Calculations for a Hg Jet Target in a Solenoid Magnet Capture System

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Hg Target for a Neutrino Factory or Muon Collider

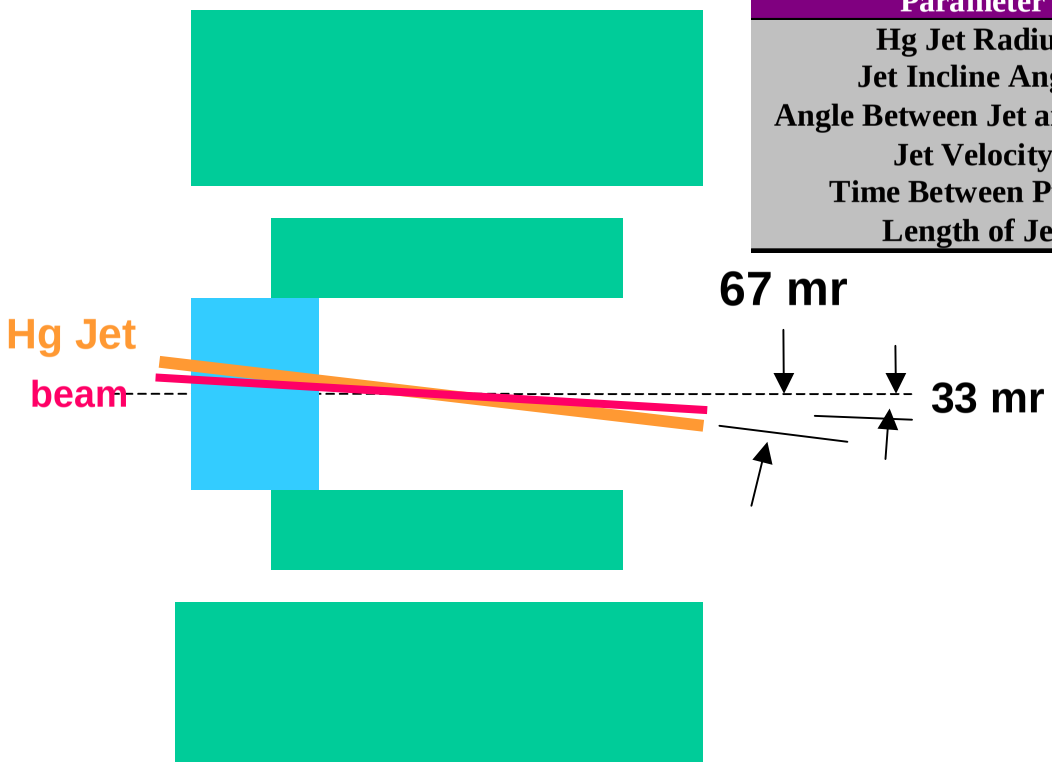
- Need an intense proton source on the target.
  - Such a source would quickly destroy most solid targets.
    - Although a C target might survive a 1 MW proton source it would not survive a 4 MW source.
  - Liquid metal target could be recycled after proton pulse.
- A high-Z material is desirable since it produces more interactions.
  - A Hg target will produce twice the number of useful pions per proton than a C target.
- A Hg jet is inserted into a 20 T solenoid field at 20-30 m/s in our design. The interaction with the field is the topic of this talk.

List of Forces, Pressures, Distortions, Deflections

- Induced azimuthal Eddy current.
- Radial forces: $J_{Eddy} \times B_z$
  - Hydrostatic Pressure
- Axial force
  - Contribution from Hydrostatic Pressure
  - Contribution from dB_z/dz
- Transverse forces and deflections
- Shear forces
- Transverse elliptical distortion

