no further cooling is possible
- this study shows first simulations of a charge recombination system
- it is only part of the cooling channel that has not been simulated at some level

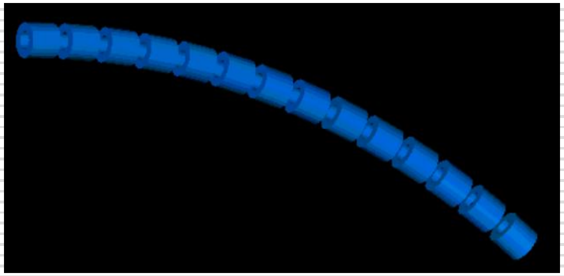
Muon collider cooling channel



- 6D cooling takes place in helical channels that requires separated charges
- final transverse cooling uses induction linacs that also prefers separated charges
- charge recombination takes place at the end of the channel
- then both charges go through same acceleration system

CHARGE RECOMBINATION

(R. Palmer)



Motion in a bent solenoid

- charge recombination uses bent solenoids
- to lowest order in Frenet coordinates

$x'' \approx h + q/p_s\,(y'\,b_s - b_0)$

$y'' \approx -q/p_s\,x'\,b_s$

h : curvature in x-z plane
q : charge
p_s : momentum along axis
b_s : solenoid filed
b_0 : dipole field

- an on-axis particle has the solution

$x(s) = -A/(\alpha\,b_s)\cos(2\pi/\lambda_L\,s) + A/(\alpha\,b_s)$

$y(s) = A/(\alpha\,b_s)\sin(2\pi/\lambda_L s) - A\,s$

$A = (h - \alpha\,b_0) / \alpha\,b_s$
$\alpha \;\; = q / p_s$
$\lambda_L = 2\pi\,p_s / q\,b_s$ (Larmor wavelength)

Consequences

(1) x motion is oscillatory with

amplitude = p_s^2 (h – q b_0/p_s) / (q b_s)^2

period = λ_L

amplitude → 0 if b_0 = h p_s / q

(2) y motion is combination of oscillatory and a term <u>linear in s</u>

oscillatory term has same amplitude and period as x, but is 90^o out of phase

linear deflection has slope = p_s (h – q b_0/p_s) / (q b_s)

Δy increases for increasing p_s, h, and s
Δy decreases for increasing b_s
Δy has opposite sign as q
<Δy> = 0 if b_0 = h p_s / q

Consequences

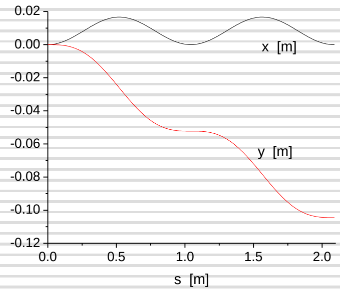
(3) vertical dispersion is present regardless of <Δy> and b_0

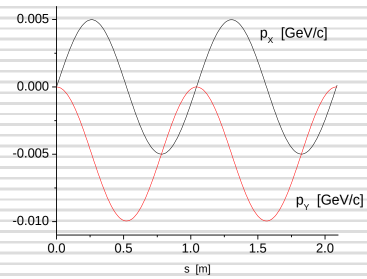
$$D(s) = -h\, p_{ref}\, s \,/\, q\, b_s$$

dispersion scales like $p \,/\, b_s$

ICOOL tracking examples

- ICOOL uses full equations of motion (no paraxial approximations)
- simplified field model used here: constant solenoid + hard-edge dipoles





