Muon Cooling Rings and Kickers

Draft 2

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Abstract

Parameters of an initial muon cooling ring are considered. The advantages of higher cooling energies, use of harmonic rf, and of a single bunch instead of a train, are discussed. Injection and extraction kicker requirements are considered. It is shown that a kicker's required current and total voltage are proportional to the normalized emittance of the beam; and that the stored energy is proportional to the square of that emittance. All three parameters are independent of the $\beta_\perp$ and of the energy in the ring; and are approximately inversely proportional to the circumference. For a 30 m circumference ring and beam with $\epsilon_n = 10\ \ \pi$ mm, the kicker energy and voltage are both far higher than in a conventional kicker. An 'induction kicker', powered by magnetic amplifiers, similar to those in induction linacs, is proposed.

1 Cooling Ring Parameters

An attraction of cooling rings, as opposed to linear cooling, is the possible savings in total length, and thus in cost, but there will be no such reduction if the circumference of the ring is not significantly shorter than the linear cooling it replaces. With study 2 parameters, continuous cooling takes place in about 100 m. We may conclude that a cooling ring circumference should be small compared with this: say 30 m.

The rotation time in 30 m is about 90 nsec, and this must be divided between:

- the bunch, or bunch train being cooled; equaling the good flat top of the injection kicker pulse t_{kick};
- the rise/fall time of the kicker pulse t_{rise}; and
- time for transients.

Let us consider dividing the time equally between them: 30 nsec each

$$t_{\text{kick}} = t_{\text{rise}} = 30 \ \text{nsec}$$

A 30 nsec long bunch train length is small compared with the 150, or 300, nsec trains in the Feasibility studies 1 and 2. At 200 MHz the train will contain only 6 bunches. For an energy spread in the ring as in Study 2 ($\sigma_p \approx 20$ MeV/c), the initial acceptance in these 6 bunches will be $\approx$ 1/5 of that in these studies, and the number of muons captured will be relatively small. Three things would help:

1. increasing the energy in the cooling ring, and
2. using harmonic rf, or
3. using low frequency rf and a single bunch.

1.1 Increasing Cooling Energy

The ratio of momentum spread to bunch length in a synchrotron bucket is given (see Appendix) by

$$\frac{\sigma_P}{\sigma_Z} \;=\; \sqrt{\frac{\omega_{rf}\ \mathcal{E}\ \cos(\phi)\ \gamma\ \mathrm{m}_\mu}{\alpha\ \mathrm{c}}}$$

where $\mathcal{E}$ is the average rf gradient, $\omega_{rf}/2\pi$ is the rf frequency, $\cos(\phi)$ is the rf phase relative to the zero crossing, and α is the momentum compaction. Here, and elswhere in this paper, the dimensions of energies, masses and momenta are in Volts, and all other units are MKS.

At low energies $\alpha \approx 1/\gamma^2$. From rf linearity and acceleration considerations σ_Z will be a fixed fraction of the bunch spacing. e.g. $\sigma_Z = f_\lambda\ c/\omega_{rf}$, where f_λ is a constant of the order of 1/3. With these assumptions:

$$\sigma_P \;\approx\; f_\lambda\ \gamma^{3/2}\sqrt{\frac{\mathcal{E}\ \cos(\phi)\ \mathrm{m}_\mu}{\omega_{\mathrm{rf}}\ \mathrm{c}}} \tag{1}$$

As the ring energy is increased, with fixed rf parameters, these rf considerations would allow $\sigma_P \propto \gamma^{3/2}$. In practice, however, the lattice acceptance will probably limit $\sigma_P \propto p$. For a string of bunches of fixed length, and fixed f_λ, the total acceptance is proportional only to σ_P. Thus acceptance $\propto p$. Also, for $\sigma_P \propto p$, equation 1 gives a requirement for $\mathcal{E}\ \cos(\phi)$ that falls with energy.

