R&D for MUCOOL

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INTRODUCTION

This note will address the question of what R&D is necessary to demonstrate cold muon beams with sufficient confidence that a proposal for a facility using such beams could enter the preliminary design stage and a major test of the design could be planned. It has always been recognized that the cooling process only involves freshman physics. Indeed, no less than Burt Richter at the FNAL IGFA Symposium in 1999, suggested that computer modeling of the process could answer many of the questions. This note will pursue that train of thought.

However, it is obvious that the technology required would be on the cutting edge, and no amount of simulation can take the place of a rigorous hardware R&D effort. What we would like to define here is the integration of the experimental program with the simulation program.

The original MUCOOL experiment integrated the hardware development and the cooling demonstration in a coherent manner. It was done at a place in the cooling chain where the emittance was small enough to fit into hardware that used fields of the order of 10 T and 800 Mhz rf. The configuration, while at the edge of technology, was still small enough to be set up in a typical test beam. The instrumentation to measure the emittance was complicated and probably would cost almost as much as the cooling hardware itself. Nevertheless the concept was developed and lead to a self-consistent arrangement that would demonstrate cooling and also develop the required hardware and provided a nice test bed for the future R&D program.

The advent of the recent interest in neutrino factories has shifted the emphasis toward the demonstration of cooling near the front end where the normalized emittance is of the order of 10^4 mm-mr. The huge emittance has made it much more difficult to set up a viable cooling experiment as the magnets required become very large and the frequency of the rf required shifts down to 200 Mhz or lower. An example of such an experiment has been proposed by K. McDonald (see his web page, MuMu/00-21). Using available equipment an experiment could be mounted that would cool the emittance by less than 9% (there is an error of a factor of two in the beta-perp used). This still is not an inexpensive experiment! So the question is, what do we need to know?

WHAT WE NEED

Even though cooling only involves well-known physics, it is still not a trivial problem to model a cooling channel on a computer! For simplicity, we adopt the model of continuously cooled beams so that we can use some simple equations for illustration. Following BNL-63615, we write:

$$\frac{d\varepsilon_n}{dz} = -\varepsilon_n \frac{dP_z}{P_z dz} + \frac{\beta_T}{2}\frac{E_s^2}{\beta^3 E m c^2}\frac{1}{X_0} \qquad \beta_T = \frac{2P_Z}{eB_z}$$

$$\varepsilon_{N_{\min}} = \frac{\beta_T E_s^2}{2mc^2 X_o \left[\frac{dE}{dx}\right]}$$

The first equation has a solution in the form of a simple exponential where the emittance approaches the equilibrium emittance from above or below depending on whether the beam is hotter or cooler than the equilibrium emittance. For 200 MeV/c muons in a 1-T. field, the equilibrium normalized emittance is 5000 mm-mr.

1. If the emittance is large compared to the equilibrium value then the cooling only depends on dP_Z/dz. It is only when the cooling approaches the equilibrium value that the focusing parameters and the scattering physics comes into play.
2. The final stages of cooling become sensitive to beta-perp, the focusing parameter, and the details of the multiple scattering, exemplified here by E_S^2.
3. We have not discussed the longitudinal coordinate, but there is heating due to the straggling in this dimension.

It is clear from above that to accurately simulate the beam, we must have an accurate model for scattering and straggling of muons in hydrogen and other low Z materials that will necessarily have to be use for windows. The accuracy must be high because the muons pass through about 100 cells. Thus a complete understanding the plural and single scattering tails is necessary in order to know the channel loss in the transverse dimension and similar considerations probably apply to the z dimension for straggling.

A cooling experiment showing a small degree of cooling does not measure these parameters with sufficient precision. In fact, I will argue that the clumsy geometry involving large angles and precession measurements in a huge volume only serve to confuse the real physics.

