Recirculating Accelerators for a Neutrino Factory

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Various designs for the recirculating accelerators for a 50 GeV neutrino factory are examined. The designs will be based primarily on longitudinal considerations, but with some thouht given to consequences in the transverse plane. The advantages and disadvantages of various designs are examined.

I. REVIEW

We will assume a racetrack geometry for the recirculating accelerators. The matched bunch length (in time) and energy spread will be kept constant in the recirculator, and this will be achieved by keeping the gradient, synchronous phase, and synchrotron tune constant from one turn to the next in the recirculator. Here we summarize results from [1] which will be used in this paper.

A Linearized Drift-Kick Map

Using the linearized drift-kick map approximation for the map around half a turn of the accelerator, and treating each linac as a drift for half the linac length, followed by an RF kick, followed by a drift for the remaining half of the linac length. The result is that the momentum compaction (assumed to be constant) in a given arc is given by

$$\alpha_C = \frac{1}{\gamma_1^2} + \frac{\beta_1^2 p_1 c^2}{L_{\rm arc}} \left[\frac{4\sin^2 \pi\nu_s}{qvL_{\rm lin}\omega \sin\phi} + \frac{1}{qvc\cos\phi} \left(\frac{1}{\beta_0} - \frac{1}{\beta_2} \right) \right], \tag{1}$$

where β_1 is the reference particle's speed divided by c, the speed of light, in the arc, $\beta_2 c$ is the reference particle's speed in the center of the linac following the arc, $\beta_0 c$ is the reference particle's speed in the center of the linac preceding the arc, $\gamma_1 = 1/\sqrt{1-\beta_1^2}$, p_1 is the reference particle's momentum in the arc, $L_{\rm arc}$ is the length of a half-arc, ν_s is the synchrotron tune for half a turn in the recirculator, q is the charge of the particles, v is the volgtage gradient in the linac, $L_{\rm lin}$ is the length of each linac (two per recirculator), ω is the angular frequency of the RF, and ϕ is the synchronous phase of the RF.

The RMS bunch length (in units of arrival time) at the center of the linac will be

$$\sqrt{\frac{2\epsilon_L \sin\pi\nu_s}{qvL_{\rm lin}\omega\sin\phi}} \sqrt{\sec\pi\nu_s}, \tag{2}$$

and the RMS energy spread at the center of the linac will be

$$\sqrt{\frac{qvL_{\rm lin}\epsilon_L\omega\sin\phi}{2\sin\pi\nu_s}} \sqrt{\cos\pi\nu_s}, \tag{3}$$

where ϵ_L is the longitudinal emittance in energy-time units. For the corresponding RMS sizes at the center of the arc, exchange $\sec\pi\nu_s$ with $\cos\pi\nu_s$ in the above formulas.

B RF Bucket

The longitudinal emittance that can be accelerated by the bucket is given approximately by

$$\epsilon_L = \frac{2}{k^2} \frac{evL_{\rm lin}\left|\sin\phi - \phi\cos\phi\right|}{\omega\sin\pi\nu_s}, \tag{4}$$

where k is the ratio of the energy half-width of the bucket to the RMS energy spread in the bunch. In this paper we will choose that parameter to be about 4.

C Transverse Beam Sizes

The vertical RMS beam size is

$$\sigma_y = \sqrt{\frac{\beta_y\epsilon_y}{\beta\gamma}}, \tag{5}$$

where β_y is the vertical β-function, ϵ_y is the vertical normalized emittance, βc is the speed of the reference particle, and $\gamma = 1/\sqrt{1-\beta^2}$. The horizontal RMS beam size is

$$\sigma_x = \sqrt{\frac{\beta_y\epsilon_y}{\beta\gamma} + \left(\frac{D_x\sigma_\Delta}{\beta pc}\right)^2}, \tag{6}$$

where β_x is the horizontal β-function, ϵ_x is the horizontal normalized emittance, where D_x is the horizontal dispersion function, σ_Δ is the RMS energy spread, and p is the momentum of the reference particle.

