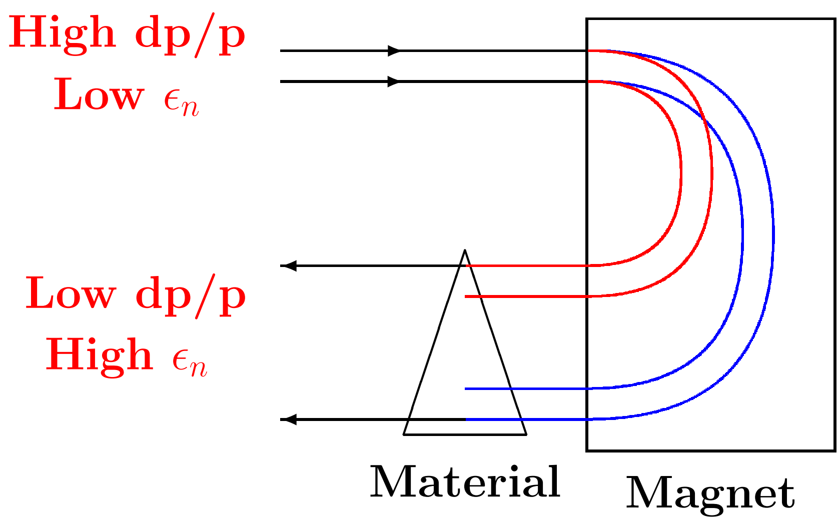
# 1 Longitudinal Cooling

## 1.1 Introduction




- dp/p reduced
- But $\sigma_y$ increased

- Long Emittance reduced
- Trans Emittance Increased

- "Emittance Exchange"

## 1.2 Partition Functions

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} \;=\; -\frac{\frac{\Delta\ (\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{1}$$

$$J_6 = J_x + J_y + J_z \tag{2}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). $\Delta p$ and $p$ refer to the loss of momentum induced by this energy loss.

In the synchrotron case, in the absence of gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In the ionization case, as we shall show, $J_x = J_y = 1$, but $J_z$ is negative or small.

### 1.2.1 Transverse

From last lecture:

$$\frac{\Delta\sigma_p}{\sigma_p} = \frac{\Delta p}{p}$$

and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} = \frac{\Delta p}{p} \tag{3}$$

and thus

$$J_x = J_y = 1 \tag{4}$$

### 1.2.2 Longitudinal

The emittance in the longitudinal direction $\epsilon_z$ is:

$$\epsilon_z = \gamma\beta_v \frac{\sigma_p}{p} \sigma_z = \frac{\sigma_p \ \sigma_z}{m_\mu} = \frac{c \ \sigma_E \ \sigma_t}{m_\mu}$$

where $\sigma_t$ is the rms bunch length in time, and $c$ is the velocity of light. $\sigma_t$ will not change as the beam passes though material.

The relative change in the rms energy spread $\sigma_\gamma$ will be given by

$$\frac{\Delta\sigma_\gamma}{\sigma_\gamma} = -\frac{\delta(d\gamma/ds)}{\delta\gamma} \Delta s$$

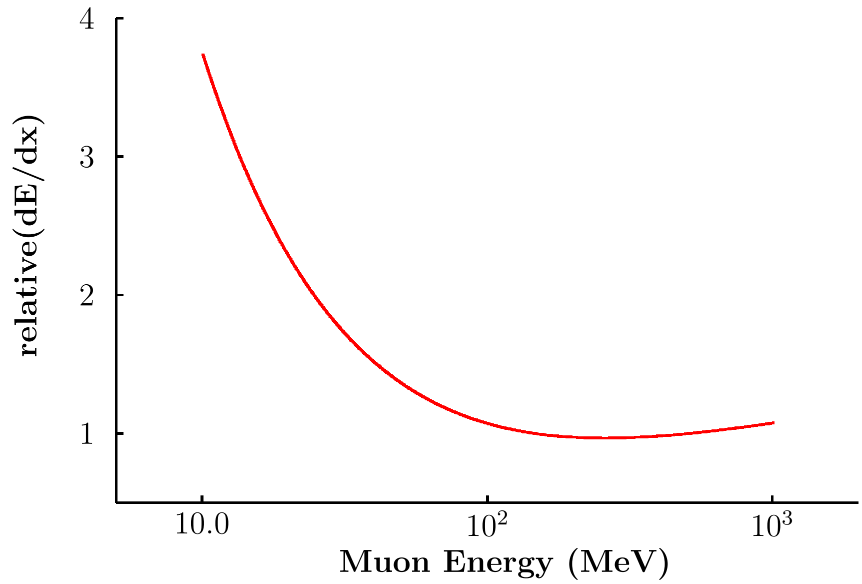
so

$$\Delta\epsilon_z = -\frac{\delta(d\gamma/ds)}{\delta\gamma} \sigma_\gamma \ \sigma_t \ c \ \Delta s$$

From the definition of the partition function $J_z$:

$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\gamma}{\beta_v^2\gamma}} = -\frac{\frac{\delta(d\gamma/ds)}{\delta\gamma}}{d\gamma/ds} \beta_v^2\gamma \tag{5}$$

## Energy Loss




A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It has a minimum at about 300 MeV, a gentle rise above and a steep rise at lower energies. It is given approximately by:

$$\frac{d\gamma}{ds} \;=\; B\,\frac{1}{\beta_v^2}\,(\frac{1}{2}\;ln(A\;\beta_v^4\gamma^4\;-\;\beta_v^2) \tag{6}$$

where

$$A \;=\; \frac{(2m_ec^2/e)^2}{I^2} \tag{7}$$

$$B \approx \frac{0.0307}{(m_\mu c^2/e)}\;\frac{Z}{A} \tag{8}$$

where Z and A are for the nucleus of the material, and I is the ionization potential for that material.

Differentiating the above:

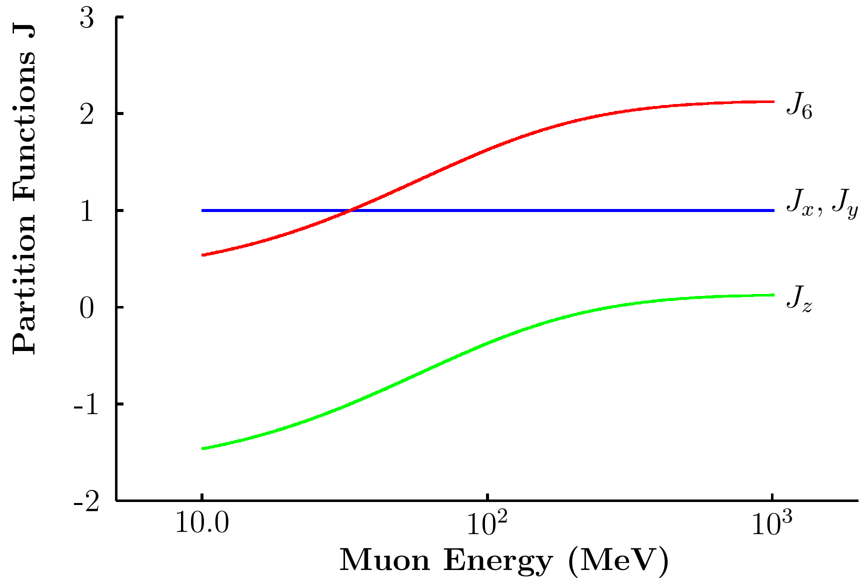
$$\frac{\delta(d\gamma/ds)}{\delta\gamma} \;=\; \frac{B}{\beta_v}\left(\frac{2}{\beta_v\gamma}\;-\;\frac{1}{(\beta_v\gamma)^3}\;ln(A\;\beta_v^4\gamma^4)\;+\;\frac{2}{(\beta_v\gamma)^3}\right)$$

Substituting this into equation 5:

$$J_z \;\approx\; -\frac{\left(\frac{2}{\beta_v \gamma} \;-\; \frac{1}{(\beta_v\gamma)^3}\; ln(A\;\beta_v^4\gamma^4) \;+\; \frac{2}{(\beta_v\gamma)^3}\right)}{\left(\frac{1}{2}\; ln(A\;\beta_v^4\gamma^4 \;-\; \beta_v^2\right)}\;\beta_v^3\gamma \qquad (9)$$

#### 1.2.3 6D Partition Function $J_6$

$J_z$, $J_{x,y}$ and $J_6 \;=\; J_x + J_y + J_z$ are plotted below




It is seen that despite the heating implicit in the negative values of $J_z$ at low energies, the six dimensional cooling $J_6$ remains positive. In fact the relative cooling for a given acceleration $\Delta E$ :

$$\frac{\Delta\epsilon_6/\epsilon}{\Delta E} \;=\; \frac{J_6}{E\;\beta_v^2}$$

rises without limit as the energy falls. This suggests that, for economy of acceleration, cooling should be done at a very low energy. In practice there are many difficulties in doing this, but it remains desirable to use the lowest practical energy.