The following steps are proposed:

1. A channel design that our present simulation indicates could “work”. This step is necessary for most of the steps that follow.
2. Expand the present scattering model that is used in DPGEANT and ICOOL and verify that the simulation results agree with analytical results in simple cases where this is possible. I think we have the framework for this step, but there are some open questions. This work can go on in parallel with step 1.
3. Use the simulation to vary the scattering and straggling parameters and thus study the sensitivity of the channel to these parameters. This step defines the accuracy required for any scattering experiment that is required, or conversely could eliminate the need for an experiment if our present knowledge is sufficiently accurate.
4. Use the simulation to study the channel to hardware errors. This step sets the accuracy required for manufacturing and measuring the magnets and the cavities. If things are too sensitive, it may indicate the need to redesign the channel.
5. Models of cavities, magnets, and an induction linac section can be started as soon as we understand what is required.
6. Tests. These include measurements to see if the criteria set in step 4 can be met, but also many other tests that are not included in the simulation and that are unique to the individual pieces of hardware…..radiation hardness, boiling of the LH_2, quench protection of the magnets, x-rays from the cavities, etc.
7. Development of instrumentation for tuning the beam can start now. Details of the tuning procedure have to await step 1.

Access to a beam will be necessary for much of this work.

THE SCATTERING MODEL

Fortunately, we have a good scattering model provided by Rutherford! However there is some work to do. The present simulation code uses the Moliere Theory to model the scattering, and the usual formulation involves using the Thomas-Fermi model for the atomic electrons. Fortunately, we know the hydrogen atom wave function, and so we can correctly insert the proper wave function into the calculations. This modifies the Rutherford Formula in the following way [the equations here follow from Bethe, PR 89,1256,(1953)]:

$$\sigma(\vartheta)\vartheta\, d\vartheta = \frac{2\chi_c^2\vartheta\, d\vartheta\left[gel(\vartheta)+ginel(\vartheta)\right]}{\vartheta^4}$$

$$\chi_c^2 = \frac{4Nte^4Z(Z+1)}{(pv)^2}$$

t is the thickness
N is the number of atoms per cm^2
p,v are the momentum and velocity of the muon

The gel and ginel are the elastic and inelastic form factors of the scattering atom. For hydrogen we can use the exact wave function. However there are molecular effects at the 2.8% level as can be seen from the PDG value for X_0 for atomic hydrogen of 63.05 g/cm^2 compared to that for molecular hydrogen of 61.25 g/cm^2. The integrals over the momentum transfer to the scatterer are similar for multiple scattering and for pair production, and so we can expect uncertainties of the order of 3%. The effect on the X_0 was measured by Bernstein and Panofsky PR 102, 522, (1956). The values of gel and ginel for other light materials are used in x-ray scattering measurements, and I have obtained extensive tables of these functions from NIST.

There are two complications that arise with the inelastic term, which is the term that describes scattering from the electron. (See MUCOOL #16 and #20 for a discussion of these effects} The first is a singularity at the origin for the inelastic process, and is easily taken care of by cutting off the integrals at momentum transfers smaller than that necessary for the 1s to 2p excitation. The elastic form factor is proportional to t^2 and therefore makes the cross section finite at the origin.

The second requires more attention. In the equation for Rutherford scattering, the old guys put in Z(Z+1) in place of Z^2 in order to account for the scattering by the electrons, and used the same form factor as the elastic term. Figure 1 shows a comparison of the elastic scattering of muons by electrons and protons. It is seen that the lab cross sections are identical out to about 5 mr that corresponds to the Jacobian peak for mu-e scattering at 90^o on the cm system. The lower branch of the curve is small compared to the upper branch and we will neglect it in what follows. The integrals for the rms scattering by the electrons then converge and once the multiple scattering is greater than about 5 mr, the electrons only contribute to the gaussian central peak by the Central Limit Theorem.

Now the Moliere theory connects the gaussian central scattering to the single scattering tail at large angles. The interpolation region is that of plural scattering. This exposes the other major trouble with the present TF model that is in GEANT. The use of Z(Z+1) in the above formula means that at large angles (i.e. greater than 5 mr) the predicted scattering cross section for hydrogen is twice as large as it should be, thus enhancing the predicted loss from the channel.