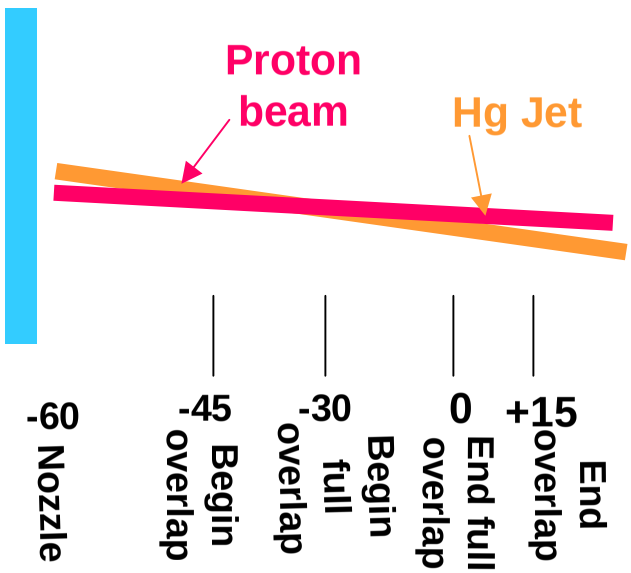
Targeting Schematic

Parameter	Symbol	Current Study
Hg Jet Radius	R_{Hg}	0.5 cm
Jet Incline Angle	θ_{jet}	67 mrad
Angle Between Jet and Beam	$\theta_{crossing}$	33 mrad
Jet Velocity	V_{jet}	30 m/sec
Time Between Pulses	Δt	20 ms
Length of Jet	L_{jet}	60 cm



Parameter	Symbol	Value
Conductivity	κ, Ω cm	10^4
Density	ρ, gm/cm^3	13.4
Surface Tension	$T_{surface}$, n/m	0.456

Obviously Not to Scale

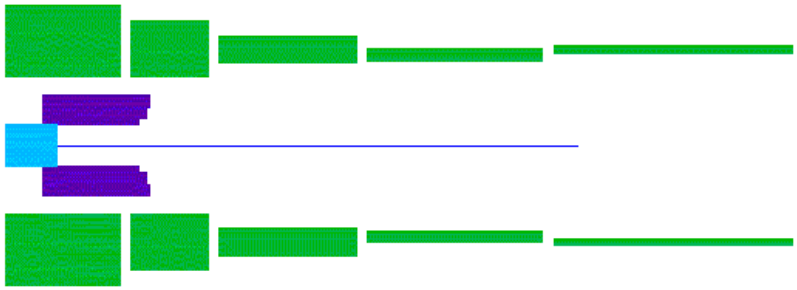


Magnet System Configuration

- Below is the coil configuration from Bob Weggel (21-Dec-00)

Coil	$Z_{central}$ cm	Half-Width cm	R_{inner} cm	R_{outer} cm	J Kamps/cm^2
HC 1	-29.35	42.75	18.00	23.43	2.796
HC 2	-26.3	45.80	23.43	34.01	2.010
HC 3	-25.05	47.05	34.01	44.19	1.475
SC 1	-54.40	50.30	60.45	123.97	2.808
SC 2	40.10	34.20	60.45	109.98	2.809
SC 3	144.75	60.45	73.53	97.03	3.011
SC 4	292.25	77.05	74.95	85.33	4.278
SC 5	484.65	105.35	82.00	88.40	5.323