Note that although higher energy implies slower cooling it does not imply much less cooling for a fixed decay loss.

$$\frac{d\epsilon}{\epsilon}\ /\ \mathrm{loss} \;\propto\; \frac{\mathcal{E}}{\beta\ \mathrm{p}}\ \gamma \;\propto\; \frac{\mathcal{E}}{\beta^2}$$

An increase in momentum by a factor of ≈ 2.5 ($200 \rightarrow 500$ MeV/c) might be reasonable, and would gain back half of that lost by having so short a train . Gaining back the full factor, however, would imply a 1 GeV ring in which it would be hard to obtain the required $\beta_\perp$'s and momentum acceptance. Straggling might also be excessive.

Another advantage of raising the energy is that, for the same lattice, the transverse acceptance increases as $\sqrt{p}$. If, in addition, the $\beta_\perp$ is allowed to increase somewhat (the first cooling ring need only cool longitudinally, so a low $\beta_\perp$ should not be needed), then the acceptance could be further increased. This might accomodate the injected and extracted beams (see section 2).

1.2 Add Harmonic rf

The addition of higher frequency rf could increase f_λ, to make the wave form more saw tooth like, increase the σ_Z's for the same rf frequency, and thus increase the acceptance. This should be studied.

1.3 Use low frequency and a single bunch

If the rf frequency were lowered to 10 MHz, then a single bunch would be about 30 nsec long, as required. The acceptance, for the same σ_P would now be about 3 times larger than that for a train, becuase of the lack of unused space between the bunches. The use of harmonics, in this case, would not increase the acceptance, but would allow a slightly higher rf frequency.

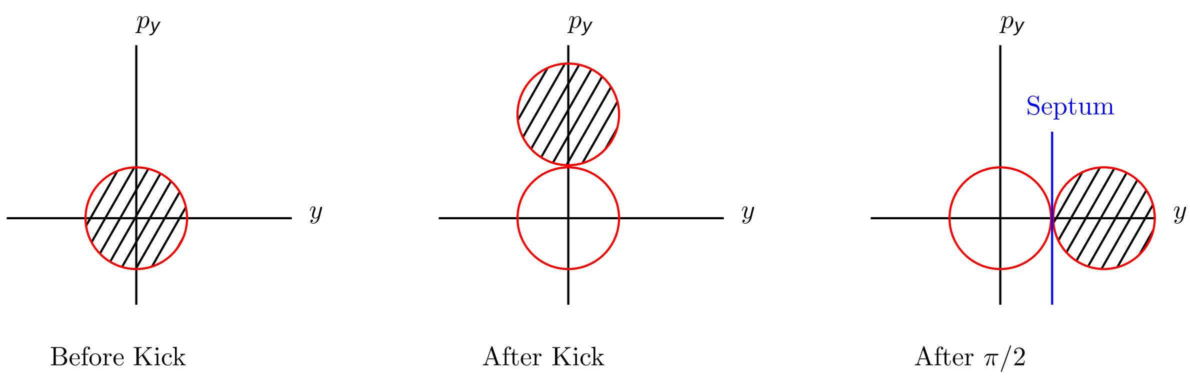
The equation 1 shows that as the frequency is lowered, then, for the same momentum acceptance, $\mathcal{E}\ \cos(\phi)$ should fall as the frequency. This means that at 10 MHz, the gradient, from bunch considerations, could be as low as 0.8 MeV/m, which might not be too expensive. But the cooling with such a low gradient would be very slow, and losses from decay would be serious.

1.4 Ring Conclusions

The choice of parameters for the first cooling ring is complicated. A ring energy higher than in study 2 seems desirable, but how high is unclear. rf with harmonics should improve performance. Low frequency single bunch systems (CERN, PJK, Balbakov) should be considered, but may not be absolutely necessary with a higher energy ring and harmonic rf.

