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Title: Multipole Calculations for Partially Keystoned Cables

Task Force: Coil Geometry Analysis

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MULTIPOLE CALCULATIONS FOR PARTIALLY KEYSTONED CABLES

R.C. Fernow

It is likely that the cable that will be used for the small diameter SSC magnet will not be fully keystoned. After a number of turns the conductor blocks constructed from these cables can in some circumstances have very irregular shapes. As a result the multipole calculation routines[1] developed for CBA, which assumed an annular conductor region, may not give answers accurate at the 10^{-4} level. For this reason a new multipole calculating routine (subroutine MULQUAD) was written to find the multipoles from an abritrary quadrilateral conductor region with constant current density. The multipole optimization programs use subroutine MULQUAD in conjunction with subroutine TRAP[2], which supplies the coordinates of each conductor turn.

The subroutine was checked by comparing MULQUAD results for a CBA dipole model with the corresponding calculation using fully keystoned, annular regions with constant current density. The two calculations agreed for all multipoles to better than 1 part in 10^5. For the partially keystoned case however, the differences in the two methods are quite dramatic as indicated in Table I, which shows the multipoles for the inner, midplane block of the LBL dipole (Model B11).

Table I

Multipoles, $b_n \times 10^4$ @ 1.33 cm

n	MULQUAD	Annular Approximation	Δ
2	797.6	781.0	-16.6
4	-136.0	-152.5	-16.5
6	-28.7	-37.2	-8.5
8	8.2	9.4	1.2
10	1.3	2.8	1.5

The annular calculation assumed the end angle of the block was the average of the angles of the upper two corners of the true block. The annular multipoles were scaled so that both calculations gave the same dipole field.

Subroutine MULQUAD uses the complex contour integral method of Halbach.[3] The complex field $H^* = H_x - iH_y$ can be expanded as

$$H^* = \sum_{n=1}^{\infty} C_n Z^{n-1} \tag{1}$$

where $Z = x + iy$, and the C_n are complex multipole coefficients. The C_n contain contributions E_n from the conductor region alone, and F_n from the image currents in the infinite permeability iron.

$$C_n = E_n + F_n \tag{2}$$

In MKS units the conductor contribution is

$$E_n = \frac{\mu_o j}{4\pi(n-1)} \oint Z^{1-n} \, dz^* \quad (n > 1) \tag{3}$$

where j is the constant current density, and the integral must be taken around the conductor boundary. For integration along straight lines as shown in Figure 1, Z and Z^* are related by

$$Z^* = Z_1^* + \Delta_{21}^* \left(\frac{Z - Z_1}{\Delta_{21}}\right) \tag{4}$$

where we define $\Delta_{21} = Z_2 - Z_1$. Equation (4) may be used to convert dZ^* in Equation (3) into dZ. The integral can then be performed analytically to give $(n > 2)$

$$E_n = \frac{\mu_o j}{4\pi(n-1)\,(2-n)} \left\{\frac{\Delta_{21}^*}{\Delta_{21}} (Z_2^{2-n} - Z_1^{2-n}) + \frac{\Delta_{32}^*}{\Delta_{32}} (Z_3^{2-n} - Z_2^{2-n}) + \ldots \right\} \tag{5}$$

where the sum continues for each side of the conductor region. For the special case n=2 Equation (3) gives

$$E_2 = \frac{\mu_o j}{4\pi} \left\{\frac{\Delta_{21}^*}{\Delta_{21}} \ln \frac{Z_2}{Z_1} + \ldots \right\} \tag{6}$$

For n=1 the appropriate integral is

$$E_1 = \frac{\mu_o j}{4\pi} \oint \frac{Z^*}{Z} \, dZ \tag{7}$$

which can be evaluated analytically to give

$$E_1 = \frac{\mu_0 j}{4\pi} \{ [Z_1^* - Z_1 \frac{\Delta_{21}^*}{\Delta_{21}}] \ell n \frac{Z_2}{Z_1} + \ldots \} \tag{8}$$

The image currents contribute the term

$$F_n = \frac{\mu_0 j}{4\pi(n+1)R^{2n}} \oint Z^{*n+1}\, dZ \tag{9}$$

where R is the inner radius of the iron. Using Equation (4) this can be integrated analytically to give

$$F_n = \frac{\mu_0 j}{4\pi(n+1)(n+2)R^{2n}} \{\frac{\Delta_{21}}{\Delta_{21}^*} (Z_2^{*n+2} - Z_1^{*n+2}) + \ldots \} \tag{10}$$

The usual BNL expansion of the field on the midplane is

$$H_y = \sum_{n=0}^{\infty} b_n X^n \tag{11}$$

while the complex expansion in Equation (1) takes the form

$$H_x - iH_y = \sum_{n=1}^{\infty} C_n X^{n-1} \tag{12}$$

Comparing Equations (11) and (12) we see that

$$b_{n-1} = - \text{Im}\,(C_n) \tag{13}$$

Subroutine MULQUAD resides on the 6600 in file MAG2PL, 1D = ZZZGFERNOW. The call sequence is

CALL MULQUAD (CXY, RFE, CD, NN, CMUL)

where

CXY: 2×4 array of x,y coordinates of corner points (cm)

RFE: iron inner radius (cm)

CD: current density (amps/cm^2)