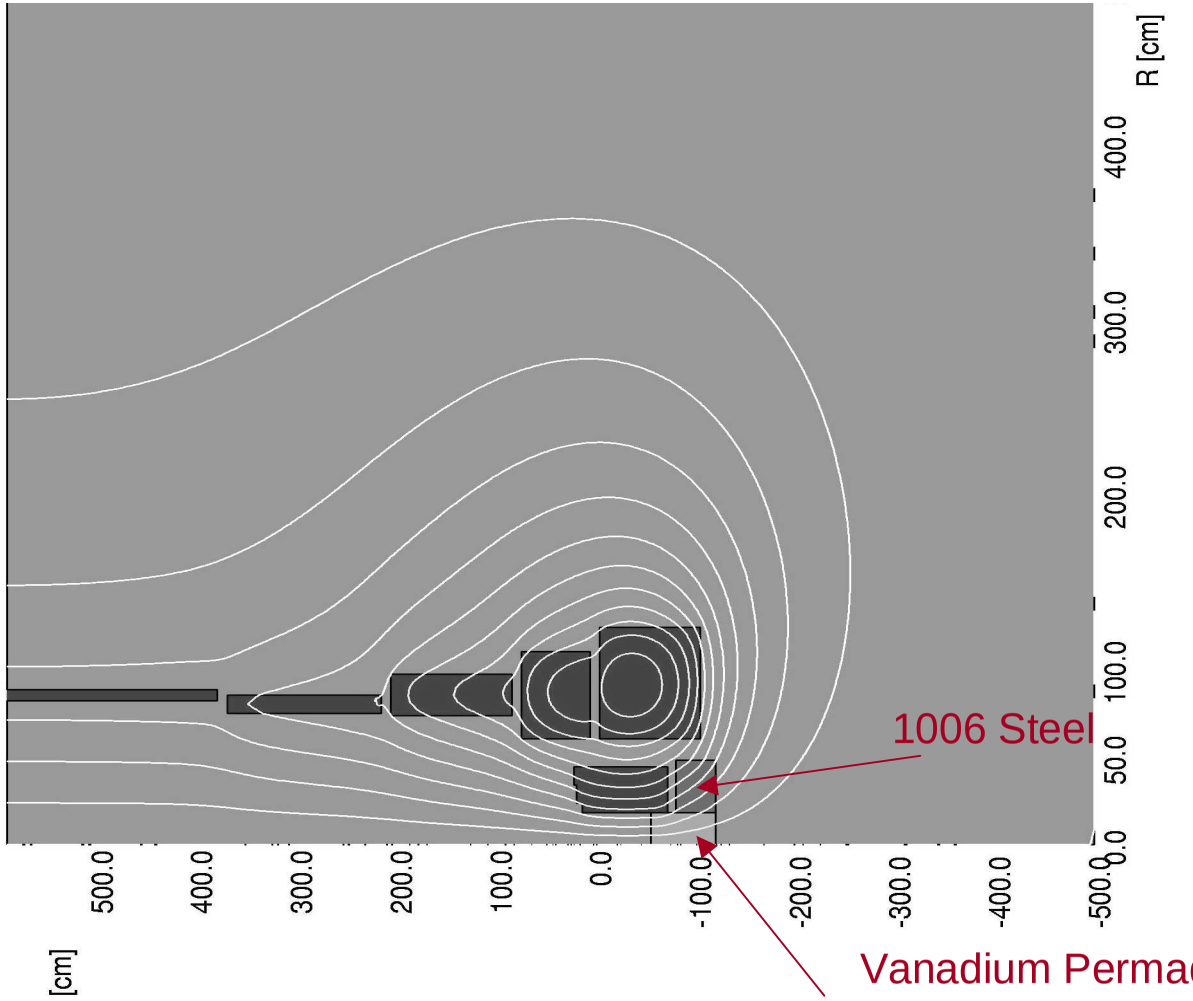
Table 1: Coil parameters for target capture magnet system.



Field Calculations

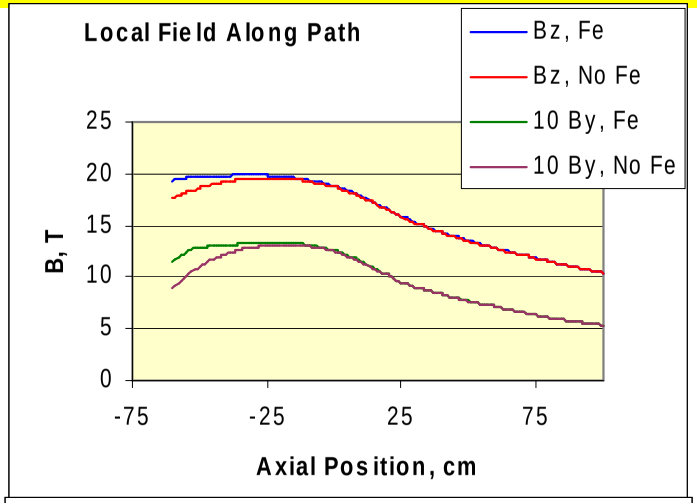
- In this model a pole made of Vanadium Permador steel is placed in the 20 T field to act as a *nozzle.*
  - This steel has M_s=2.4 T. (Figure shown on next transparency).
  - The nozzle is present to insure that the Hg jet enters the field intact.
- Opera-2D is used for field calculations.
  - This is a 2D finite element field solver that solves the cylindrical symmetric problem.
  - Only the hole in the nozzle breaks cylindrical symmetry. This only has effects in the vicinity of its aperture.
- The beam path is inclined with respect to the magnet axis at 67 mrad.
  - Justification for minimum perturbation of the beam path will be mentioned later.