### 1.2.4 Longitudinal Heating Terms

and from Perkins text book, converted to MKS:

$$\Delta(\sigma_\gamma^2) \approx 0.06\ \frac{Z}{A}\ \left(\frac{m_e}{m_\mu}\right)^2\ \gamma^2\ \left(1-\frac{\beta_v^2}{2}\right)\ \rho\ \Delta s\ =\ 2\sigma_\gamma\ \Delta\sigma_\gamma$$

$$\epsilon_z\ =\ \sigma_\gamma\ \sigma_t\ c$$

Since $t$ and thus $\sigma_t$ is conserved

$$\frac{\Delta\epsilon_z}{\epsilon_z}\ =\ \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

and using eq. **??**:

$$\Delta s\ =\ \frac{\Delta p}{p}\ \frac{\beta_v^2\ E}{dE/ds}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z}\ =\ \frac{0.06}{2\sigma_\gamma^2}\ \frac{Z}{A}\ \left(\frac{m_e}{m_\mu}\right)^2\ \gamma^2\ \left(1-\frac{\beta_v^2}{2}\right)\ \rho\ \frac{\beta_v^2\ E}{dE/ds}\ \frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z}\ =\ -\ J_z\ \frac{dp}{p}$$

giving an equilibrium:

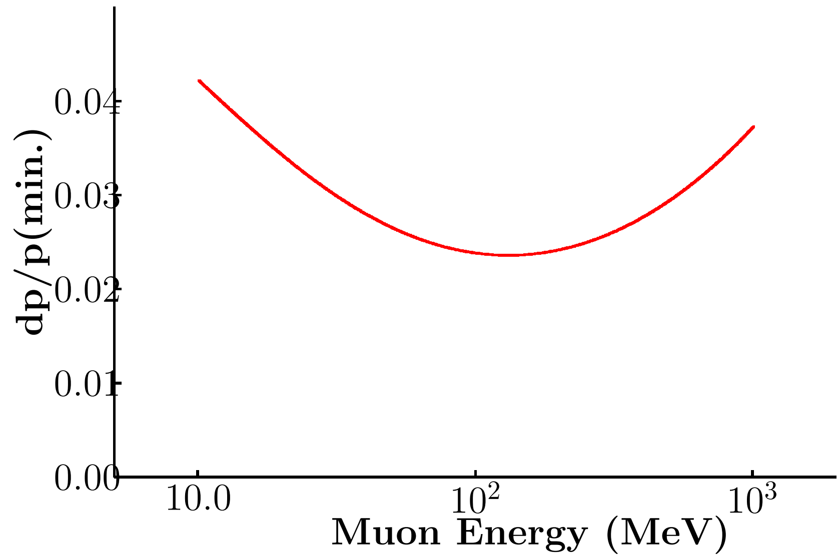
$$\frac{\sigma_p}{p}\ =\ \left(\left(\frac{m_e}{m_\mu}\right)\ \sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right)\ \sqrt{\frac{\gamma}{\beta_v^2}\ \left(1-\frac{\beta_v^2}{2}\right)\ \frac{1}{J_z}} \tag{10}$$

For Hydrogen, the value of the first parenthesis is ≈1.45 %.

If there is no coupling between transverse and longitudinal emittances then $J_z$ is small or negative, and the equilibrium does not exist or is large.

However, since $J_6$ is always greater than 0, we can use wedges to redistribute the $J$'s to allow $J_z\ =\ J_6/3$.

The following plot shows the dependency for hydrogen




It is seen to favor cooling at around 300 MeV/c, but has a broad minimum.

### 1.2.5 rf and bunch length

To obtain the Longitudinal emittance we need $\sigma_z$.

If the rf acceleration is relatively uniform along the lattice, then we can write the synchrotron wavelength:

$$\lambda_s \;=\; \sqrt{\frac{\beta_v \gamma \; \lambda_{rf} \; (m_\mu)}{\alpha \mathcal{E}_{\rm rf} \; \cos(\phi)}} \tag{11}$$

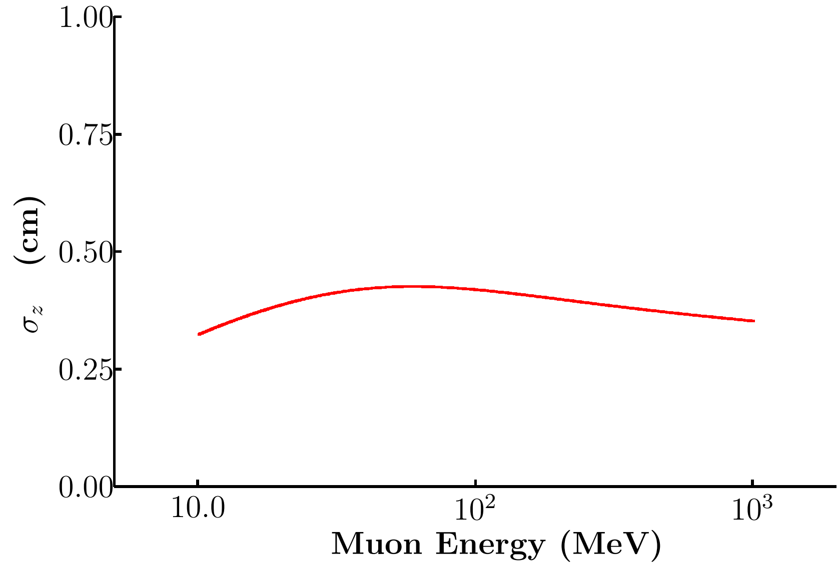
where, in a linear lattice

$$\alpha \;=\; \frac{1}{\gamma^2} \tag{12}$$

and the field $\mathcal{E}_{\rm rf}$, ie it is the rf accelerating field; $\phi$ is the rf phase, defined so that for $\phi = 0$ there is no acceleration.

The bunch length, given the relative momentum spread $dp/p = \delta$, is given by:

$$\sigma_z \;=\; \delta \; \frac{\beta_v \; \alpha \; \lambda_s}{2\pi} \;\propto\; \frac{\beta_v^{3/2}}{\gamma} \; \sqrt{\frac{\lambda_{rf}}{\mathcal{E}_{\rm rf}}} \tag{13}$$




It is seen to be only weakly dependent on the energy.

## 1.3 Simulation

Several "local" codes, but
**2 Documented codes**
**(GEANT & ICOOL)**

**Both have:**

- Global fields
  unlike MAD, TRANSORT etc.
- Choices of scattering and straggling formulations
- Standing Wave RF fields
- allow use of both
  1. Maxwellian, or
  2. "hard edged" magnetic fields
- Flexible Geometries

- Good tracking

### 1.3.1 GEANT

- CERN code
- **Works in Cartesian Coord's**
- Uses field maps in 3D
- Requires tweaking to get reference orbit
- Good graphics
- 3 versions:

  1. GEANT 3 is in Fortran single precision (not suitable)
  2. GEANT DP has been modified and has been much used
  3. GEANT 4 is new, C++, and good, but lacks some ease of use

### 1.3.2 ICOOL

- BNL (Rick Fernow) Fortran code
- **Works in Transport" Coords**
- Uses field maps in 2D, OR
- Field multipoles about a reference orbit
- No tweaking needed
- But does not specify exact coil locations needed
- Poor graphics
- Some Optimization Capability with "OPTICOOL"

## 1.4 Emittance Exchange Studies

- Attempts at separate cooling & exch.
  - Wedges in Bent Solenoids
  - Wedges in Helical Channels[1]

  Poor performance & problems matching between them

- Attempts in rings with alternate
  cooling & exchange
  - Balbakov[2] with solenoid focus
    achieved Merit=38-94

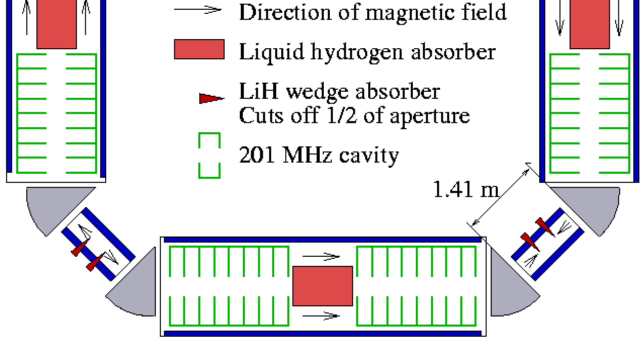
- Attempts in rings with combined cooling & exchange
  - Garren et al[3] Quadrupole focused ring
    achieved Merit ≈15
  - Palmer et al[4]
    achieved Merit ≈160

## 1.5 Example 1

## Balbekov 6D Cooling Ring

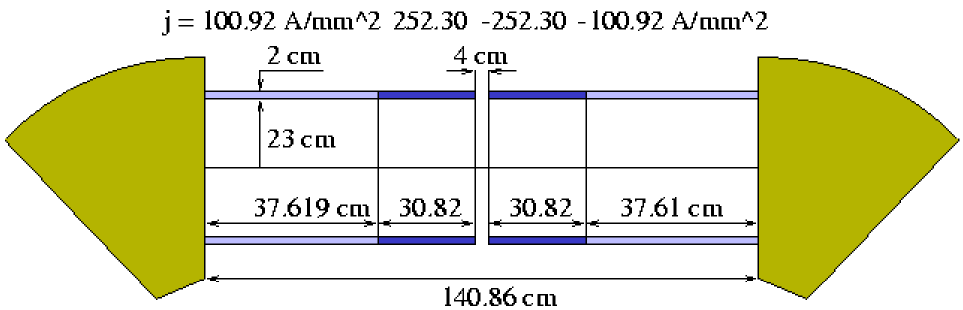
Alternate transverse cooling with H2 with emittance exchange in Li wedge