single particle motion

p = 0.100 GeV/c

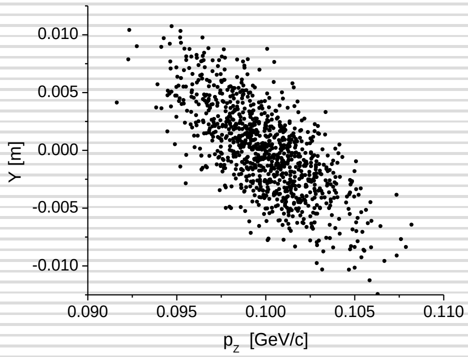
L = 2 λ_L

b_s = 2 T

b_0 = 0

h = 0.3 m^{-1}

q = +



same channel with a <u>beam</u>

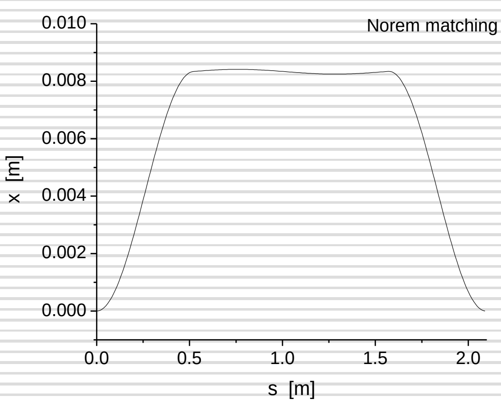
b_0 = 0.1 T

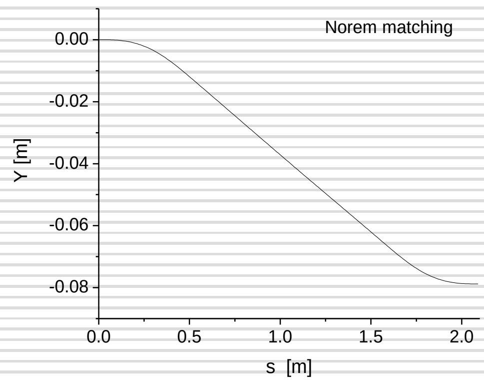
get dispersion with mean deflection = 0

Matching

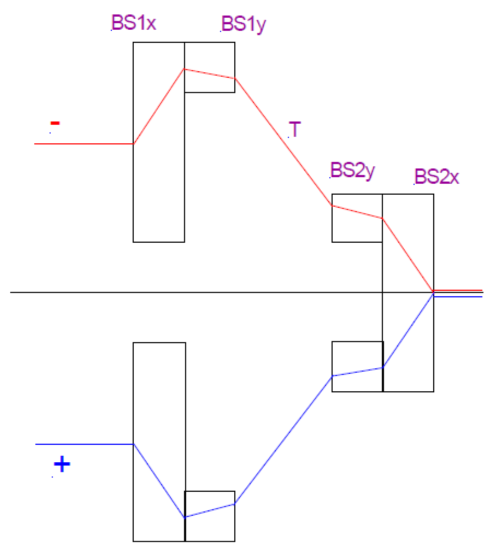
- careful matching is necessary to avoid emittance growth if $b_0 = 0$
- Norem matching uses tapered curvature at entrance and exit

part 1	$L = \lambda_L / 2$	$\kappa = h / 2$
part 2	L is arbitrary	$\kappa = h$
part 3	$L = \lambda_L / 2$	$\kappa = h / 2$