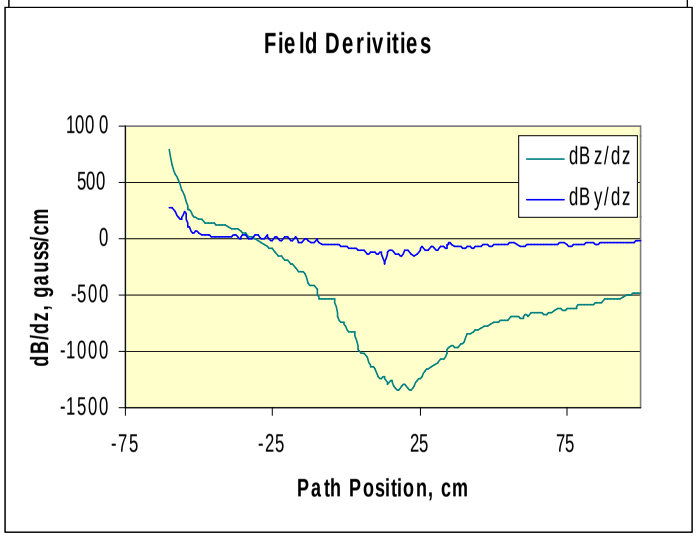
2D Axial Symmetric Model



Magnetic Field in Local Coordinates

- Figure show B_z and B_y in the local coordinate system of the Hg jet.
  - Local system is inclined 67 mrad to solenoid axis.
  - Field shown with and w/o iron pole present:
    - Pole keeps field reasonably uniform over targeting region.
    - Pole is Vanadium Permadur Steel.
    - There is a 2T difference due to pole.
- Field Derivitives dB_z/dz and dB_y/dz in local coordinate frame.





Axial Forces

Equation for axial force on jet in the field:

Force Density

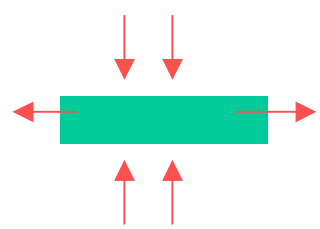
$$f_z = -\frac{r^2}{4} v\kappa \left(\frac{dB_z}{dz}\right)^2 - \left(\frac{r_o^2 - r^2}{4}\right) v\kappa \frac{d}{dz}\left(B_z \frac{dB_z}{dz}\right) - \frac{rv\kappa}{2} B_y \frac{dB_z}{dz} \sin\phi$$

Direct term

From $j_\phi B_r$

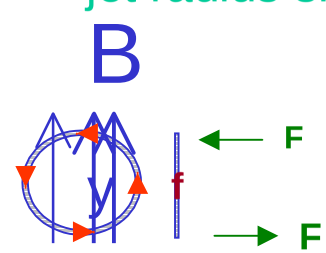
Hydrostatic term

From $j_\phi B_z$ transferred to axial direction since Hg is liquid



Shear term

Can be reduced by keeping incline angle small and by jet radius small



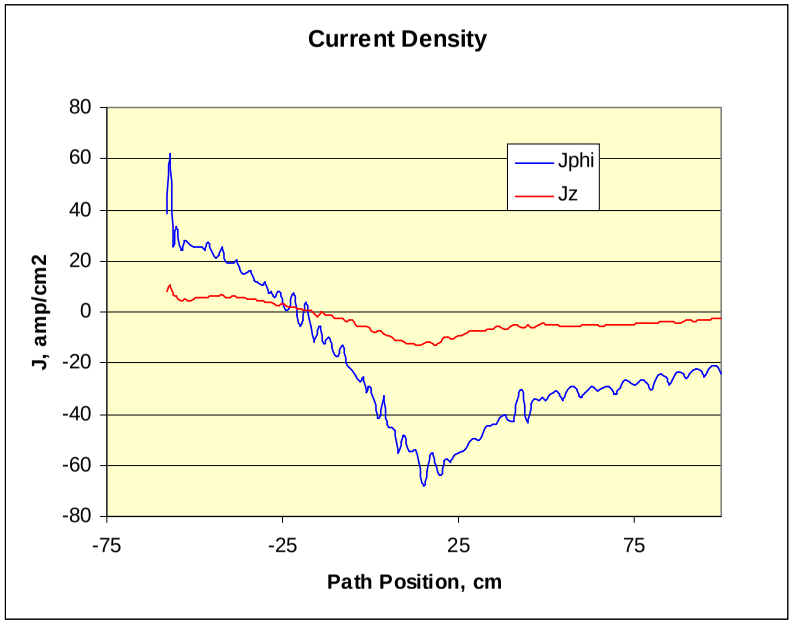
Current Densities Induced in Hg Jet

- An azimuthal current density is induced with the form:

$$j_\phi = \frac{rv\kappa}{2}\frac{dB_z}{dz}$$

- An axial current density is induced in regions of changing B_y:

$$j_z = xv\kappa\frac{dB_y}{dz}$$



Force Densities

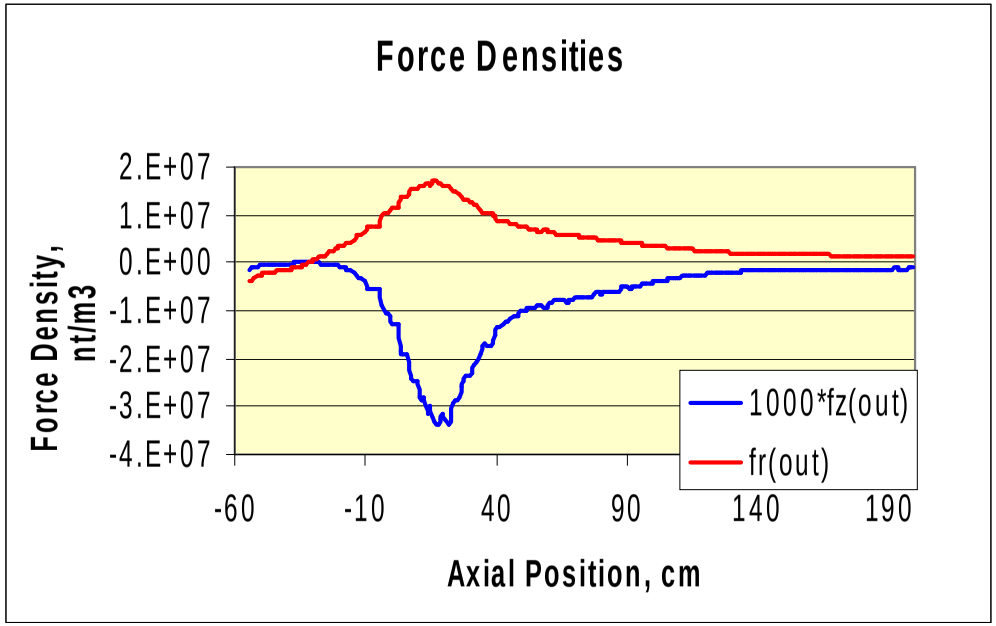
- Axial force density
  - averaged over radius