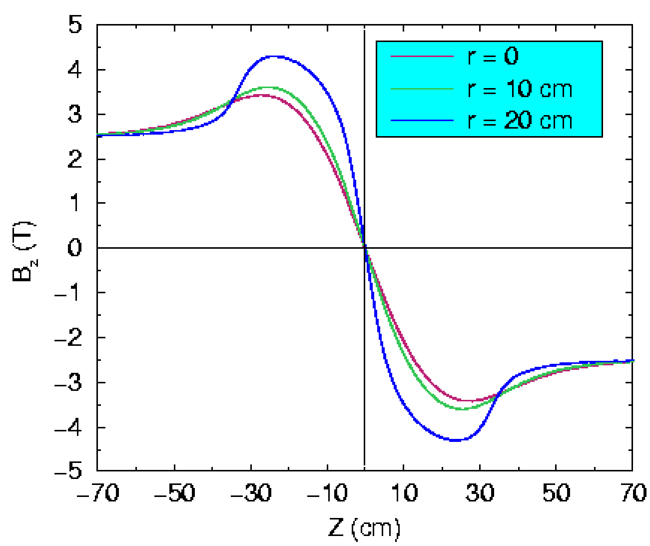
[1] MUC-146, 147, 187, & 193
[2] MUC-232 & 246
[3] Snowmass Proc.
[4] MUC-239




| | |
|---|---|
| Circumference | 36.963 m |
| Cell Length | 2.27+6.97=9.25 m |
| Energy | 250 MeV |
| Max $B_z$ | 5.155 T |
| RF Frequency | 205.69 MHz |
| Gradient | 15 MV/m |

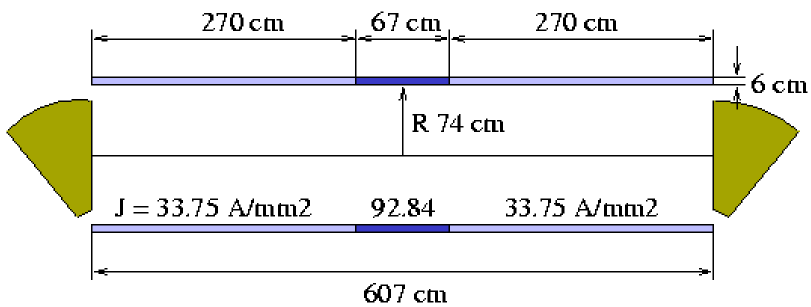
## Short Straight

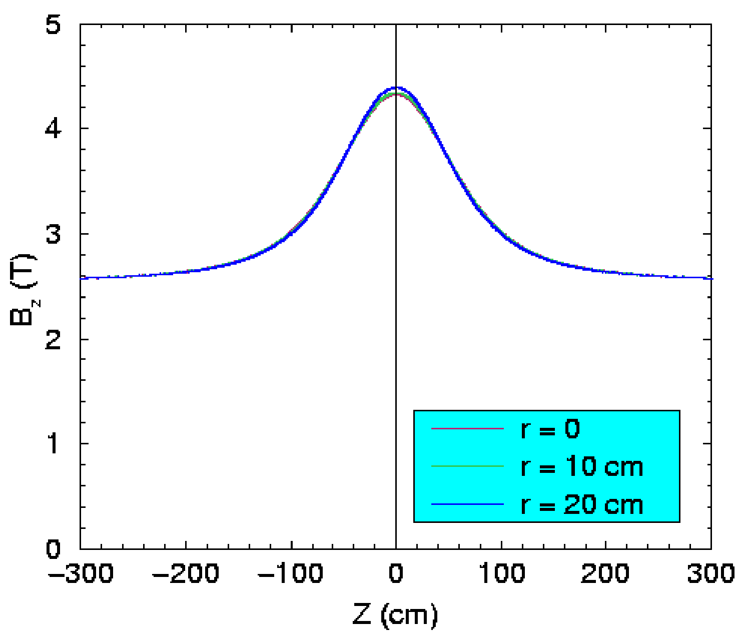


- Field flip in center
- Max Dispersion in center
- LiH Wedge in center

## Long Straight

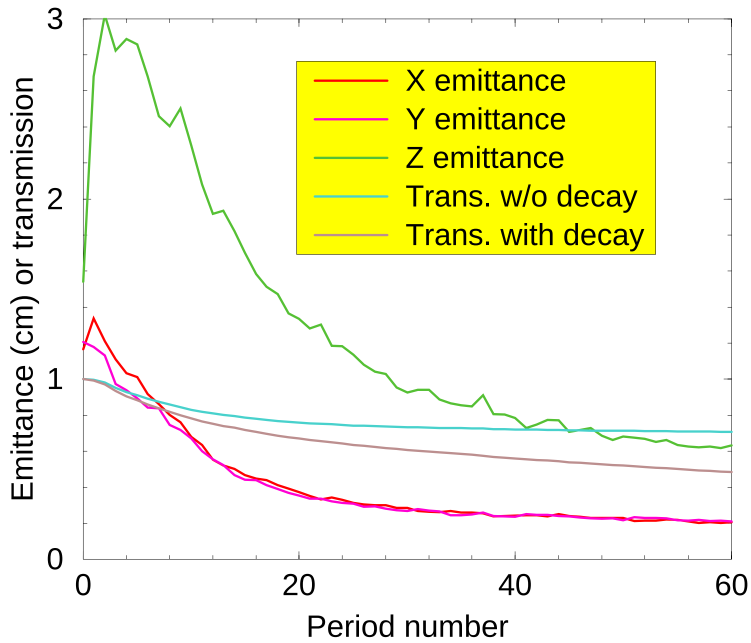



- No flip & No Dispersion
- Higher field & lower $\beta$ at center
- Hydrogen Absorber at center
- RF on either side

### 1.5.1 Performance

| | Before | After |
|---|---|---|
| $\epsilon_{\perp}$ (cm) | 1.2 | 0.21 |
| $\epsilon_{\parallel}$ (cm) | 1.5 | 0.63 |
| $\epsilon_6$ (cm$^3$) | 2.2 | 0.028 |
| $\epsilon_6/\epsilon_{60}$ | 1 | 79 |
| $N/N_0$, no decay | 1 | 0.71 |
| $N/N_0$, inc. decay | 1 | 0.48 |
| Merit | 1 | 38 |




### 1.5.2 Conclusion for Balbakov

- Good cooling in all dimensions
- Merit Factor 38
  c.f. Study 2 Linear: Merit=15

**BUT**

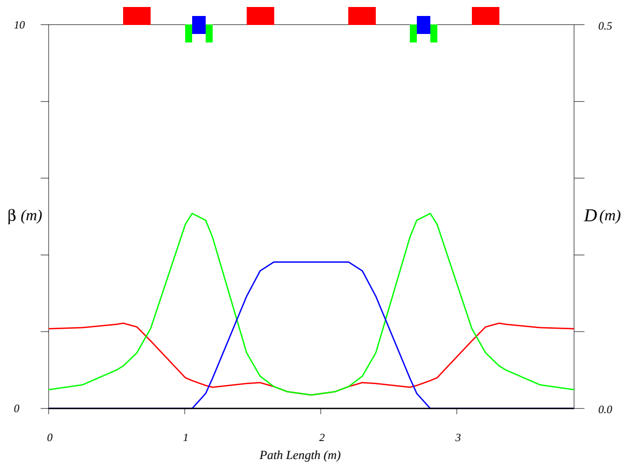
- Calculated without Maxwellian fields
- Design of bends proving hard
- Injection and extraction very hard
  Merit $\rightarrow 3.9$ with missing rf
- Upward spiral an alternative

### 1.6 Example 2: Quadrupole Ring

Garren, Kirk

- Motivation
  - Easier to design lattice (dispersion suppression, etc.)
  - More experience than with solenoids
- Thick wedge: both cooling and longitudinal/transverse coupling
- Long. and Transverse in same Cell
- Limit phase advance per cell
- Avoid crossing resonances