note that oscillations are removed



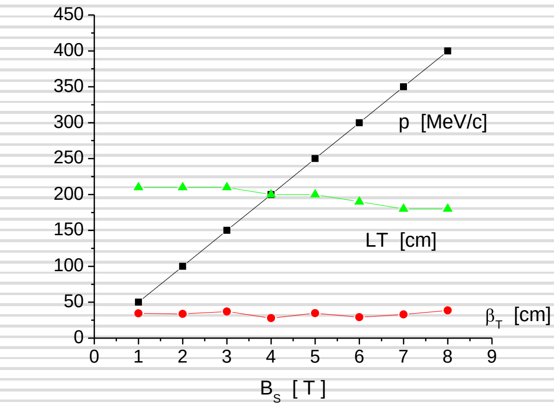
Vertical slice

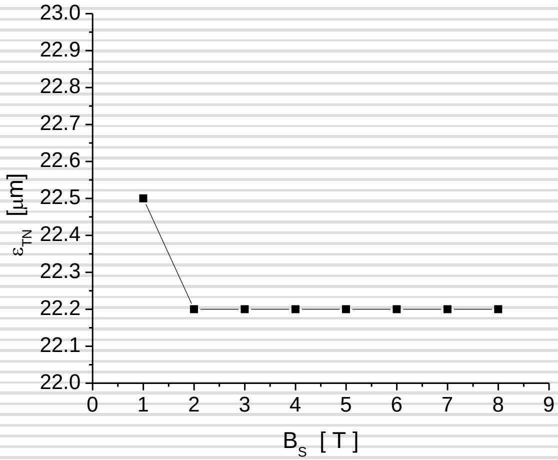
Design strategy

- use ICOOL to design basic system
- use G4beamline to check practical issues
  i.e. space for coils, current densities
- combine bent solenoids in pairs with opposite curvature
  removes dispersion
- use Norem matching
  reduces emittance growth
- insert second BS pair in the perpendicular plane
  get adequate beam separation going into BS2X
- don't put RF in channel
  it disrupts the dispersion cancellation
- try to avoid flux leakage at solenoid interfaces
- allow adequate separation for incoming beam lines
- assume upstream matching optics can adjust β_T
- assume upstream RF system can accelerate beams and adjust σ_Z
  want minimum $\sigma_{PZ} \rightarrow$ maximum $\sigma_Z \leq \lambda_{RF} / 24$

notes:
1) actual recombination takes place in **BS2x**
2) this whole system can be rotated 90^o around **z** if desired,
x $\leftrightarrow$ **y**

Scaling





- found that you can keep $\Delta\varepsilon_{TN}$ ~constant, if
- keep p/B_S constant
  => constant λ_L and constant dispersion
- keep β_T constant for incident beam
- make fine adjustments with L_T
- start seeing small ε_{TN} growth below 2 T

Input beam parameters

ε_{TN}	22	μm
ε_{LN}	72	mm
p	100	MeV/c
σ_X	2.8	mm
σ_{PX}	0.83	MeV/c
β_T	34	cm
σ_Z	300	cm
σ_{PZ}	2.47	MeV/c

- σ_Z is at maximum for 4 MHz following RF

Solenoid channel parameters

BS1x	
B_S	2 T
B_Y	0.1 T
L_e	0.52 m
L_c	0.50 m
h	0.30 m^{-1}
Δy	± 5.1 cm

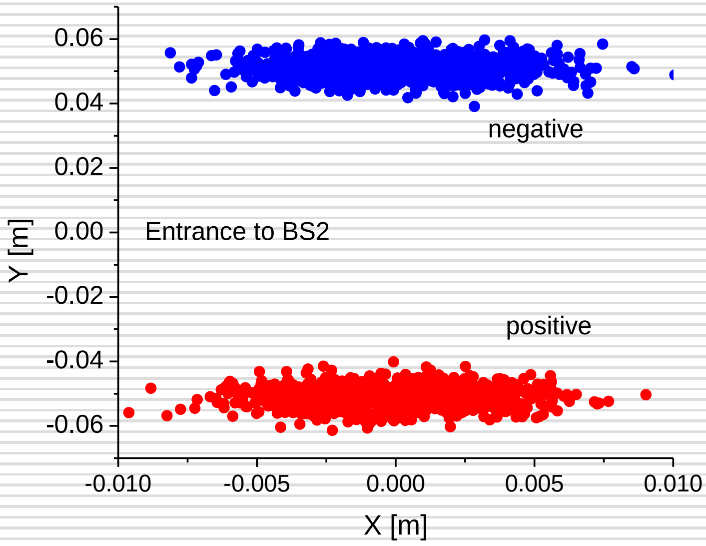
BS1y	
B_S	2 T
B_X	-0.023 T
L	1.04 m
g	0.07 m^{-1}
Δx	± 1.1 cm