$$< f_z >= \frac{r^2}{4} v\kappa \left(\frac{dB_z}{dz} \right)^2$$

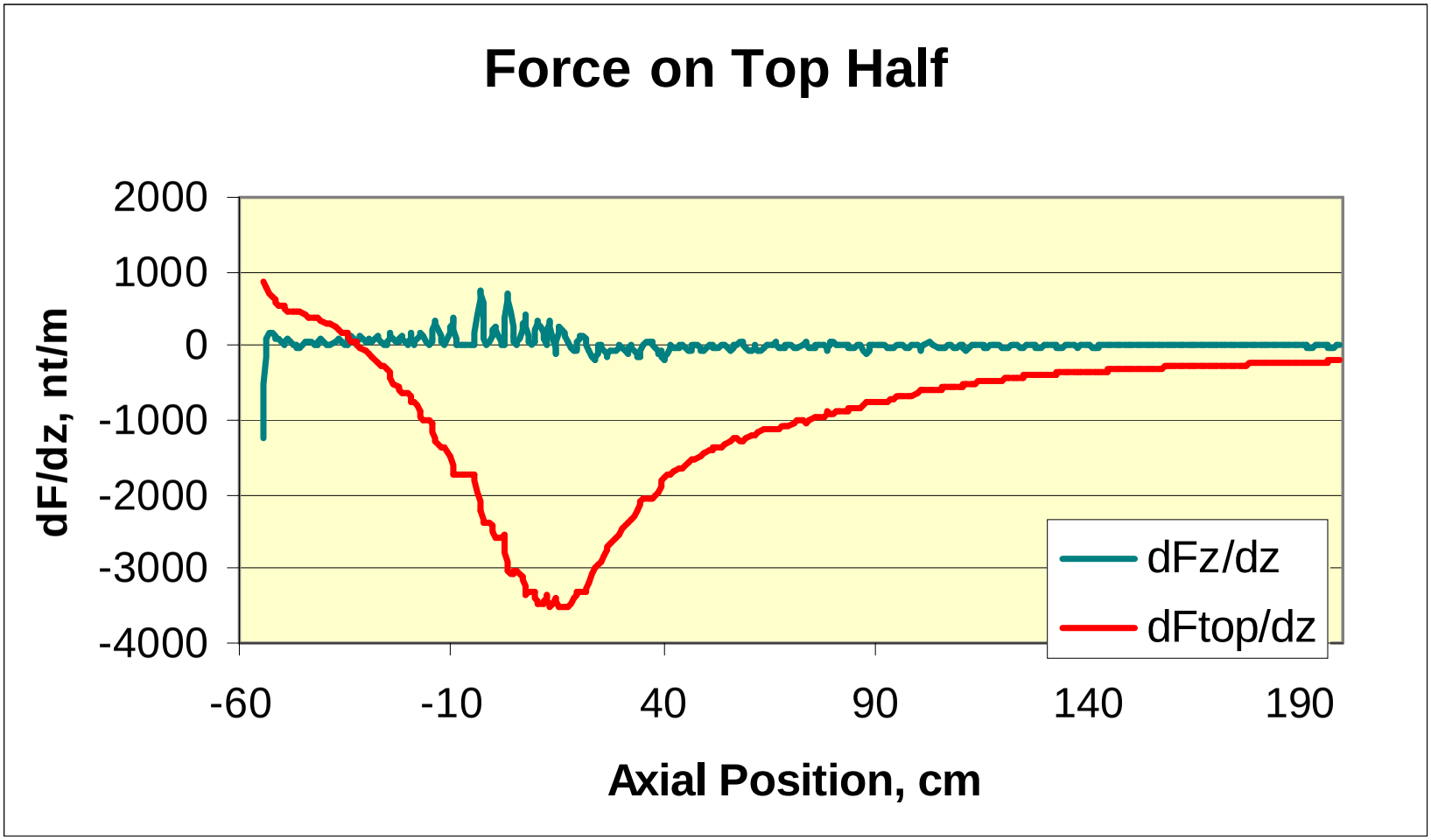
- Axial hydrostatic pressure from radial force
  - averaged over radius