N: multipole number n

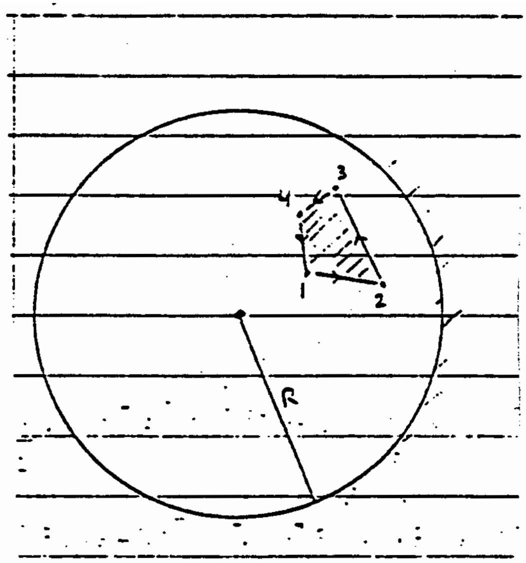
CMUL: complex multipoles C_n(G/cm^{n-1})

References

1. R. Fernow, MAG2, Field Computation Note 23, 1983.

2. G. Morgan, Geometry of Partially Keystoned Cables, Analysis Section Note 47, 1983.

3. K. Halbach, Fields and First Order Perturbation Effects in 2D Conductor Dominated Magnets, NIM 78 185 (1970).



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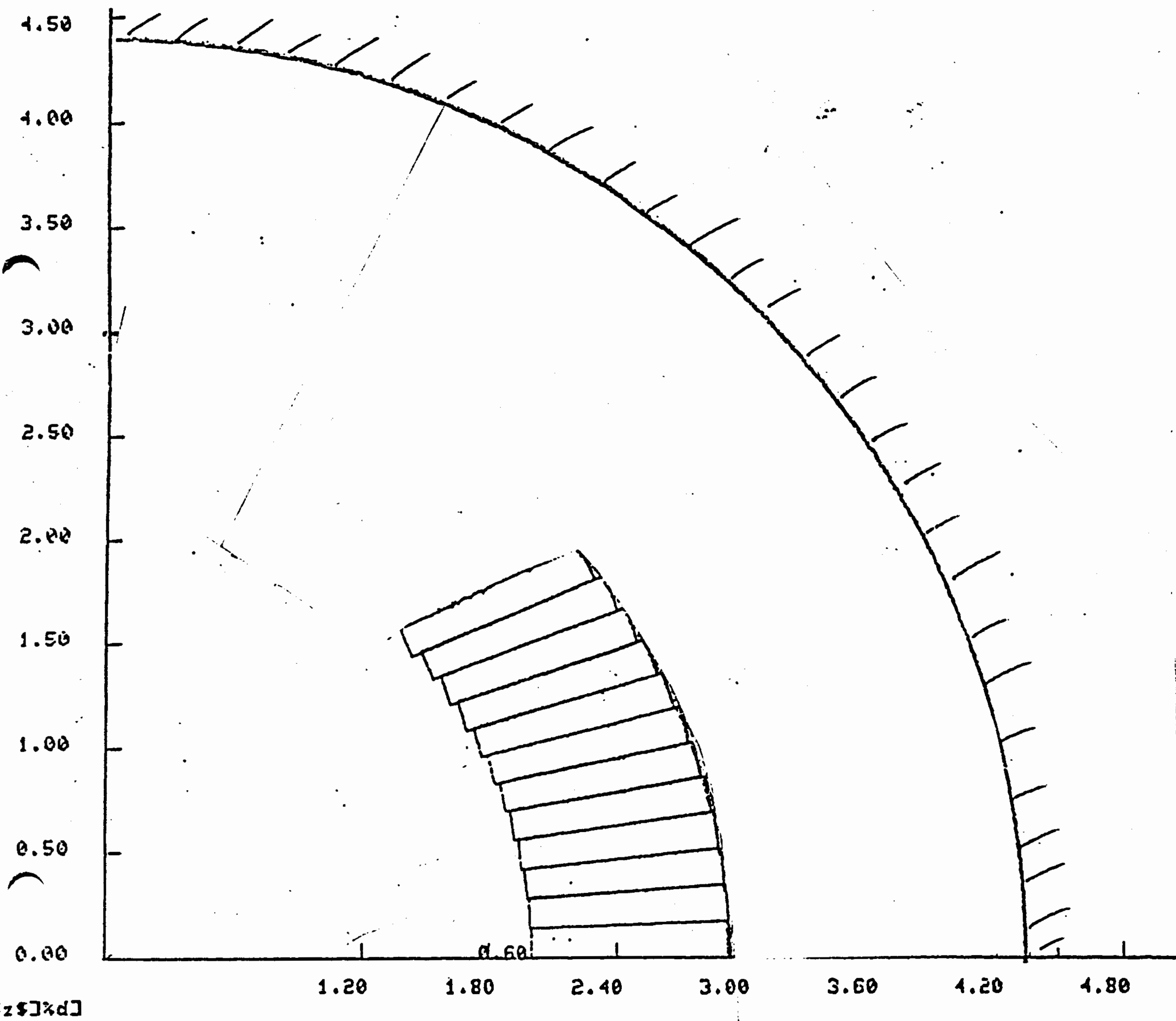

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