This problem can be seen in Figure 2 and 3 where the scattering is shown for samples of H_2 that are 5 mm and 32 cm thick. These two values correspond to roughly the smallest step size in DPGEANT and a typical absorber size in a cooling channel. DPGEANT builds the 32 cm thick case by combining 64 small steps. It is seen that for the smaller step, that indeed the Z(Z+1) formulation would be correct as 5 mr is well outside the gaussian peak, while for the 32 cm case, the electron contribution is completely hidden within the central peak. The cutoff of the electron at 5 mr makes complications in the simulation program as these integration programs vary the step size to suit the environment. This in turn causes the scattering formulation to vary. One solution would be to fix the step size at a minimum value (4 mm?) and thus fix the scattering algorithm. In any case, there is a problem that needs solving.

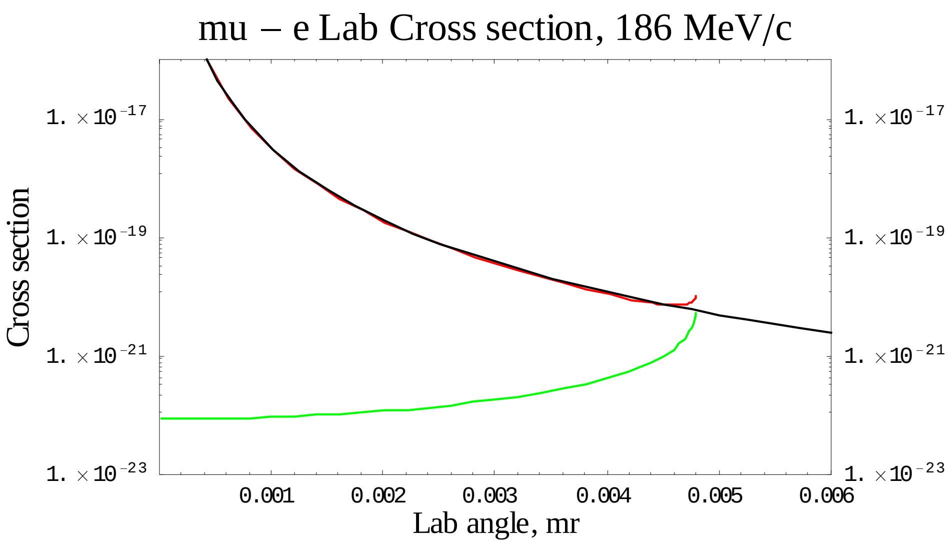

Figure 1. The above figure shows a comparison of the lab cross section for muons scattering from electrons (red, green) and muon proton scattering. The Jacobian peak that occurs at 90^0 in the CM system can be seen at a lab angle of m_e/m_{mu} = 5 mr.

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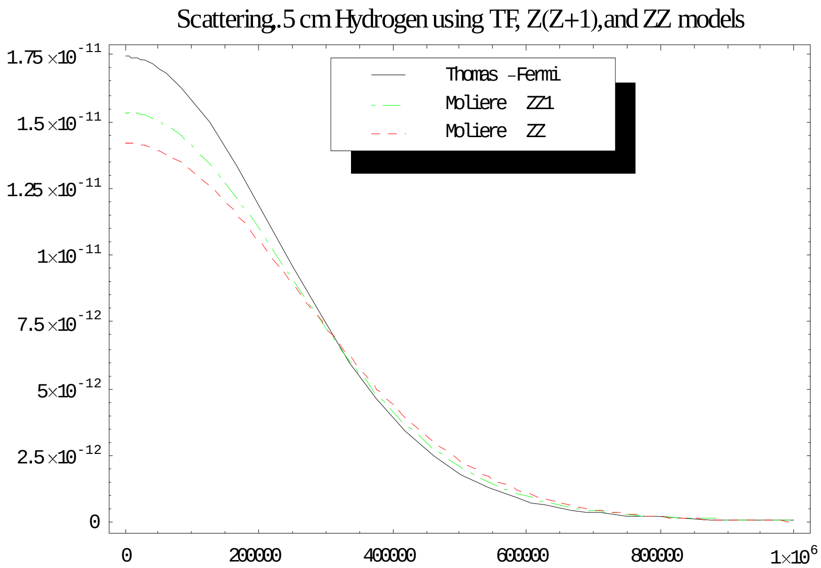

Figure 2 . Scattering in 0.5 cm of hydrogen for the three different models. Figure 3 below shows the scattering for a 32 cm thick target.