$$< f_p >= -\left(\frac{r_o^{\,2}}{8} \right) v\kappa \frac{d}{dz} \left(B_z \frac{dB_z}{dz} \right)$$

- Numerical evaluation uses
  - r_0 = 0.5 cm
  - v_z= 20 m/sec
  - κ= 1×10^6 ohm^{-1} m^{-1} (Hg)

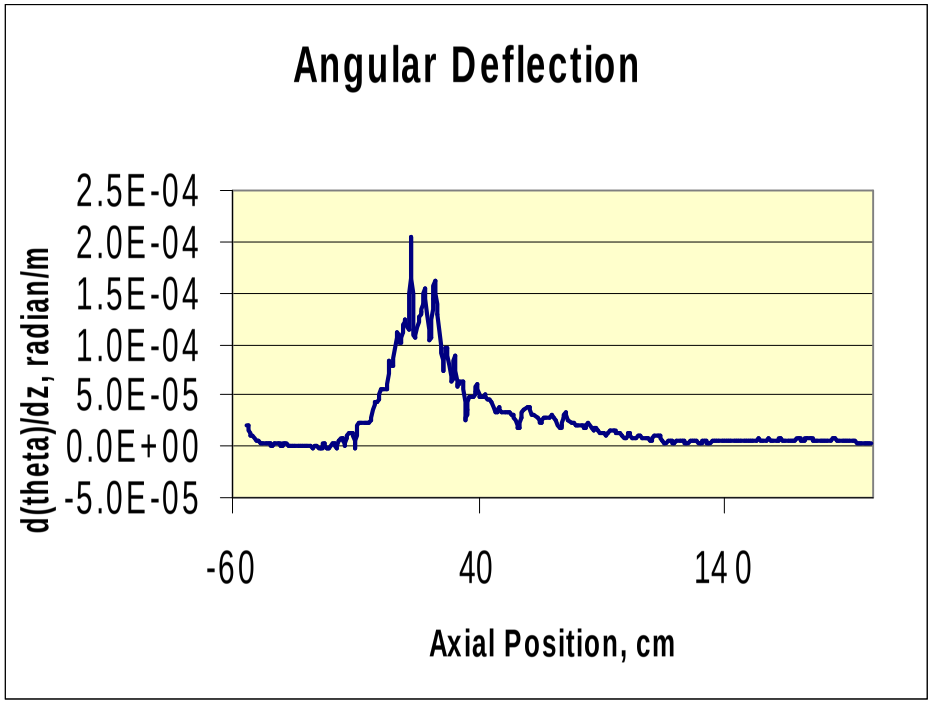


Shear Forces



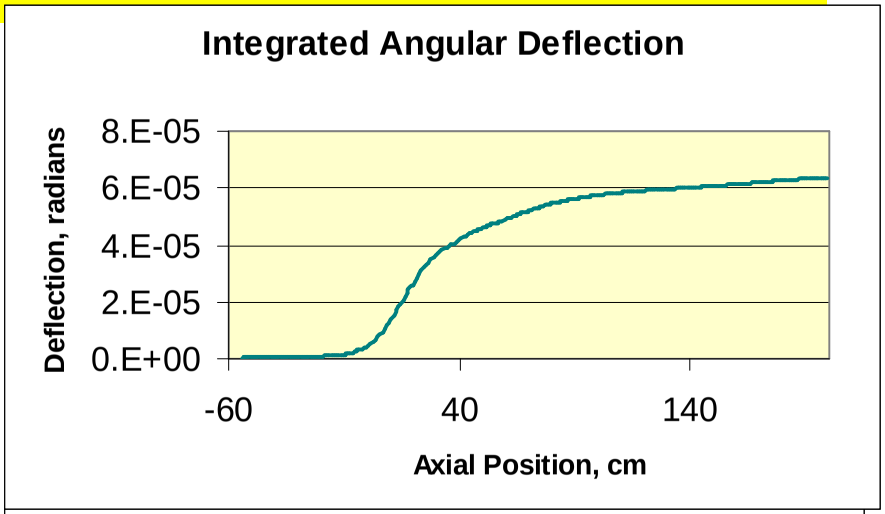
Angular Deflection from Transverse Force

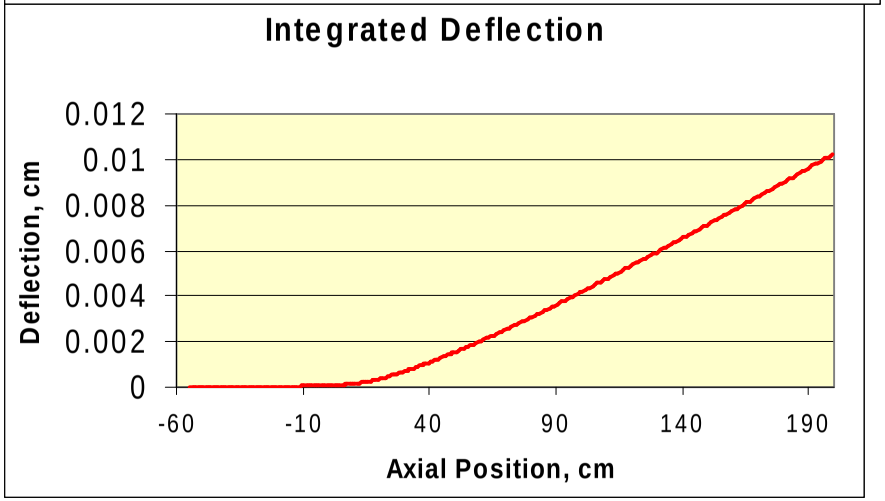
- The vertical force density is
$$f_y = f_r \sin(\phi)$$
- This gives
$$\frac{dF_y}{dz} = \frac{\pi}{8} v \kappa r^4 \frac{dB_y}{dz} \frac{dB_z}{dz}$$
- The resultant deflection per unit length is
$$\frac{dv_y}{dz} = \frac{\kappa r^2}{8\rho} \frac{dB_y}{dz} \frac{dB_z}{dz}$$
  - This is plotted in figure
$$\frac{d\theta}{dz} = \frac{\kappa r^2}{8 v \rho} \frac{dB_y}{dz} \frac{dB_z}{dz}$$