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Lower bounds for the maxima over the arc of these beam sizes can be computed by computing what the transverse parameters would be if the focusing and bending were constant:

$$\beta_x = \beta_y = \rho\sqrt{\alpha_C} \qquad D_x = \rho\alpha_C, \tag{7}$$

where ρ is the radius of curvature of the arc.

D Decays Losses

For a bunch at constant energy, the number of particles as a function of longitudinal position s is given by

$$N(s) = N(0)e^{-ms/p\tau} \tag{8}$$

where τ is the decay lifetime of the particles, and m is their mass. If the particles are undergoing acceleration at constant gradient, the result can be written in terms of initial and final energy/momentum as

$$N_f = N_i \left(\frac{E_f + p_f c}{E_i + p_i c}\right)^{-\frac{mc}{\tau q v \cos\psi}}. \tag{9}$$

Here the f subscripts refer to the final values, and the i subscripts refer to the initial values.

E Power Requirements

If the gradient is to be kept constant in the linacs, the power extracted by the beam in one pass must be resupplied before the next time the beam passes through the linac. The minimum peak power required to run in this state is thus

$$P_{\text{beam}} = NqvL_{\text{lin}}\cos\phi\frac{\beta c}{(L_{\text{lin}} + L_{\text{arc}})}, \tag{10}$$

where N is the number of particles in the beam. In this case βc is the maximum speed of the reference particle in the recirculator.

The maximum possible RF to beam efficiency that one can hope to achieve is given by

$$\varepsilon = \frac{nNqv\cos\phi}{(n-1)Nqv\cos\phi + \dfrac{v^2}{\omega(r_s/Q)}}, \tag{11}$$

where r_s is the shunt impedance per unit length (with transit time factor and filling factor, since v contains those factors), and n is the number of turns in the recirculator.

TABLE I: Neutrino factory parameters relevant to recirculating accelerators.

Initial momentum	p_i	4 GeV/c
Final momentum	p_f	50 GeV/c
Longitudinal emittance	ϵ_L	8.36 meV-s
Normalized transverse emittance	ϵ_n	2 mm-rad
Number of particles	N	3×10^{12}
Bunch frequency		200 MHz
Linac filling factor		0.65

II. RECIRCULATING ACCELERATOR SCENARIO

A Parameters

Some relevant parameters for a neutrino factory are given in Tab. I. The initial momentum is based on the difficulty of constructing an arc for the large relative energy spreads which will necessarily occur at lower momenta [2]. Below that energy, a straight linac will be used to accelerate the particles. The maximum gradient and r_s/Q, both without filling factor but with transit time, are expected to scale with frequency as

$$v = 30\ \text{MV/m}\sqrt{\frac{f}{800\ \text{MHz}}} \tag{12}$$

$$\frac{r_s}{Q} = 1000\ \Omega/\text{m}\frac{f}{800\ \text{MHz}}. \tag{13}$$

B Scenarios

The first thing to note is that a single recirculating accelerator would not be appropriate for the full energy range. First of all, the racetrack geometry requires that all arcs have a similar length, that length being determined by the arc length required for the highest energy. Thus, if a single racetrack were used for all energies, the lower energy arcs would be needlessly long, and the number of decays would end up being excessive. Secondly, the full range of energies in the beam must go through the linac. Since the focusing strength of a quadrupole is inversely proportional to the particle's momentum, and the quadrupoles in the linac must focus all particles over the energy range in the recirculator, it would be difficult to impossible to choose a quadrupole layout which would give stability over the entire energy range and keep the beam size under control. Finally, a larger energy range will presumably require more turns around the recirculating accelerator, increasing the complexity of the switchyard where the single linac meets several arcs.

For the purposes of this discussion, it is assumed that for each pass through the recirculator, the particles will pass through a different arc. Strictly speaking, this is not necessary, and many of the computations here will still

be valid if the particles make multiple passes through the same large energy acceptance arc.

We will choose to use two recirculators: the transition from one recircirculating accelerator to the next will be chosen for now so that each accelerator stage increases the particles' momentum by the same factor; this leads to recirculating accelerators accelerating from 4 GeV to 14 GeV, then 14 GeV to 50 GeV.