2 Kickers

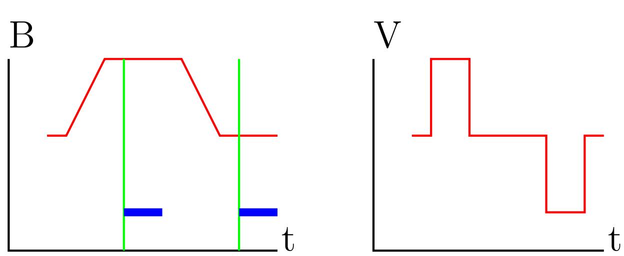
2.1 Introduction



In order to kick a beam into or out of a ring we need to displace it's phase space so that it is separate from that of the stored beam. This can be done in transverse momentum (with a conventional kicker) or longitudinal momentum (with acceleration as discussed by Neuffer) directions. The beam can then be manipulated by a transverse 90 deg. phase space advance, or with dispersion, so that the kicked beam is transversely displaced from the stored beam, and a septum can be introduced between them. This note will consider only transverse kickers.

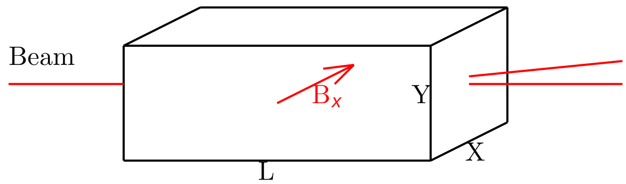
The kick should ideally occur locally, in a distance L small compared to the $\beta_\perp$; but could also be distributed around the ring at n locations spaced by any numbers of half integers of the betatron tune, each with length L/n.

The idealized magnetic and voltage pulse shapes are plotted below. Symmetric shapes are shown, that would allow the same kicker to be used for injection and extraction. The beam is shown for injection.



2.2 Formulae

Consider a kicker with horizontal field B_x, length L, height Y, and depth X.



With the transverse twiss parameter in the kick direction β_x, relativistic parameters $\beta\gamma$, normalized emittance ϵ_n (assumed equal in x and y), the half acceptances in sigmas f_σ, the ratio of beam size in the y over x directions R, the muon mass in Volts m_μ, and the velocity of light c: the required minimum transverse momentum kick is:

$$\Delta p_y \;=\; B_x\ L\ c \;=\; m_\mu\ 2\ f_\sigma \sqrt{\frac{\epsilon_n\ \beta\gamma}{\beta_y}}$$

We can use this to determine a minimum L for a given maximum B (set by ferrite saturation), or fix L at a reasonable maximum fraction (say 10% of circumference C) and calculate a minimum B.

The x aperture of the kicker X should be:

$$X \;=\; 2\ f_\sigma \sqrt{\frac{\epsilon_n\ \beta_x}{\beta\gamma}}$$

$$Y \;=\; R\ X$$

Assuming a total flux $\Phi \;=\; f_\Phi\ B_x\ L\ Y$ to allow for leakage flux, and $\int Bdl/\mu \;=\; f_\mu\ B\ X$ to allow for finite μ's in the flux return, then the current I, single turn Voltage V, and total kicker stored energy J, are given by:

$$I \;=\; \frac{f_\mu\ B\ Y}{\mu_o} \;=\; \frac{f_\mu\ 4\ f_\sigma^2\ R}{\mu_o\ c} \quad \frac{\epsilon_n}{L}$$

$$V \;=\; \frac{f_\Phi\ B\ X\ L}{t_{\rm rise}} \;=\; \frac{f_\Phi\ 4\ f_\sigma^2\ m_\mu}{c} \quad \frac{\epsilon_n}{t_{\rm rise}}$$

$$J \;=\; f_{Bdl}\ f_\Phi\ \frac{B^2\ L\ X\ R}{2\ \mu_o} \;=\; f_{Bdl}\ f_\Phi\ \frac{m_\mu^2\ 16\ f_\sigma^4\ R}{2\ \mu_o\ c^2} \quad \frac{\epsilon_n^2}{L}$$

where $t_{\rm rise}$ is the linear rise time of the pulse. We see that, for a fixed L and $t_{\rm rise}$, neither the stored energy, current or total Voltage are dependent on the beam energy or β_x. If L and $ct_{\rm rise}$ are set at fixed fractions of the circumference, then all three are approximately inversely proportional to that circumference.