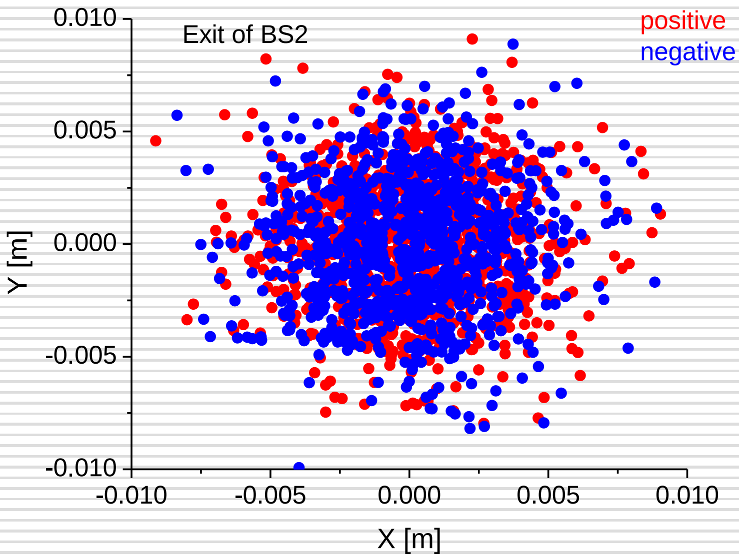
BS2y	
B_S	2 T
B_X	0.023 T
L	1.04 m
g	-0.07 m^{-1}
Δx	± 1.1 cm

BS2x	
B_S	2 T
B_Y	0
L_e	0.52 m
L_c	0.50 m
h	-0.30 m^{-1}
Δy	± 5.1 cm

- central transport line length = 2.1 m
- incoming beam lines are offset by ±27 cm