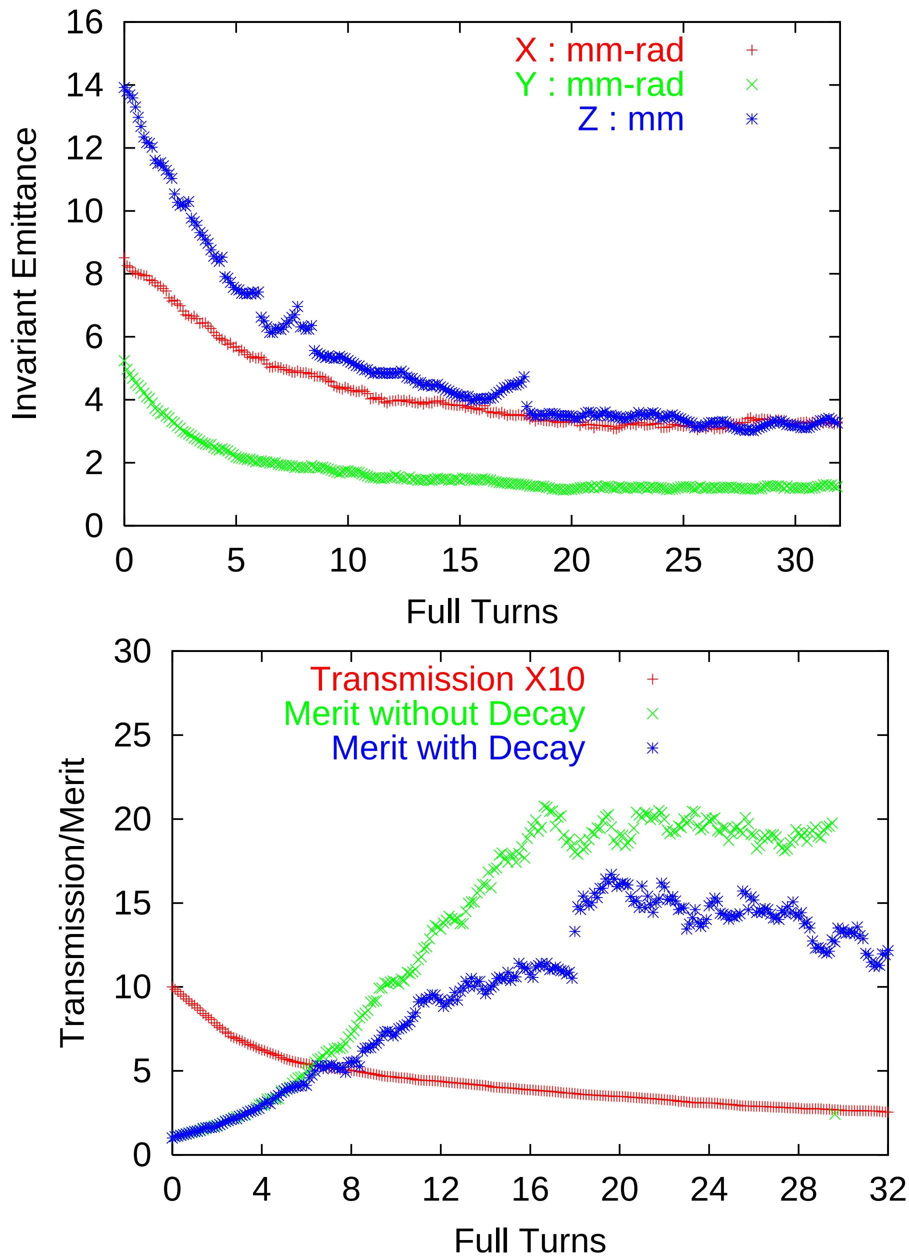
# One of 8 Cells




# Parameters

| | |
|---|---|
| Circumference | 31 m |
| Cell Length | 3.8 m |
| Momentum | 250 MeV/$c$ |
| Magnet aperture (full) | 40 cm |
| Max pole tip field | 2 T |
| RF Frequency | 200 MHz |
| RF Gradient | 16 MV/m |

# Performance



Invariant Emittance
X : mm-rad
Y : mm-rad
Z : mm
Full Turns
Transmission/Merit
Transmission X10
Merit without Decay
Merit with Decay
Full Turns

| | Before | After | ratio |
|---|---|---|---|
| $\epsilon_x$ (mm) | 8.5 | 3.4 | 2.5 |
| $\epsilon_y$ (mm) | 5.2 | 1.2 | 4.2 |
| $\epsilon_\parallel$ (mm) | 14 | 3.8 | 3.7 |
| $\epsilon_6$ (mm$^3$) | 0.62 | 0.015 | 39 |
| $N/N_0$, inc. decay | 1 | 0.41 | .41 |
| Merit | 1 | 16 | **16** |

# Conclusion

- Final trans. emittance similar to Balbakov
- Longitudinal emittance lower than Balbakov

**BUT**

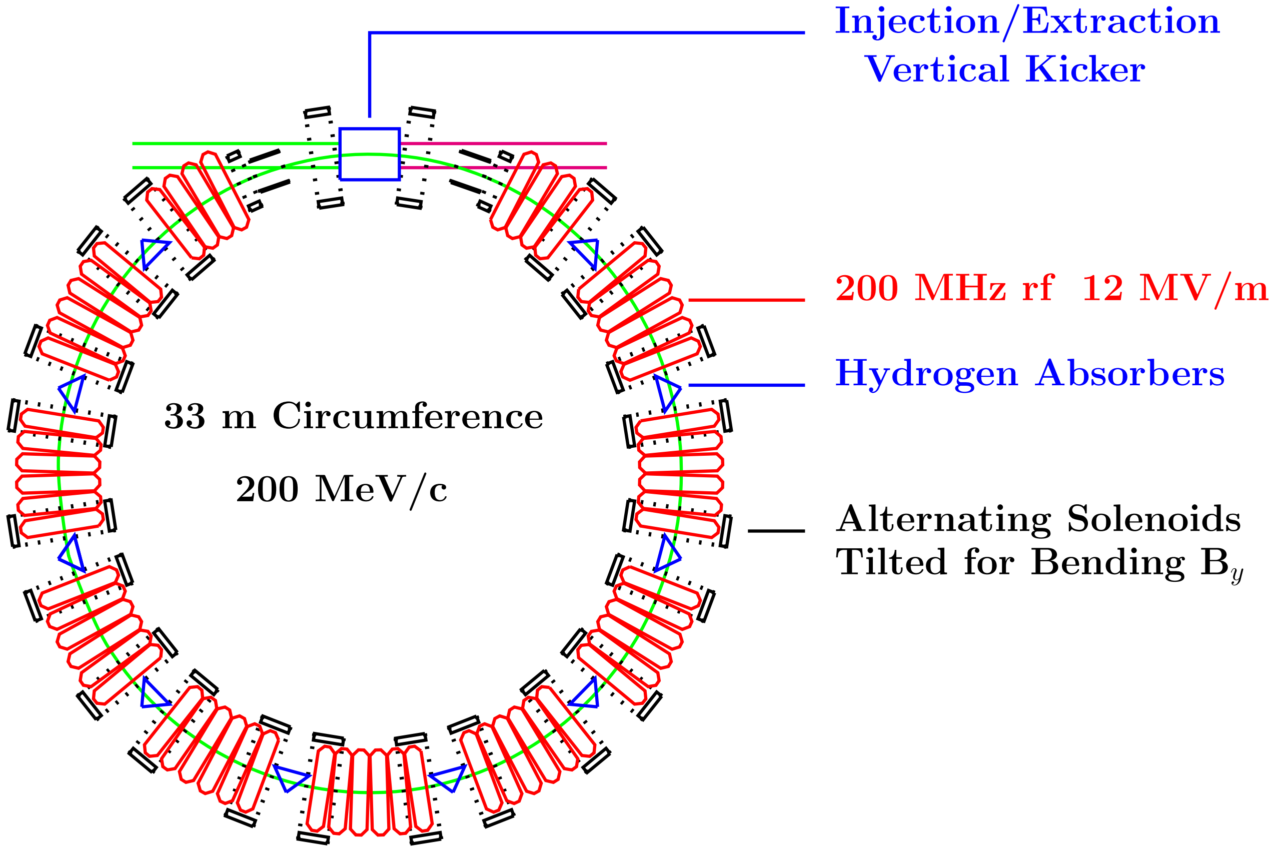
- Currently Less acceptance and thus less Merit
- Probably due to use of Quads vs Solenoids

## 1.7 Example 3

## RFOFO Ring[5]

[5] MUC-232

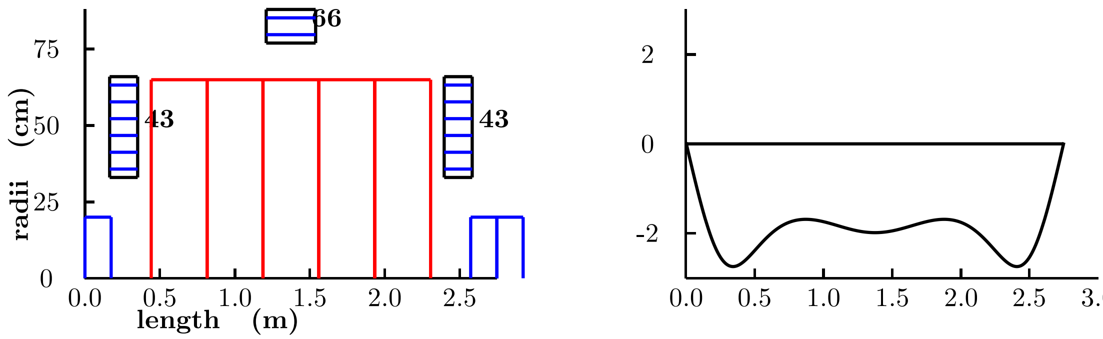
### 1.7.1 Introduction



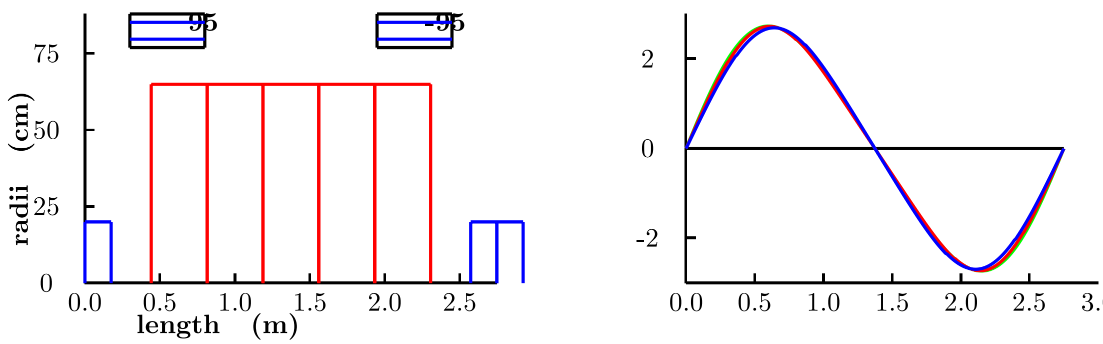

### 1.7.2 Lattice

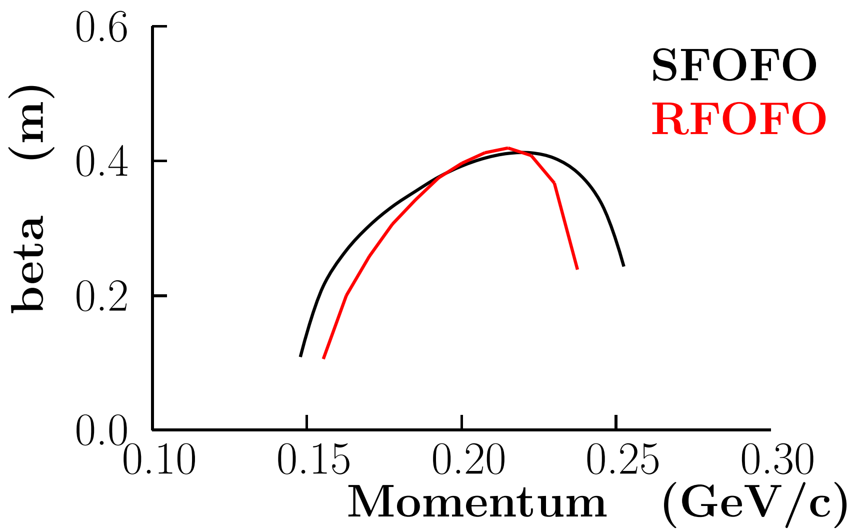
- Make all cells ≈same
  avoid matching problems