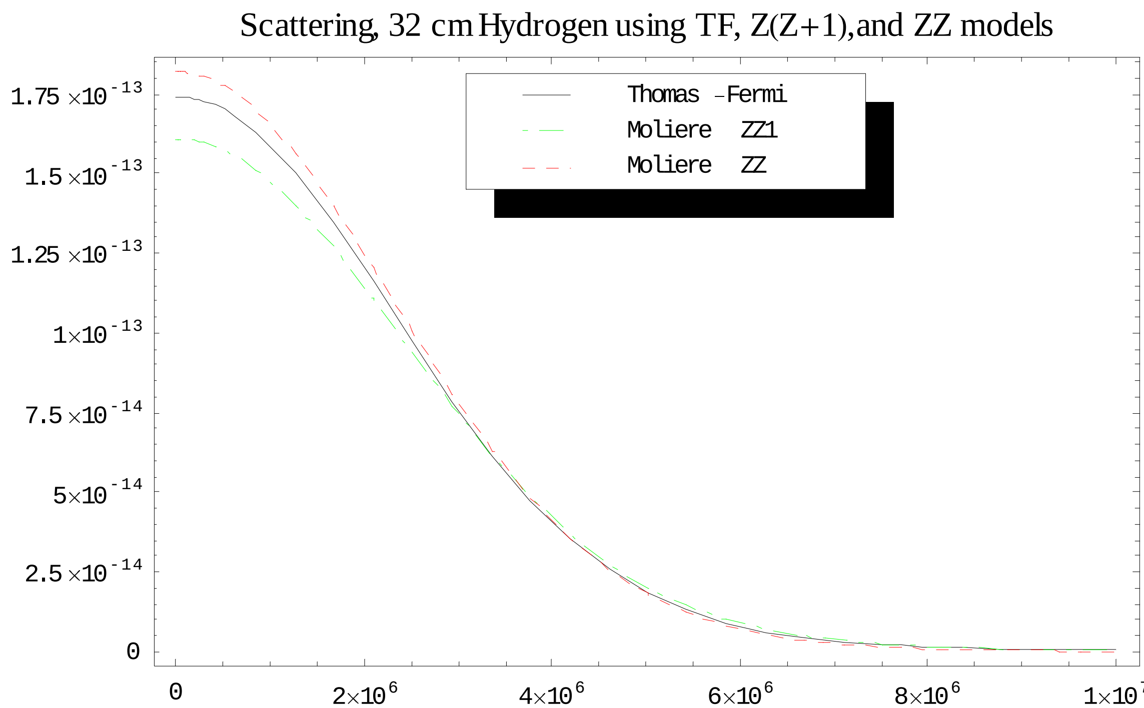

Fig. 3. The above curves show the scattering of 200 MeV/c muons by 32 cm of H_2 for the three models of the hydrogen atom. The vertical scale must be multiplied by P_t dP_t in MeV/c to get the probability for scattering into dP_t at a value P_t. All the curves are normalized to one. The Jacobian limit for the electron is at about 1 MeV/c.