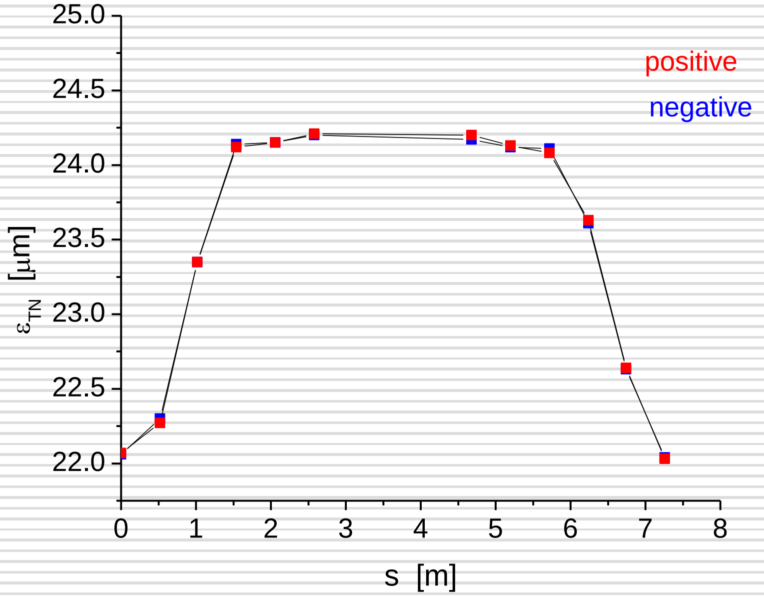
Beam at BS2x





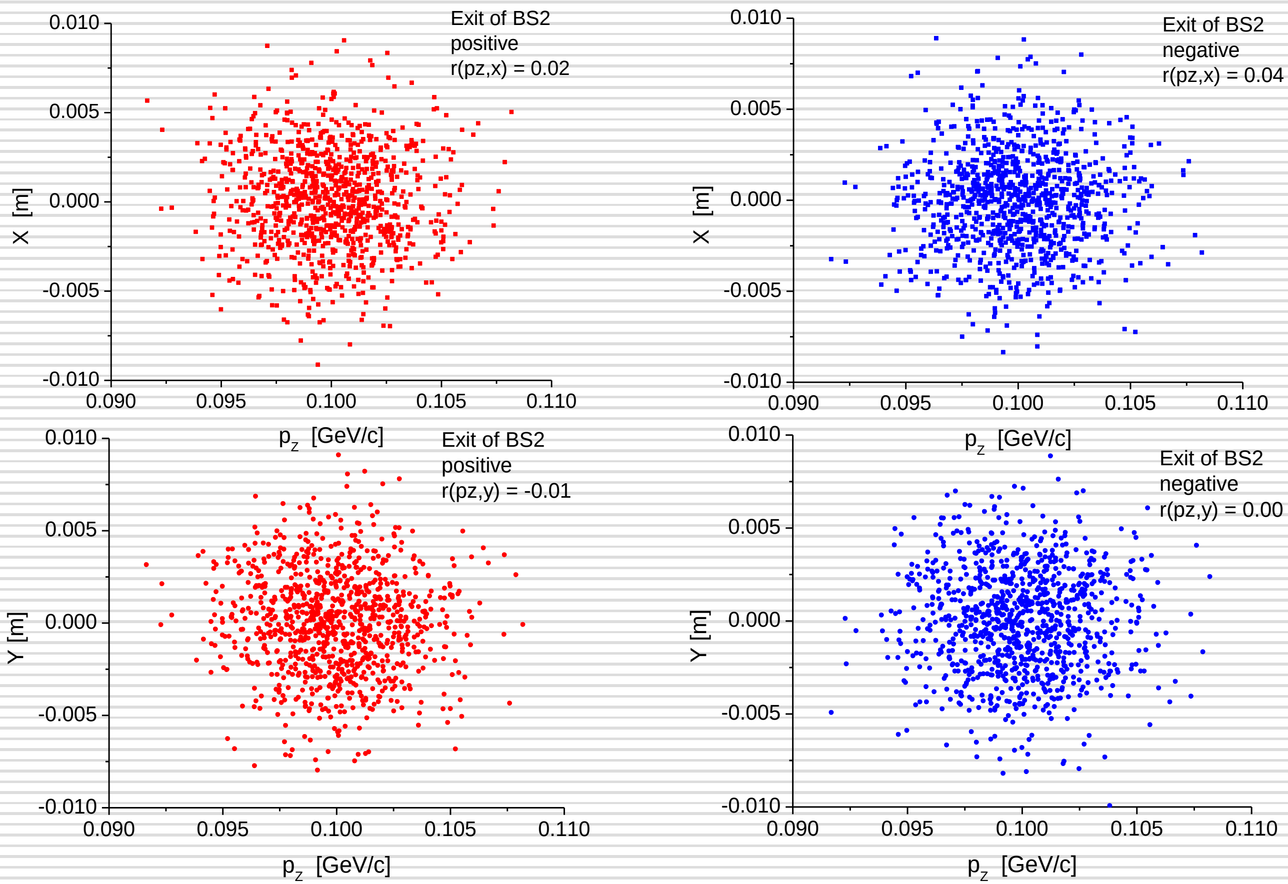
- exit beam bunches are symmetric
- combined transversely
- separate longitudinally by ½ λ_{RF}

Emittance



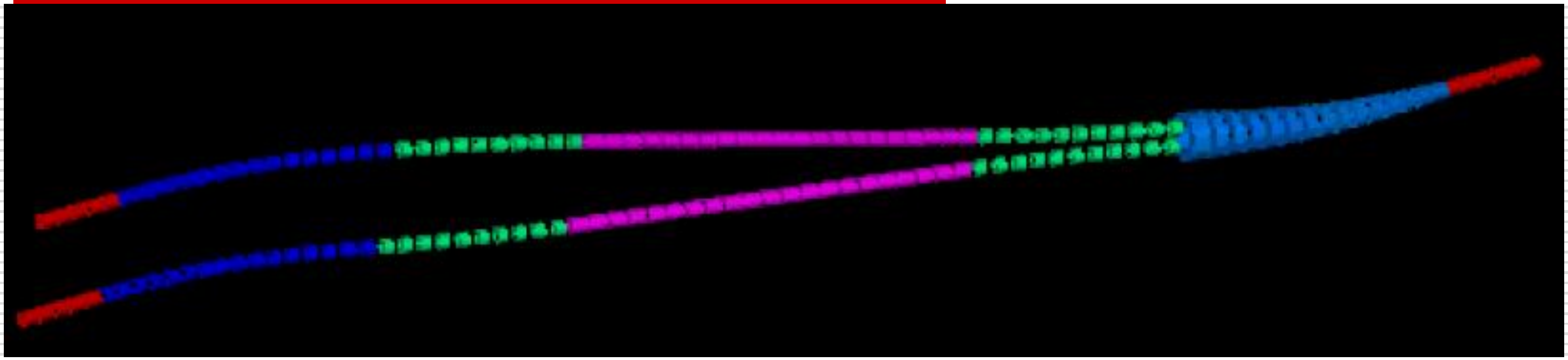
- transverse emittance growth small < 0.1%
- longitudinal emittance growth small < 0.1%
- transmission very good ~99%