Preliminary computations indicate that the arcs will have and average bending field of about 1 T (this is based approximately on the lattice design of Carol Johnstone); the reason for this low number is the large energy acceptance that will be required in each arc, combined with the complexity of the switchyard.

The synchrotron tune per half turn will be initially chosen to be 0.15. This is the maximum value the synchrotron tune can possibly have. Reasons for such a choice are outlined in [1]. If appropriate, smaller synchrotron tunes will be examined.

The results of several trial sets of parameters are given in Tab. II. We will examine the results based on several criteria:

- The momentum compaction cannot be too high; above 0.03 is difficult.
- Larger energy spreads are more difficult
- We would like to limit the total decay losses in the recirculating accelerators to 10% or so.
- We would like to keep the beam size down as much as possible.
- We would like to minimize peak power requirements and maximize efficiency.
- Higher frequencies are probably preferable from a cost standpoint.

Let's examine the results of Tab. II. First, consider the low energy recirculator. If we consider 800 MHz, the RMS relative energy spread is at least 5%, which is probably too high. Going down to 400 MHz, the energy spreads drop down as low as 2.5%, which may be more reasonable for a large energy acceptance arc. Howveer, if we try to limit the decay losses to 5%, one cannot use a 12 turn design, and one is in fact forced down to 6 turns. In fact, switchyead complexity makes a large number of turns difficult, and even 6 turns may be difficult. An existing design [2] has only 4 turns. With 6 turns, the RMS energy spread is not too much worse, 3.1%. At 200 MHz, the RMS energy spreads are even lower, but the maximum momentum compactions required start to grow substantially, and reach a size that is prohibitive. However, the momentum compaction could be reduced at the cost of a slightly increased energy spread and substantially reduced synchrotron tune. In the end, 6 turns at 400 MHz seems to be a good compromise scenario.

For the higher energy recirculator, similar arguments suggest using 800 MHz linacs at 7 turns. To reduce the energy spread in the arcs, one could also consider going to 400 Mhz linacs for 6 turns; however, if we were capable of making the arcs for the low energy recirculator, this should not be necessary (although the arcs would be more well-behaved).

C Alternate Scenario

Let's consider another scenario, probably more appropriate for superconducting linacs, where the r/Q is based on TESLA and is 988 Ω/m at 1300 MHz and scales linearly in frequency from there. Furthermore, assume the same gradients but a filling factor of 0.5. The lower energy recirculator will be from 3 GeV/c to 12 GeV/c, and the higher energy recirculator will be from 12 GeV/c to 50 GeV/c. Tab. III gives the results.

The numbers and conclusions here turn out very similar to the numbers and conclusions for the previous scenario. The only cavaet is that the relative energy spread in the beam for the first turn will be larger than it would have been in the previous example due to the lower energy. Thus, a reasonable scenario would be 5 turns at 400 MHz for the low energy recirculator and 6 turuns at 800 MHz for the high energy recirculator.

REFERENCES

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[1] J. Scott Berg. "Acceleration for a High Energy Muon Collider." To appear in the proceedings of *Studies on Colliders and Collider Physics at the Highest Energies: Muon Colliders at 10 TeV and 100 TeV—HEMC '99*, Montauk, NY, 27 September–1 October 1999. Also BNL report BNL-67115.

[2] Carol Johnstone. Private Communication.

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TABLE II: Neutrino factory parameters relevant to recirculating accelerators.

