Matrix Description of Cooling

- Linear transfer matrix for cooling section M
- Matrix has eigenvalues λ:

$$M\boldsymbol{v}_k = \lambda_k \boldsymbol{v}_k$$

- Six eigenvalues
  - Symplectic
    - "Stable" oscillations: 3 complex conjugate pairs
    - "Unstable" oscillations: one or more pairs real, unequal
  - Non-symplectic (absorber):
    - Absolute value of complex eigenvalues
    - Greater than 1, unstable (no cooling)
    - Less than 1, stable (cooling)
- Cooling in all planes:
  - All eigenvalues in complex conjugate pairs
  - All have absolute value less than 1

Ellipses Rotated in Phase Space

- Find matrix that diagonalizes M:

$$A^{-1}MA = R$$

$$R = \begin{bmatrix} \alpha_x \cos\mu_x & \alpha_x \sin\mu_x & 0 & 0 & 0 & 0 \\ -\alpha_x \sin\mu_x & \alpha_x \cos\mu_x & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_y \cos\mu_y & \alpha_y \sin\mu_y & 0 & 0 \\ 0 & 0 & -\alpha_y \sin\mu_y & \alpha_y \cos\mu_y & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_t \cos\mu_t & \alpha_t \sin\mu_t \\ 0 & 0 & 0 & 0 & -\alpha_t \sin\mu_t & \alpha_t \cos\mu_t \end{bmatrix}$$

  - α_k are the absolute values of the eigenvalues
  - A can be found easily from the $\boldsymbol{v}_k$
- Matched ellipses are found from A:

$$\boldsymbol{x}(\phi) = A \begin{bmatrix} \cos\phi \\ -\sin\phi \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

  - Similarly for other components
  - These ellipses scale down in dimension by α_k
  - Shape otherwise preserved

Straight Absorber

- Damps p_x, p_y, antidamps E:

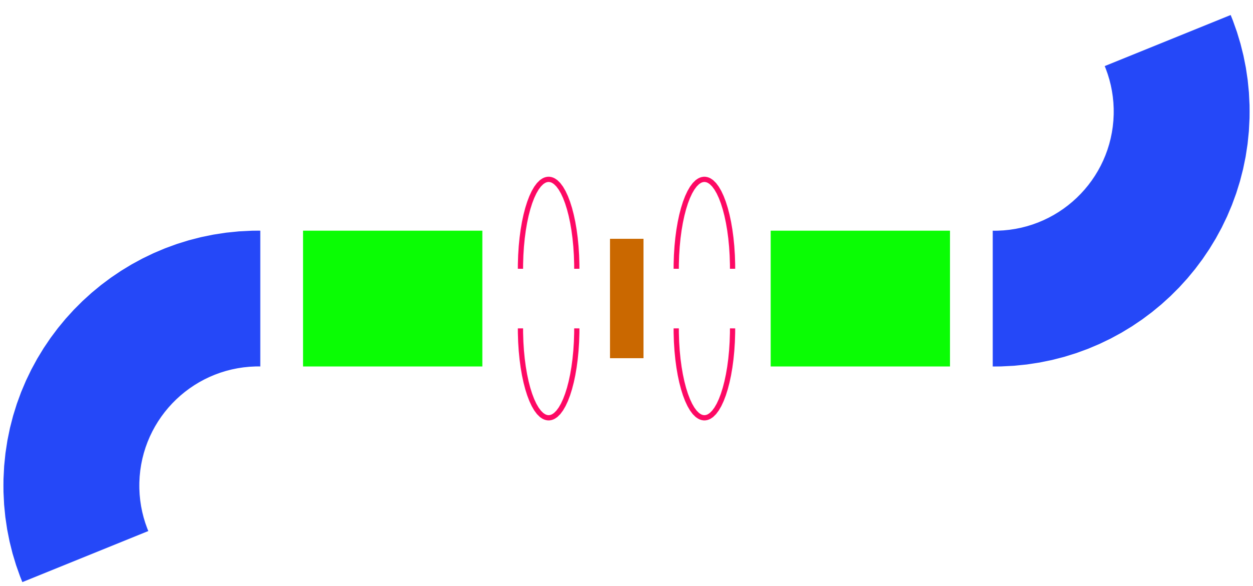
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_\perp^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_\perp^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_\parallel^2 \end{bmatrix}$$

  - $\alpha_\perp < 1$, $\alpha_\parallel > 1$
- Damping in all three planes w/o absorber
  - Ellipse has p_x or p_y projection greater than E projection (hand wave) at absorber
  - Dispersion by itself won't achieve this (shearing, not rotation)
  - Need dispersion while in RF cavities

- Benefits:
  - Don’t need strong focusing onto wedges
  - Simpler dispersion generating objects
  - Everything else “convnetional” straight cooling
- Disadvantage: spatial projections larger?
- Need to set up matched ellipse
  - Will damp onto matched ellipse
    - Ugly transients
    - Hit nonlinearities

Test Lattice

- 2-D example (x, t)
- Linear only
- Ignore longitudinal growth (irrelevant)
- Ignoring RF, want D'_x nonzero at absorber (turns energy into p_x)
- Layout:
  - Appropriate symmetry in lattice
  - Bends to generate dispersion, momentum compaction
  - Bends go opposite direction for D'_x
  - Generic transverse linear map, reversed on opposite sides of absorber
  - RF cavities around absorber



Results

- Can choose parameters so both eigenvalues have equal absolute values (maximum sharing)
- Far from linear resonances
- Ellipses
  - Assume equal ellipse areas
  - If significantly different areas
    - To what extent will this work if I only project small fraction?
    - My biggest concern.
- RF can be arbitrarily small.
  - Bends get longer.
  - Energy spread increases (good: straggling?).
  - Bucket smaller
- Lattice doesn't have small β at absorber
  - Maybe a better region of parameter space exists
  - Probably need adjustments to lattice
- Two transverse planes?