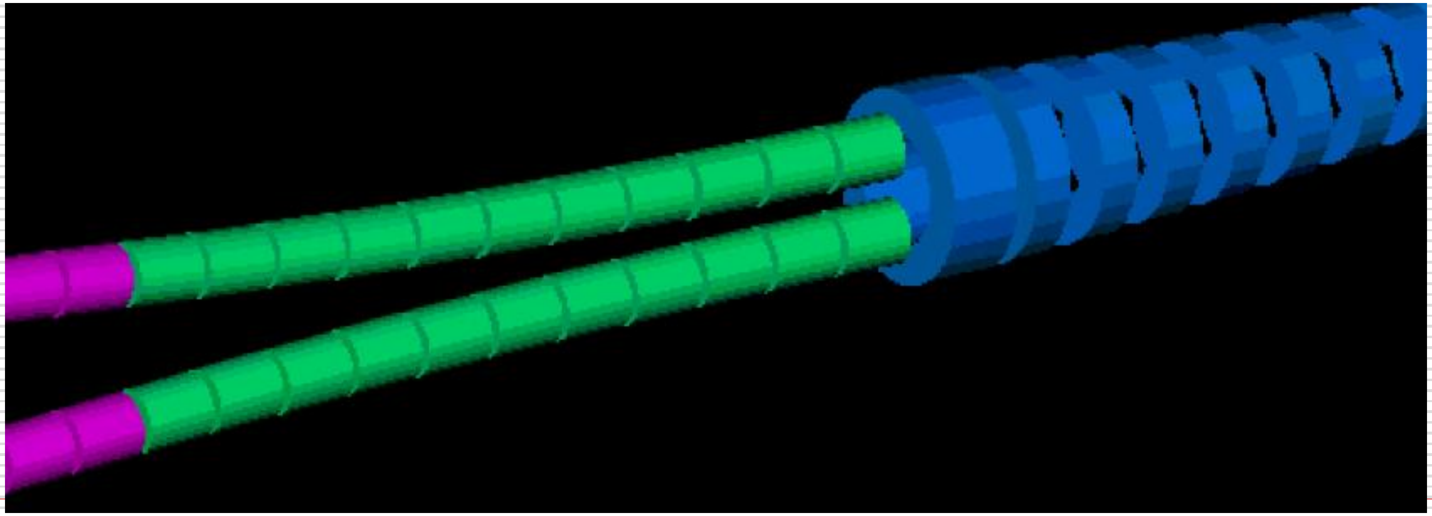
Dispersions at end



- dispersion is almost completely removed

G4beamline model





Coil properties

- adjusted coil dimensions to avoid geometric overlap
- adjusted current densities to give ~2 T along whole channel

	L [cm]	**a [cm]**	**b [cm]**	**J_E [A/mm^2]**
straight	9	2	4	90
small BS	9	2	4	91
large BS*	5 - 10	7 – 2.2	12 – 4.6	70 - 105

* BS2X, tapered

Summary

- these are the first simulations of the last piece of the muon collider cooling channel
- the design is built around pairs of symmetric bent solenoids
- with ICOOL idealized-field approximations used here
  no problem with transmission or emittance growth
  dispersion is eliminated in the exit beam bunches
- with G4beamline geometry description used here
  no problem with overlapping coils
  current densities in all coils are reasonable
- issues for further study
  fields in real channel will have variations due to use of discrete coils, flux leakage at BS2X
  need to determine the actual fields seen along the reference orbits
  what effect will this have on the emittance growth ?