Integrated Angular Deflection

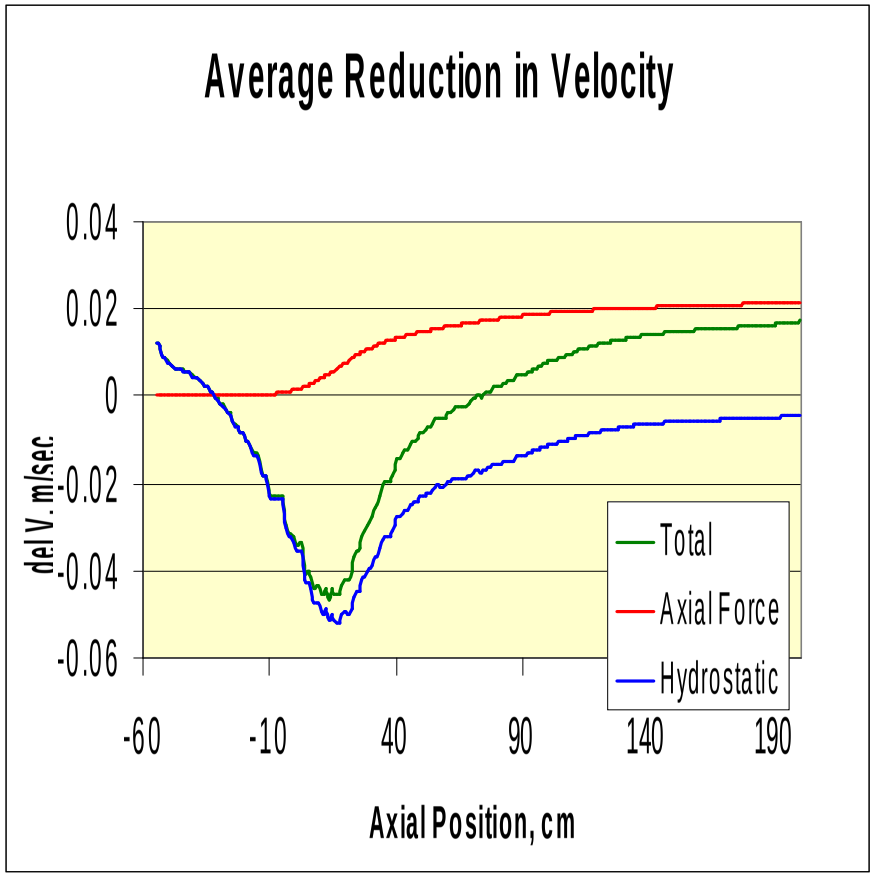
- The figure shows $d\vartheta/ds$ and δ_x integrated over the path length.
- The Hg jet will be deflected ~0.1 mrad over the trajectory.
- The spacial deflection is less than 0.15 mm over the 1.5 meter path.
- Adjusting the field for the changing path has not been done. This probably is not important.





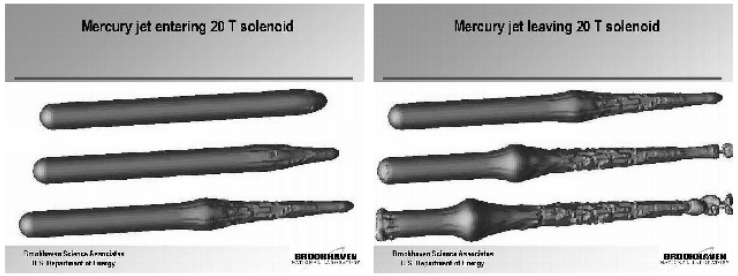
Deceleration of Hg Jet

- Green curve in figure is the total axial deceleration of the Hg jet entering the magnets: ΔV
  - $V(z) = V_0 - \Delta V$
- Blue curve shows the contribution from axial force, $\langle f_z \rangle$.
- Red curve shows the contribution from the hydrostatic pressure resulting from the radial force, $\langle f_p \rangle$.



Magnetohydrodynamics

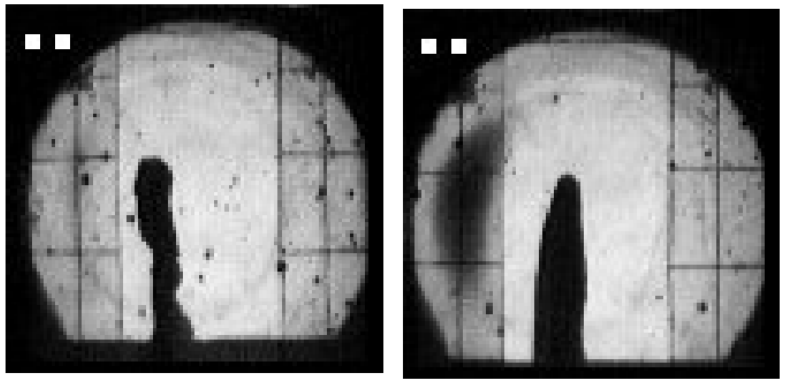
Eddy currents may distort the jet as it traverses the magnet.



Analytic model suggests little effect if jet nozzle inside field.

1 cm diam. jet, $v = 4.6$ m/s, $B = 0$ T; $v = 4.0$ m/s, $B = 13$ T:

No Field



13 T Field

$\Rightarrow$ Damping of surface tension waves (Rayleigh instability).

These are results of Magnetohydrondynamics calculations done with the FronTier code. (R. Samulyak)

These are initial results of an experiment performed at Grenoble by our collaborators.