**SFOFO as in Study 2**




**RFOFO has Reversed Fields**





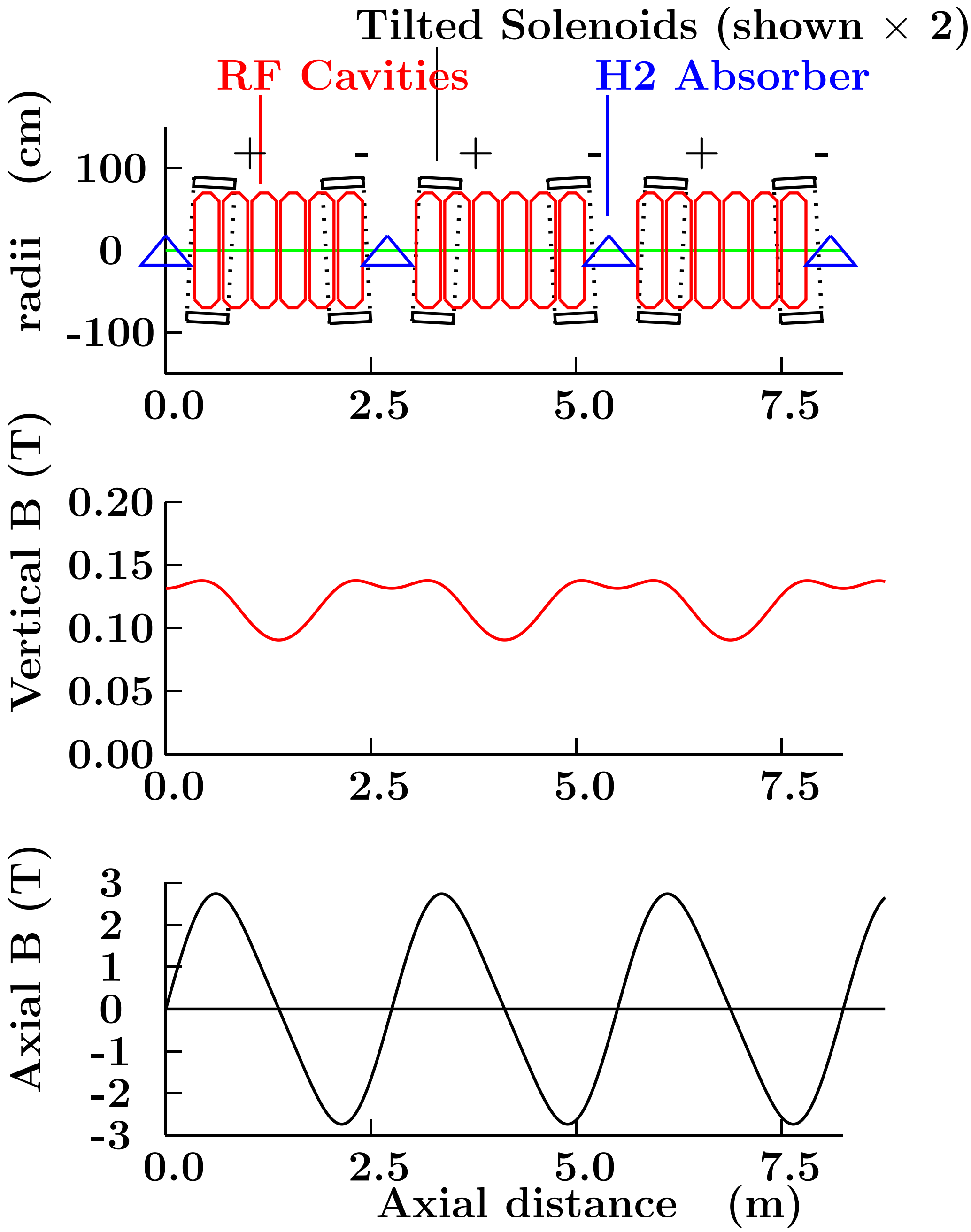

- RFOFO Mom acceptance worse

BUT

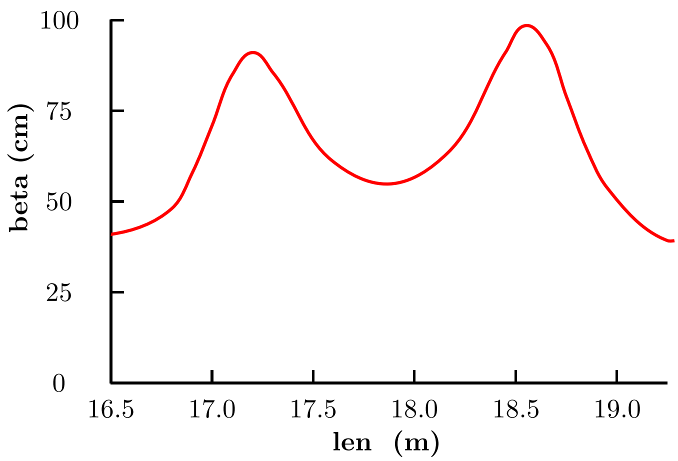
- All cells the same
- Fewer resonances
- Choose RFOFO

### 1.7.3 Tilt Coils to get Bend

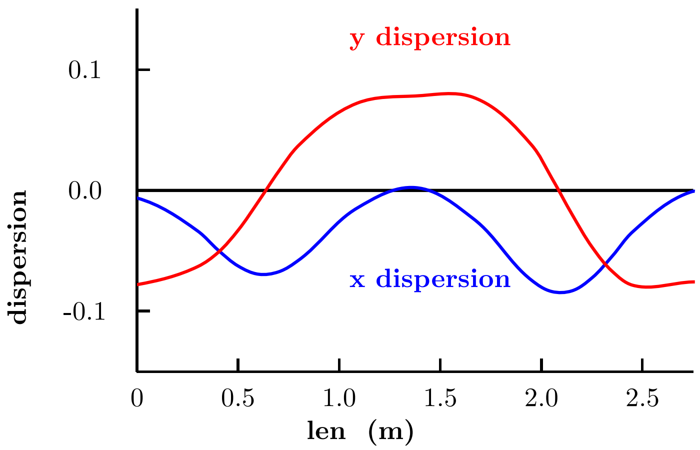
.