$p_{\min}$ GeV/c	$p_{\max}$ GeV/c	ν_s	f MHz	n	L_{lin} m	σ_τ ps	σ_Δ MeV	P_{beam} MW	decay %	ε %	$\alpha_{\min}$ 10^{-3}	$\alpha_{\max}$ 10^{-3}	$\sigma_{x,\max}$ mm
4	14	0.15	200	4	142	108	87	625	4.3	3.5	12.8	29.3	18
4	14	0.15	200	6	97	122	77	492	5.5	5.1	15.1	39.0	19
4	14	0.15	200	12	52	152	62	302	9.0	9.0	20.9	61.9	23
4	14	0.15	200	25	28	189	50	165	16.0	15.5	30.3	97.3	26
4	14	0.075	200	4	137	83	104	635	4.3	3.7	4.8	9.9	12
4	14	0.15	400	4	106	67	140	713	3.8	9.1	5.4	11.4	14
4	14	0.15	400	6	74	76	124	544	5.0	12.5	6.2	15.1	15
4	14	0.15	400	12	41	93	101	319	8.5	20.3	8.3	23.4	17
4	14	0.15	800	4	82	41	227	788	3.4	21.2	2.4	4.4	11
4	14	0.15	800	6	59	47	201	585	4.6	27.3	2.8	5.8	12
14	50	0.15	200	4	482	71	132	645	4.3	3.7	5.2	12.7	13
14	50	0.15	200	5	388	76	123	569	4.9	4.6	5.7	15.0	14
14	50	0.15	200	22	96	123	76	190	14.2	16.4	12.5	41.2	20
14	50	0.15	400	4	349	45	211	743	3.7	9.9	2.1	5.0	10
14	50	0.15	400	6	238	51	185	568	4.9	13.8	2.5	6.7	11
14	50	0.15	400	22	73	76	123	198	13.7	34.3	4.8	15.9	15
14	50	0.15	800	4	257	28	338	830	3.3	23.8	0.8	2.0	8
14	50	0.15	800	7	154	33	283	547	5.0	34.2	1.1	2.9	9
14	50	0.15	800	22	58	47	200	203	13.3	57.8	1.9	6.0	11

TABLE III: Neutrino factory parameters relevant to superconducting recirculating accelerators.

$p_{\min}$ GeV/c	$p_{\max}$ GeV/c	ν_s	f MHz	n	L_{lin} m	σ_τ ps	σ_Δ MeV	P_{beam} MW	decay %	ε %	$\alpha_{\min}$ 10^{-3}	$\alpha_{\max}$ 10^{-3}	$\sigma_{x,\max}$ mm
3	12	0.15	200	4	167	111	84	554	5.4	2.2	13.3	31.2	19
3	12	0.15	200	6	115	126	74	449	6.7	3.1	15.4	41.6	21
3	12	0.15	200	11	67	153	61	306	9.7	5.2	19.8	62.4	24
3	12	0.15	200	22	38	188	50	181	16.1	9.0	27.4	95.8	27
3	12	0.15	400	4	125	69	136	645	4.6	5.6	5.9	12.2	15
3	12	0.15	400	5	103	74	127	567	5.3	6.8	6.3	14.2	16
3	12	0.15	400	6	88	78	120	506	5.9	7.8	6.7	16.1	16
3	12	0.15	400	11	53	94	100	330	9.0	12.3	8.3	23.6	18
3	12	0.15	800	4	97	43	220	727	4.1	13.6	2.9	4.7	12
3	12	0.15	800	5	81	46	206	627	4.8	15.8	3.1	5.5	13
12	50	0.15	200	4	660	70	135	578	5.3	2.3	4.6	12.2	13
12	50	0.15	200	6	446	80	118	471	6.5	3.4	5.4	16.4	15
12	50	0.15	200	11	250	97	97	322	9.5	5.9	7.2	25.1	17
12	50	0.15	200	23	125	123	77	185	16.3	11	10.8	40.9	20
12	50	0.15	400	4	478	44	215	683	4.5	6.2	1.9	4.8	10
12	50	0.15	400	6	325	50	188	537	5.7	8.8	2.2	6.4	11
12	50	0.15	400	11	185	61	155	351	8.7	14.6	2.9	9.8	13
12	50	0.15	400	23	96	76	123	192	15.6	24.8	4.2	15.8	15
12	50	0.15	800	4	351	27	343	782	3.9	15.6	0.8	1.9	8
12	50	0.15	800	6	241	31	302	596	5.1	21.2	0.9	2.5	9
12	50	0.15	800	11	140	38	250	375	8.2	31.5	1.1	3.8	10
12	50	0.15	800	23	76	47	201	199	15.1	46.0	1.6	6.0	11