2.3 Example

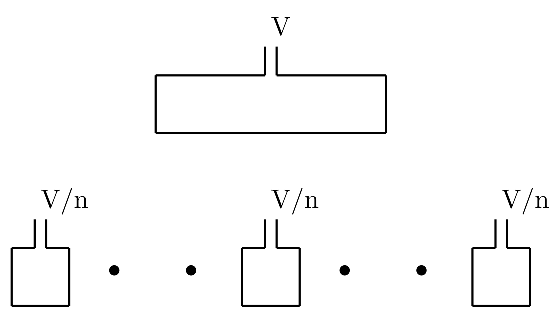
Consider the case of a first cooling ring with circumference ≈30 m and initial normalized emittance of 10 π mm (acceptance at 3 sigma of 90 mm). The energy and $\beta_\perp$ are as in Study 2.

f_Φ		$\sqrt{2}$
f_μ		$\sqrt{2}$
f_σ		3
m_μ	V	$1.05\ 10^8$
c	m/s	$3\ 10^8$
ϵ_n	π mm	10
β_x	m	1
$\beta\gamma$		2

Then:

	Kicker		c.f. Ind Acc
L	m	3.0	3.0
t_{rise}	ns	30	100
B	T	0.1	0.6
X	m	0.42	N.A.
J	Joules	4.2	4.8
I	kA	48	189
V	MV	5.9	0.19
P	GW	286	35