## Beta and Dispersion .



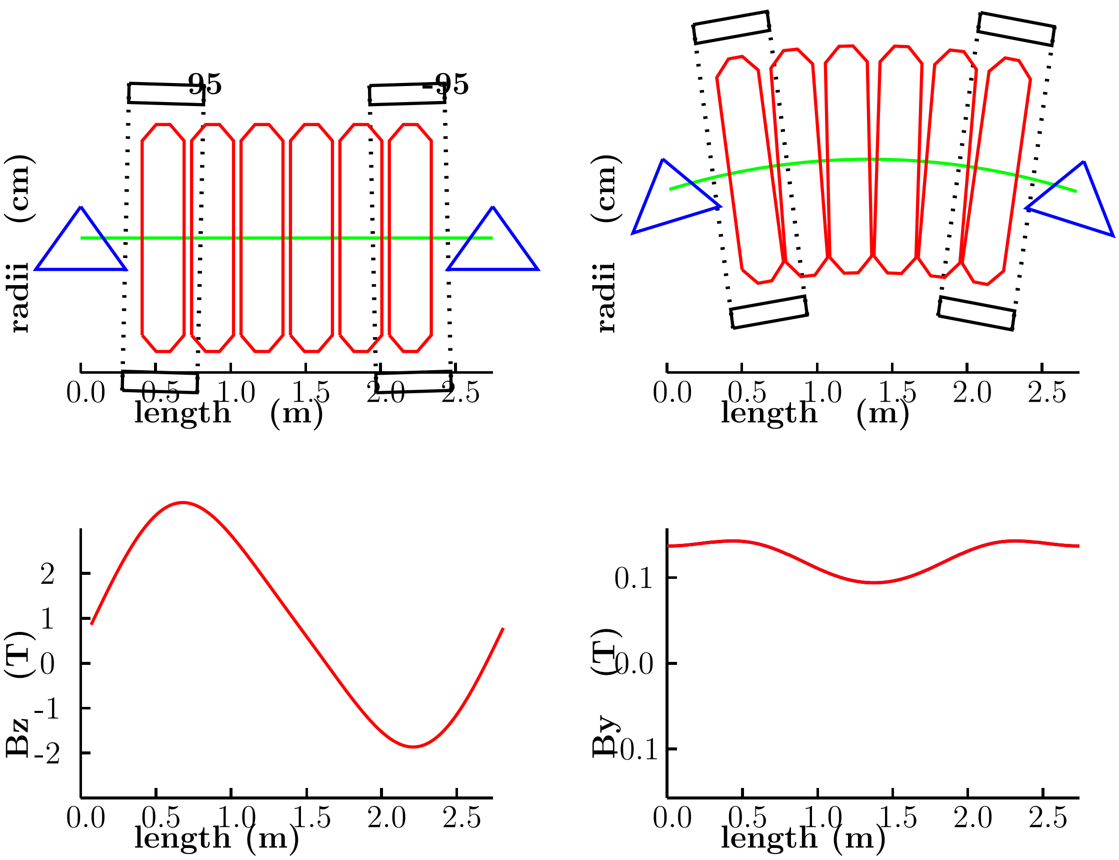

beta is ≈ straight case




Dispersion is rotating
back and forth

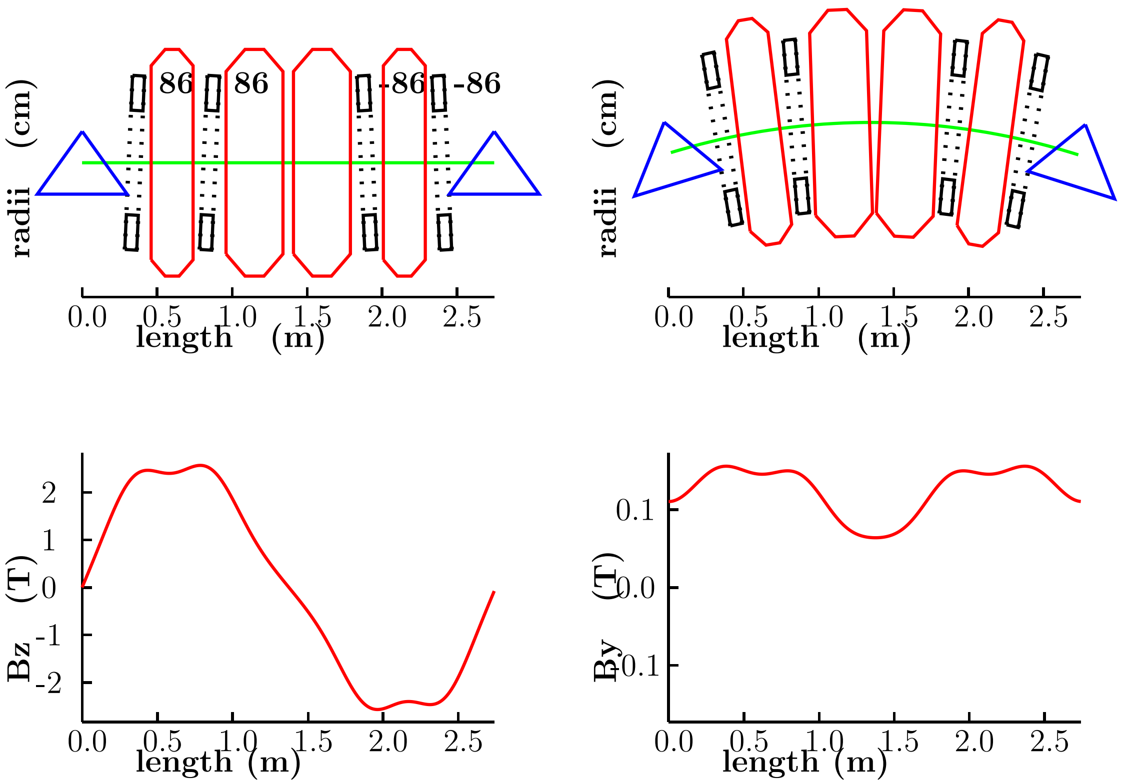
### 1.7.4 Cell Layouts

## a) Coils outside RF




- Wedges shown 0 and 90 deg.
  true angle 30 deg
- Amp-turn-length = 54 MAm/cell
- RF Grad = 12 MV/m

## b) Coils between Cavities




- Amp-turn-length = 14 MAm/cell
- RF Grad = 16 MV/m

- Performance the same
- Choice not yet made

### 1.7.5 Params for Simulation

Coils

| gap m | start m | dl m | rad m | dr m | tilt rad | I/A $A/mm^2$ |
|---|---|---|---|---|---|---|
| 0.310 | 0.310 | 0.080 | 0.300 | 0.200 | 0.0497 | 86.25 |
| 0.420 | 0.810 | 0.080 | 0.300 | 0.200 | 0.0497 | 86.25 |
| 0.970 | 1.860 | 0.080 | 0.300 | 0.200 | -.0497 | -86.25 |
| 0.420 | 2.360 | 0.080 | 0.300 | 0.200 | -.0497 | -86.25 |

amp turns 5.52 (MA)
amp turns length 13.87326 (MA m)
cell length 2.750001 (m)

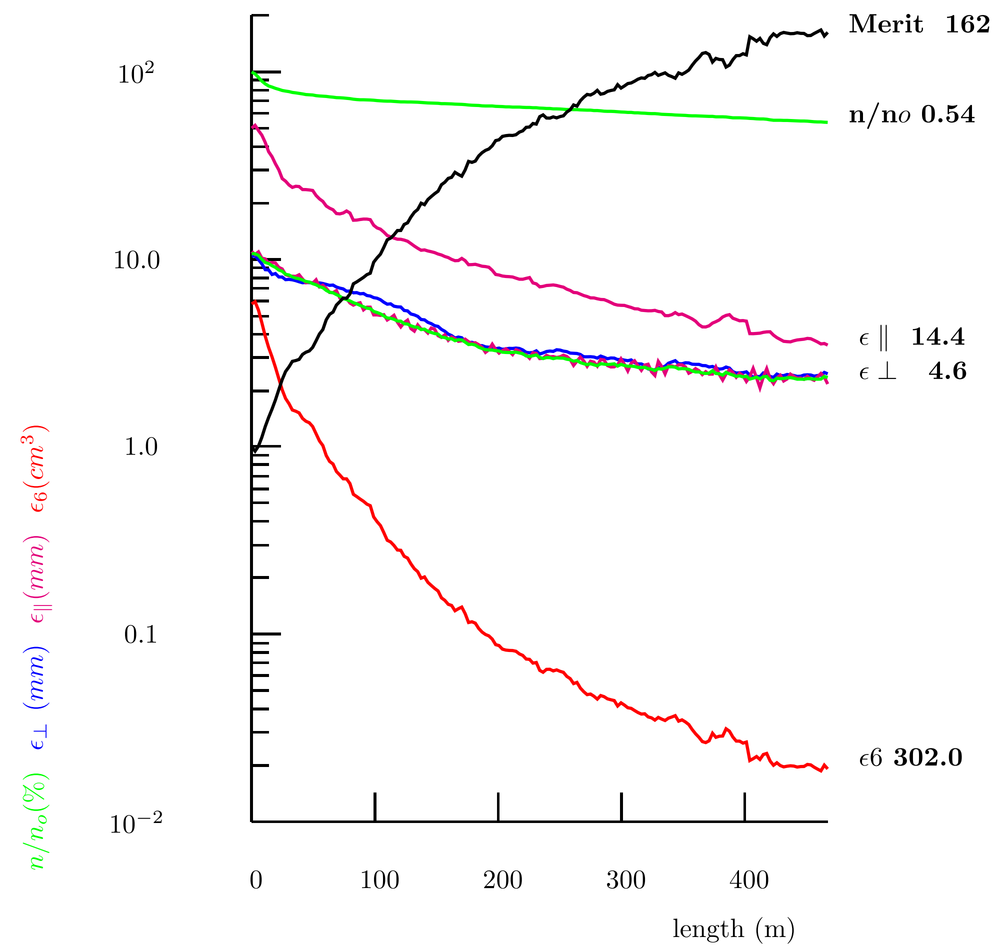
Wedge

| | | |
|---|---|---|
| Material | | H2 |
| Windows | | none |
| Radius | cm | 18 |
| central thickness | cm | 28.6 |
| min thickness | cm | 0 |
| wedge angle | deg | 100 |
| wedge azimuth from vertical | deg | 30 |

RF

| | | |
|---|---|---|
| Cavities | | 6 |
| Lengths | cm | 28 |
| Central gaps | cm | 5 |
| Radial aperture | cm | 25 |
| Frequency | MHz | 201.25 |
| Gradient | MV/m | 16 |
| Phase rel to fixed ref | deg | 25 |
| Windows | | none |

### 1.7.6 Performance




| | len | trans | $\epsilon_\perp$ | dp/p | $\epsilon_\parallel$ | $\epsilon_6$ | max Q | merit |
|---|---|---|---|---|---|---|---|---|
| | m | % | $\pi$ mm | % | $\pi$ mm | $\pi^3$cm$^3$ | | |
| final | 468 | 54 | 2.3 | 4.0 | 3.5 | 0.019 | 24 | **162** |
| initial | | | 10.7 | 11.2 | 50.1 | 5.787 | | |
| ratio | | | 4.6 | 2.8 | 14.4 | 302.0 | | |

If $J_\perp = 1$ then:

$$\epsilon_\perp(\min) \;=\; \frac{38\ 10^{-4}\ 0.4}{0.85} \;=\; 1.8 \;\; (\pi mm\ mrad)$$

So here

$$J_\perp \approx \frac{1.8}{2.3} \; = \; 0.78$$

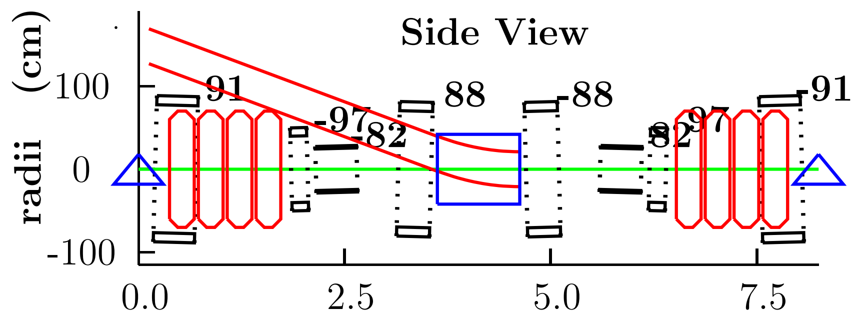
$$J_\parallel \approx 2 - 2\; 0.78 = 0.43$$

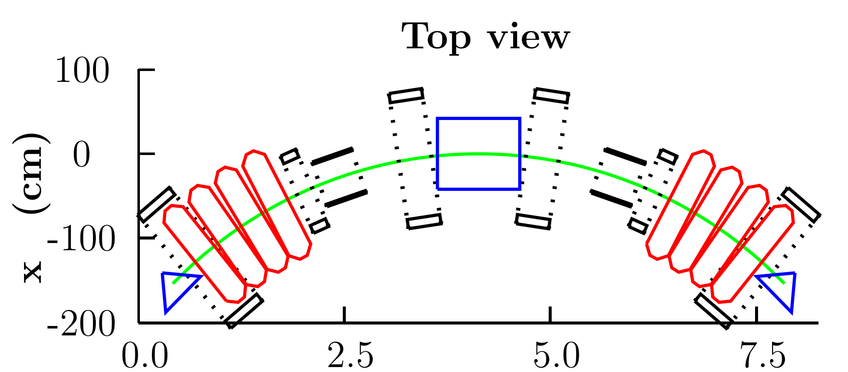
From equation 10 we expect

$$\frac{dp}{p}(\mathrm{min}) \;\approx 3\%$$

The observed value is 4%, but it is still falling.
An equilibrium of 3 % appears reasonably correct

### 1.7.7 Insertion for Inject/Extract

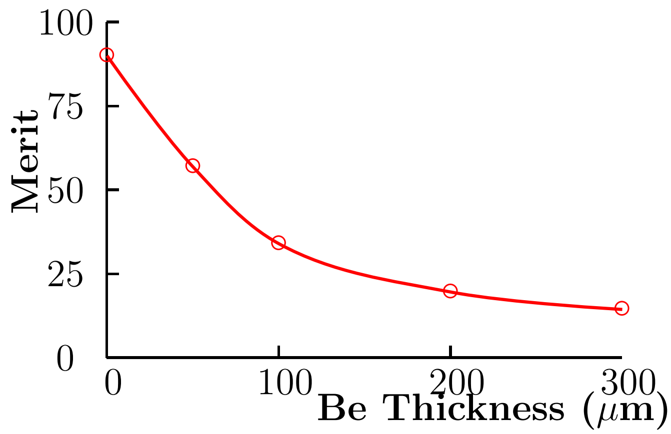


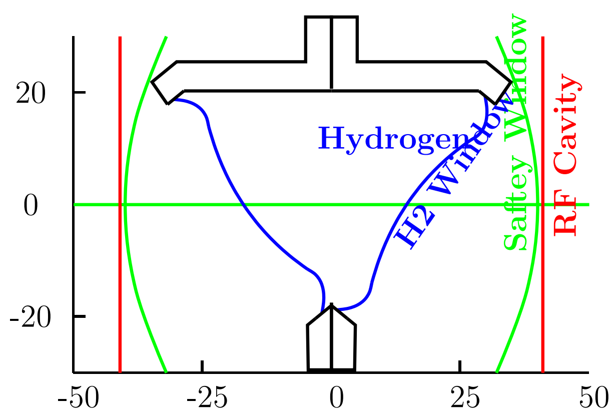

- First Simulation gave Merit = 10
  Synchrotron tune = 2.0: Integer
- Increase energy, wedge angle, and add matching.
- Merit 160 → 110

### 1.7.8 Unanswered Questions

- RF windows must be very thin
- RF at 70 deg will help




- Design of wedge absorber




- But best with apex inside aperture

### 1.7.9 Conclusion for RFOFO Ring

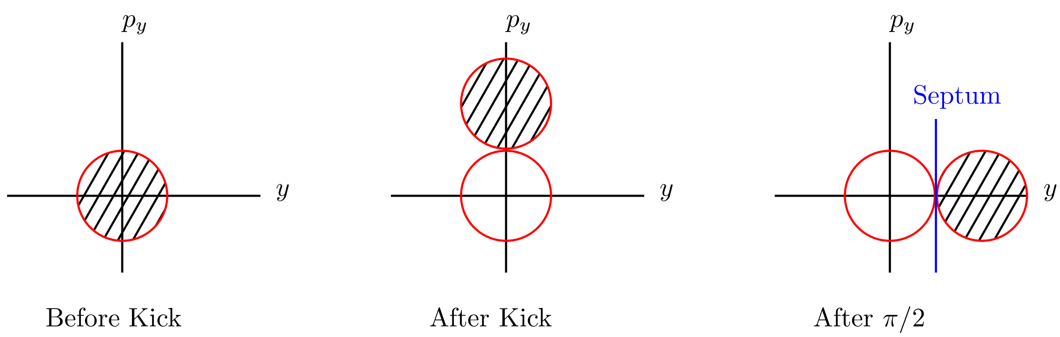
- RFOFO Ring Cools better than linear a channel
  Merit 160 for ring vs 15 for Study 2

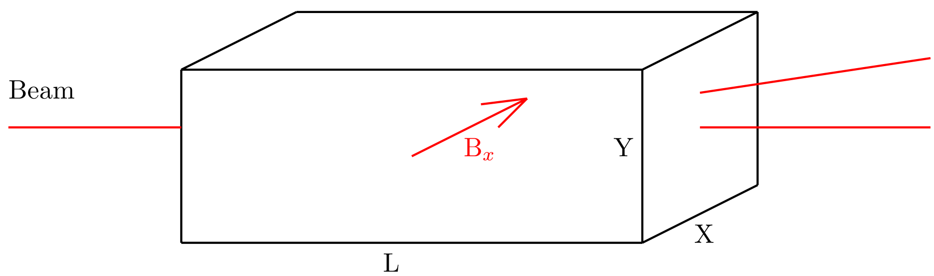
- Uses fewer components
  33 m ring vs. 108 m Study 2
- Simulation done with Maxwellian Fields
  But exact coil positions need determining
- Simulation with GEANT Needed
- Injection insertion details not designed
- Kicker still problematical

# 2 Injection/Extraction

## 2.1 Kickers

### 2.1.1 Minimum Required kick




$$f_\sigma = \frac{\mathrm{Ap}}{\sigma} \qquad \mu \;=\; \mathrm{inf} \qquad F \;=\; \frac{Y}{X}$$

$$I = \left( \frac{4 \; f_\sigma^2 \; m_\mu}{\mu_o \; c} \right) \quad \frac{\epsilon_n}{L}$$

$$V = \left( \frac{4 \; f_\sigma^2 \; m_\mu \; R}{c} \right) \quad \frac{\epsilon_n}{\tau}$$

$$U = \left( \frac{m_\mu^2 \; 8 \; f_\sigma^4 \; R}{\mu_o \; c^2} \right) \quad \frac{\epsilon_n^2}{L}$$

- muon $\epsilon_n$ $\gg$ other $\epsilon_n$'s
- So muon kicker Joules $\gg$ other kickers
- Nearest are $\bar{p}$ kickers

## Compare with others

For $\epsilon_\perp = 10\ \pi$mm, $\beta_\perp = 1$m, & $\tau$=50 nsec:
After correction for finite $\mu$ and leakage flux:

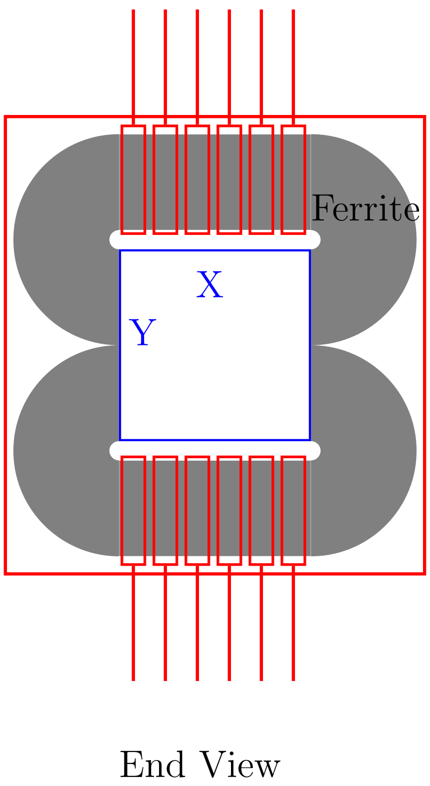
| | | $\mu$ Cooling | CERN $\bar{p}$ | Ind Linac |
|---|---|---|---|---|
| $\int Bd\ell$ | Tm | .30 | .088 | |
| L | m | 1.0 | $\approx$5 | 5.0 |
| $t_{\mathrm{rise}}$ | ns | **50** | **90** | **40** |
| B | T | **.30** | **$\approx$0.018** | **0.6** |
| X | m | .42 | .08 | |
| Y | m | .63 | .25 | |
| $\mathrm{V}_{\mathrm{1turn}}$ | kV | **3,970** | **800** | **5,000** |
| $\mathrm{U}_{\mathrm{magnetic}}$ | J | **10,450** | **$\approx$13** | **8000** |