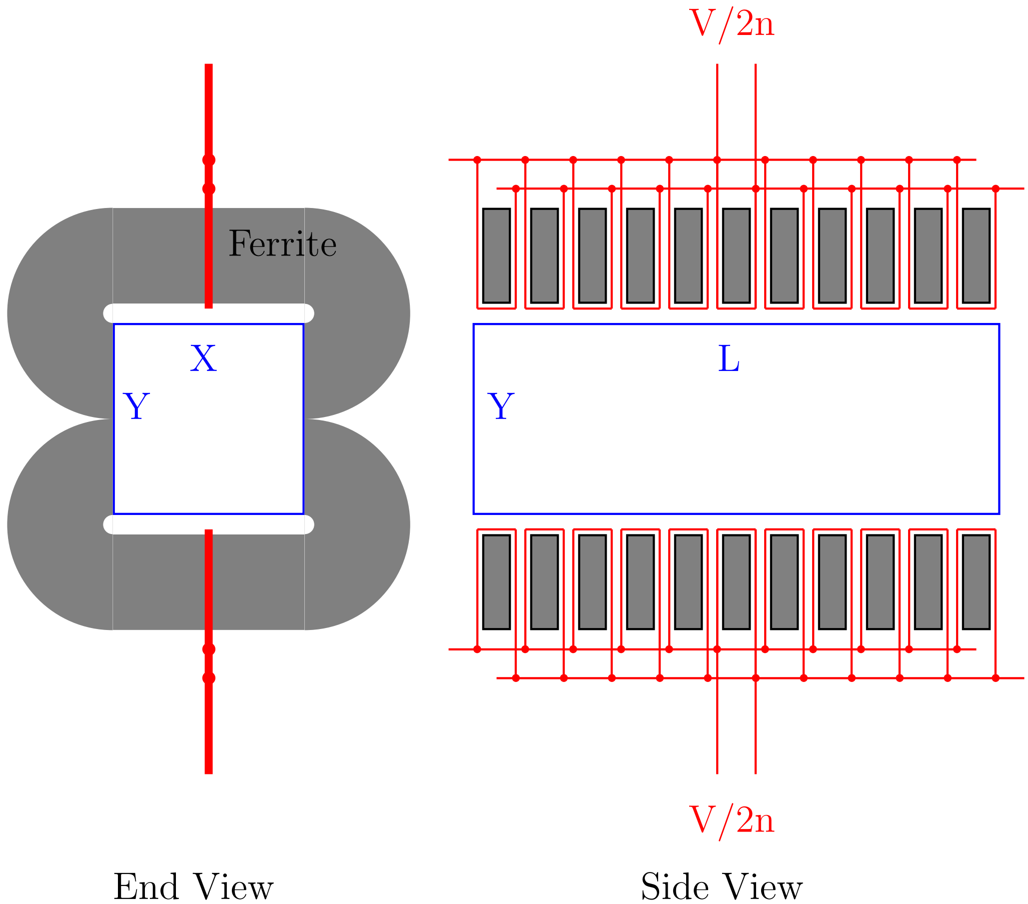
2.4 Practicality



These parameters are far beyond those of typical kickers, but the stored energy is actually less than that supplied by magnetic amplifiers to the same length of an (the second) induction linac in Feasibility Study 2 (see above table). The peak power is higher, but the pulse length is correspondingly shorter. Magnetic amplifiers with more pulse compression may well be possible that would provide the needed peak power. But the Voltage (5.9 MV) is far too high. Since the kicker length is ≈7 times as long as its width, one could break it into 7 parts, each with 840 kV. This is still very high.

The comparison with induction linacs suggests a better solution.

2.5 Induction Kicker



Multiple conductors could loop groups of the ferrite field returns (see figure above), much as the conductors in an induction linac loop individual sub sections of ferrite. The ferrite could be divided in any number of parts thus allowing the Voltage to be chosen arbitrarily. If such a kicker has not been named before, I would call it an 'Induction Kicker'.

It is far from clear that this will work, but needs study.

3 Other Problems

Even if the required kicker were practicable, there remain problems in designing the lattice.

If the kick is done in a single shot, then the length of the kicker must be short compared with the local $\beta_\perp$. In the current lattices, with maximum $\beta_\perp$ of the order of 1 m, this implies a shorter kicker, with higher field and stored energy, than specified above. A special injection/extraction section with a higher beta could be designed, but matching this into the rest of the ring could be difficult. As noted in section 1.1, somewhat higher $\beta_\perp$'s may be acceptable if the ring is only cooling in the longitudinal direction. This would help.

The alternative is to break the kicker into several parts, spaced by an integral number of 90 deg. phase advances. This is not natural with the existing phase advances per cell, and will be complicated by the variations of advance over the large initial momentum spread. Another problem is the requirement of ever increasing transverse acceptances to transmit the injected or extracted beams. As noted in section 1.1, raising the ring energy could help the transverse acceptance.

4 Conclusion

This study suggests that the first cooling ring injection and extraction problems may be soluable, but much work remains.

Appendix on Synchrotron motion

Taking the relativistic factors β and γ to be their average values, we define the distance and momentum displacements from a bunch center:

$$Z = z \ - \ \beta \ ct$$

$$P \ = \ p \ - \ \beta\gamma \ m_\mu$$

The momentum compaction α is defined by:

$$\frac{dv}{v} \ = \ \alpha \ \frac{dp}{p}$$

so

$$\frac{dZ}{dt} \ = \ \frac{c \ \alpha}{\gamma \ m_\mu} \ \ P$$

If $\mathcal{E}$ is the average acceleration gradient, ω_{rf} is the rf frequency, and ϕ is the rf phase from the zero crossing, and since:

$$\Delta P \ = \ \frac{\Delta E}{\beta}$$

so

$$\frac{dP}{dt} \ = \ \frac{dE \ c}{dz} \ = \ -\mathcal{E} \ \cos(\phi) \ \omega_{\mathrm{rf}} \ \ \mathrm{Z}$$

and

$$\frac{d^2 Z}{dt^2} \ = \ -\left(\frac{c \ \alpha}{\gamma \ m_\mu} \ \ \omega_{rf} \ \mathcal{E} \ \cos(\phi)\right) \ \ Z$$

The synchrotron frequency ω_s, and ratio of dP to dZ are then:

$$\omega_s \ = \ \sqrt{\frac{c \ \alpha \ \omega_{rf} \ \mathcal{E} \ \cos(\phi)}{\gamma \ \mathrm{m}_\mu}}$$

$$\frac{\sigma_P}{\sigma_Z} \ = \ \sqrt{\frac{\omega_{rf} \ \mathcal{E} \ \cos(\phi) \ \gamma \ \mathrm{m}_\mu}{\alpha \ \mathrm{c}}}$$