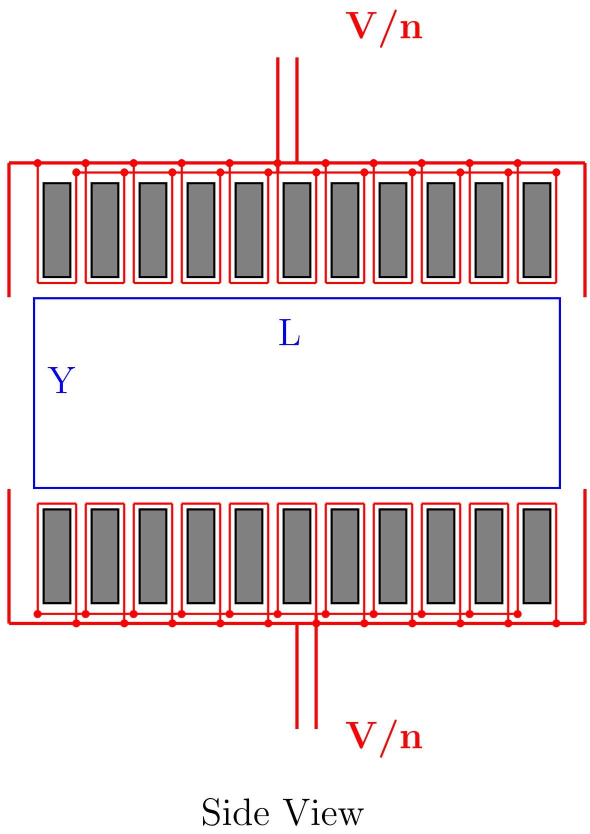
# Note

- U is 3 orders above $\bar{p}$
- Same order as Induction
- And t same order
- But V is too High

### 2.1.2 Induction Kicker

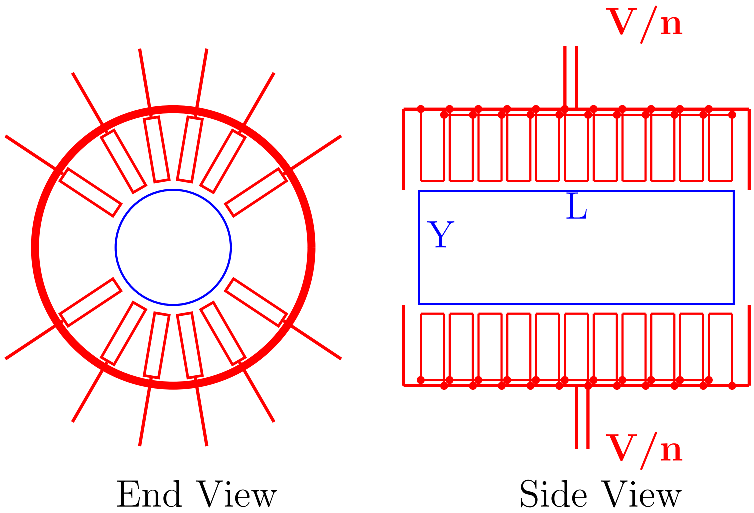
- Drive Flux Return
- Subdivide Flux Return Loops
  Solves Voltage Problem
- Conducting Box Removes
  Stray Field Return



# Works with no Ferrite

- V = the same
- U≈ 2.25×
- I≈ 2.25×
- No rise time limit
- Not effected by solenoid fields

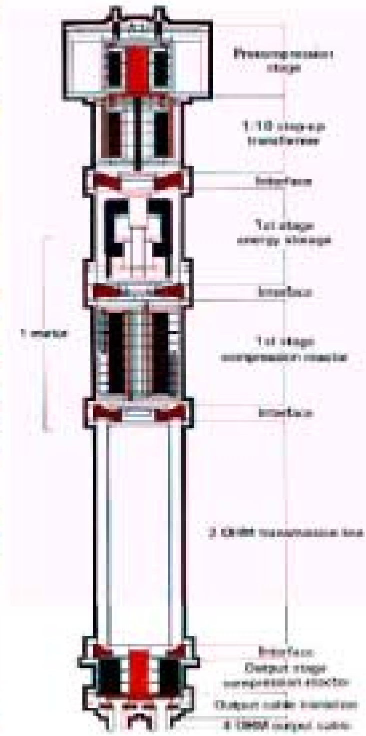


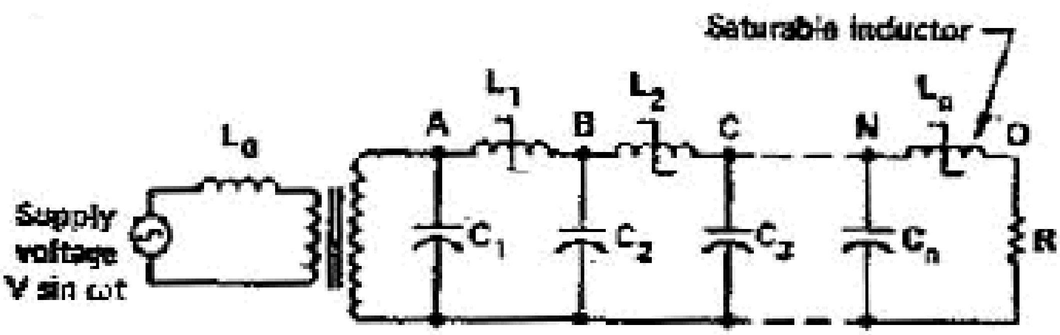

- If non Resonant: 2 Drivers
  for inj. & extract.
  Need 24 ×2 Magamps (≈ 20 M$)
- If Resonant: 1 Driver, 2×efficient
  Need 12 Magamps (≈ 5 M$)

.

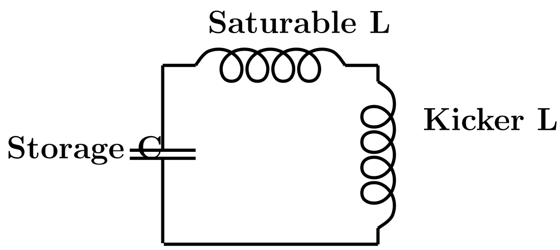
## 2.2 Magnetic Amplifiers

Used to drive Induction Linacs
similar to ATA or DARHT


### Magamp principle




Initially Unsaturated, $L = L_1$ is large:

$$\tau_L \;=\; \sqrt{(L+L_1)C} \quad \text{is slow}$$

The current $I$ rises slowly:

$$I \;=\; I_o \; \sin\left(\frac{t}{\tau_L}\right)$$

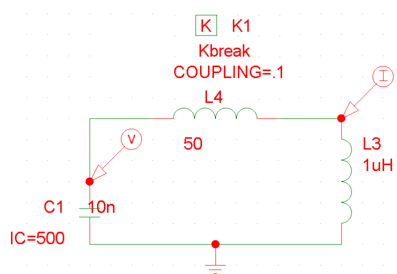
When the inductor saturates
$L = L_2$ is small:

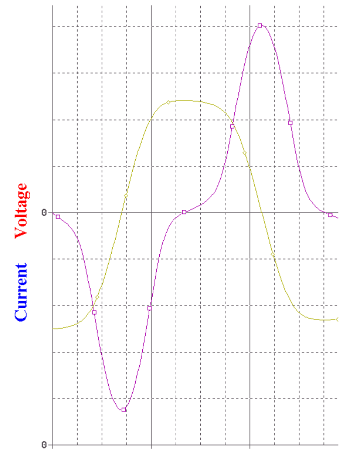
$$\tau_S \;=\; \sqrt{(L+L_2)C} \qquad \text{is fast}$$

After approx $\pi$ phase
Inductor regains its high inductance
The oscillation slows before reversing.

# Pspice Simulation

## a) Single stage




Circuit Model (Reginato)

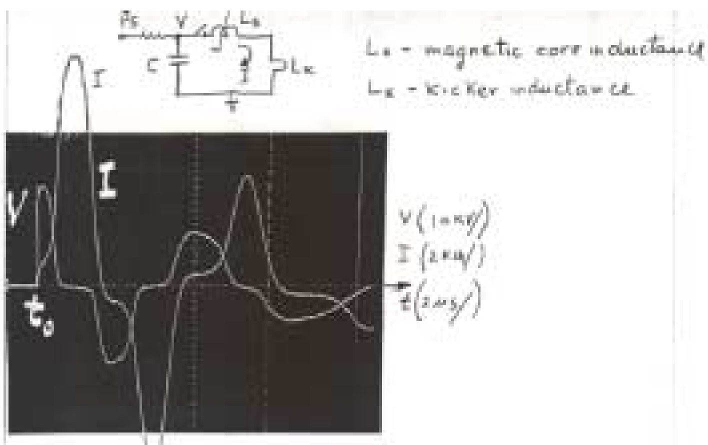
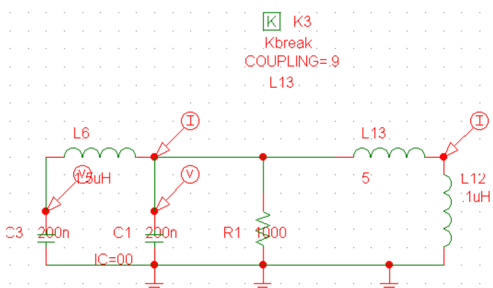
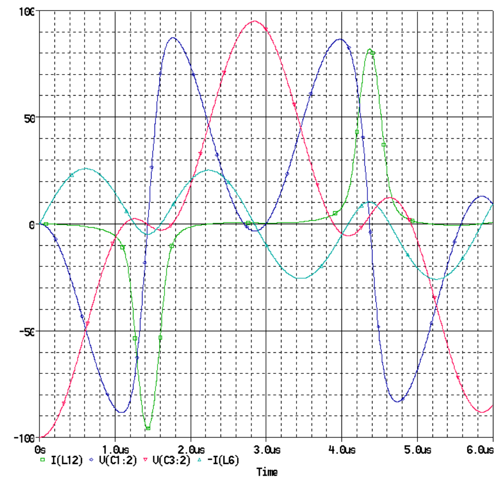



fig. 1

## a) 2 Stage




# 3 Ring Cooler Conclusion

- Rapid Progress has been made.
- Need for very thin windows is greater than for linear coolers
- Work needed on Hydrogen wedge design
- Much Work needed on Insertion
  but probably doable
- The Kicker is the least certain
- Need pre-cooler or other ideas to match phase space into short bunch train

BUT

- Performance better than linear coolers
- Might lower acceleration cost
- Real hope that Collider requirements may be met