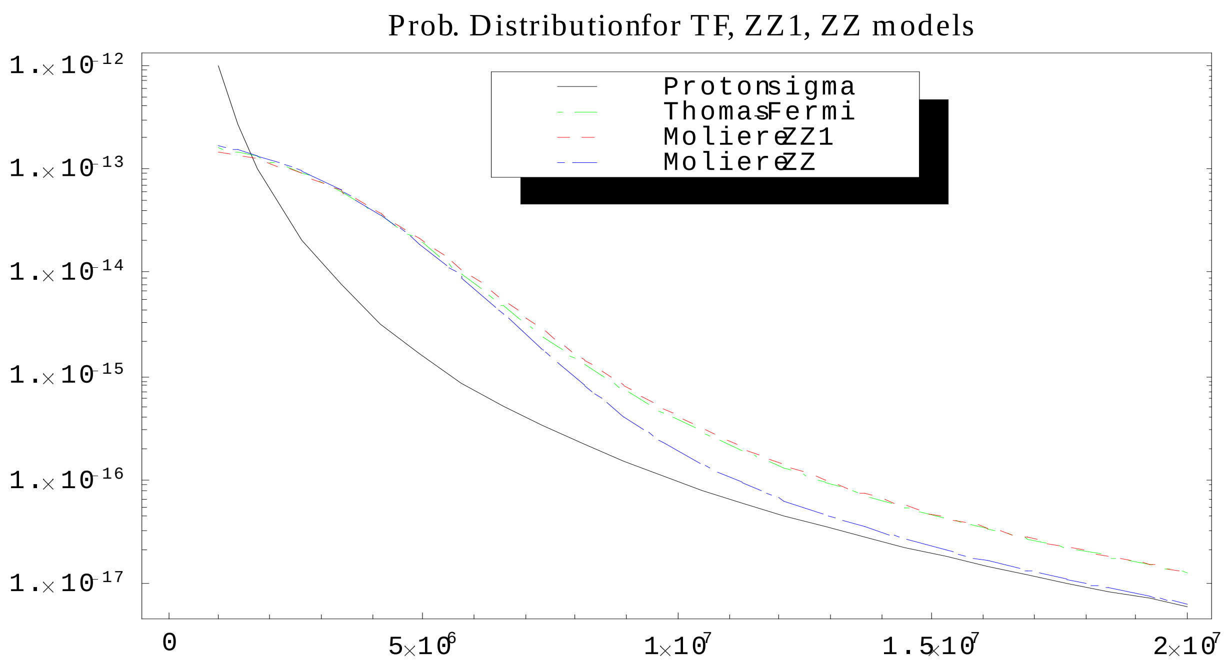

Fig. 4. The single scattering Rutherford cross section and the plural scattering region is shown for comparison. The ZZ curve matches onto to the single scattering for high P_t where as the TF and ZZ! Models are twice the correct value.

The above figure shows the region of transition between multiple and single scattering. The T-F and the Z(Z+1) models both have tails that are twice the Rutherford cross section which is also shown. Note that the Z^2 model correctly matches on to this limiting cross section. Plural scattering extends from a P_t of 5 MeV/c to about 15 MeV/c for 200 MeV/c muons. The above curves also give an impression of the uncertainties that exist in the calculations. The question that needs answering is: "Do they matter?".

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SCATTERING EXPERIMENT

First: Do we need an experiment?

Below, we will look at the possibility of doing a scattering experiment to an accuracy of about 0.1% , but first we should discuss the issue of do we need an experiment and if so how accurate. At this point, we do not know the answer, but the information may already be there to make this decision. In any case it will be available.

Lets assume we have, from the present work, a channel that looks promising in its simulation. At this point, one can investigate how sensitive it is to the various scattering parameters. The parameters involve the rms scattering angle, the large angle scattering, dE/dz, straggling, and correlation's between scattering and straggling. This study can be used to **define** the required accuracy. At the same time this study should examine the sensitivity to the magnetic field parameters and the RF configuration, and define the accuracy that the hardware must achieve in order to match the simulation. This is a crucial step in my assumption about how the R&D program should be configured.

I will assume that the experiment should measure the following with an accuracy of the order of 0.1%:

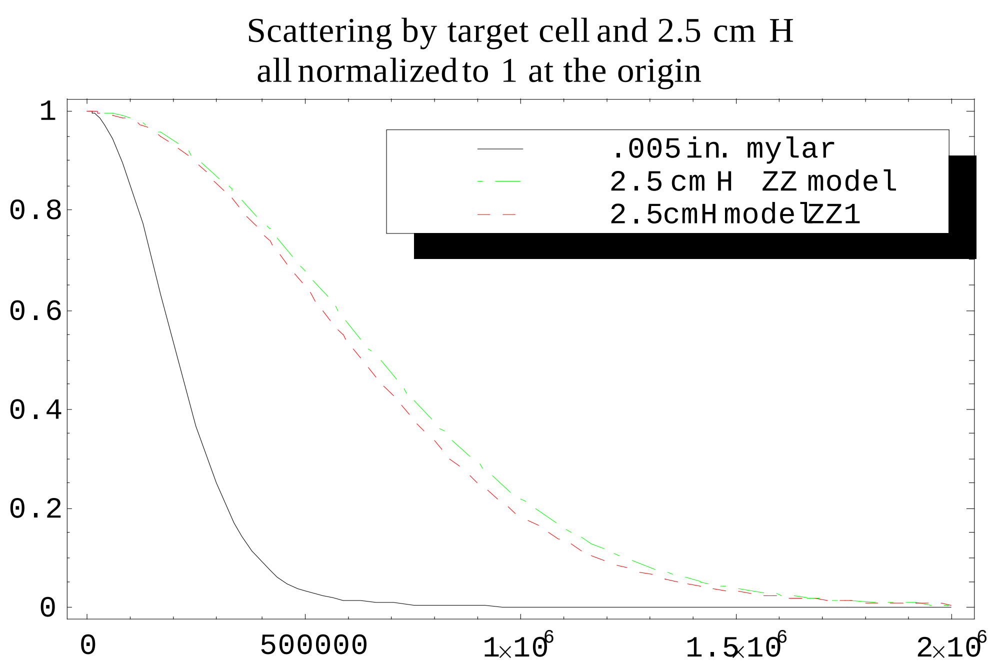
1. The central nearly gaussian distribution.
2. The plural scattering region out to the single scattering tail.
3. The effect of the electron.
4. Identify the molecular effect.
5. Measure straggling.

The above accuracy would be enough to convince us that we really do understand the underlying physics and allow a precession check of the simulation program. On the other-hand, such accuracy may be completely unnecessary and we should be guided by our simulation studies as mentioned above.

I think a somewhat higher energy than 200 MeV/c could be used since the coulomb scattering depends mainly on P_t, and not P. This could make the instrumentation somewhat easier. I will assume in what follows that we use 500 MeV/c muons and that the chamber resolution is 0.5 mr in POLAR angle. If we wish to see the effects of the electron at 5 mr, the rms angle should be perhaps 0.4 * 5 mr = 2 mr. Such a gaussian central part should be pretty well measured with the chambers. If the rms angle from the hydrogen is 2 mr, that implies a target thickness equal to $1/19.23^2\ X_0$ or a thickness of about 2.5 cm of LH_2. If the windows of mylar can be kept to .005" total, they will contribute an rms scattering angle of 0.8 mr.

Next we would like a thicker target where the electrons are well contained in the gaussian peak. Suppose we pick a target 50 cm long. This has an rms angle of 8.42 mr which is nicely bigger than 5 mr. The windows could be thicker for this target without interfering with the accuracy of the measurement and the thickness provides about 15 MeV energy loss which should be sufficient to make a good measurement of the straggling and its correlation with the scattering angle. I would think that an error on the energy measurement of 0.2% would be good enough. Note that for straggling, scattering at the down stream end of the target is not important, but scattering in the final angle after an analysis magnet will dominate the errors. However, for correlation between straggling and scattering, we may have to be more careful.

Below we show some distributions from these targets. The first shows the scattering in the 2.5 cm target. The black curve is the normalized distribution caused by the mylar windows and the two models are for Z^2 and $Z(Z+1)$. Since the electron cuts off at P_t = 2.5 MeV/c, it is proper to use $Z(Z+1)$ for the thin target.



The next set of curves shows the same information on a logarithmic plot with a wider scale. It is seen that the two models Z^2 and $Z(Z+1)$ start to differ at a P_t of around 2 MeV/c and the ratio is 2 at larger angles as it should be. So measurements in this region will indicate how the plural scattering matches on to the single scattering tail. The blue line is at the 10^{-3} level and shows that runs more than 10^4 muons will be necessary to trace out this region. This is better shown in tables below.

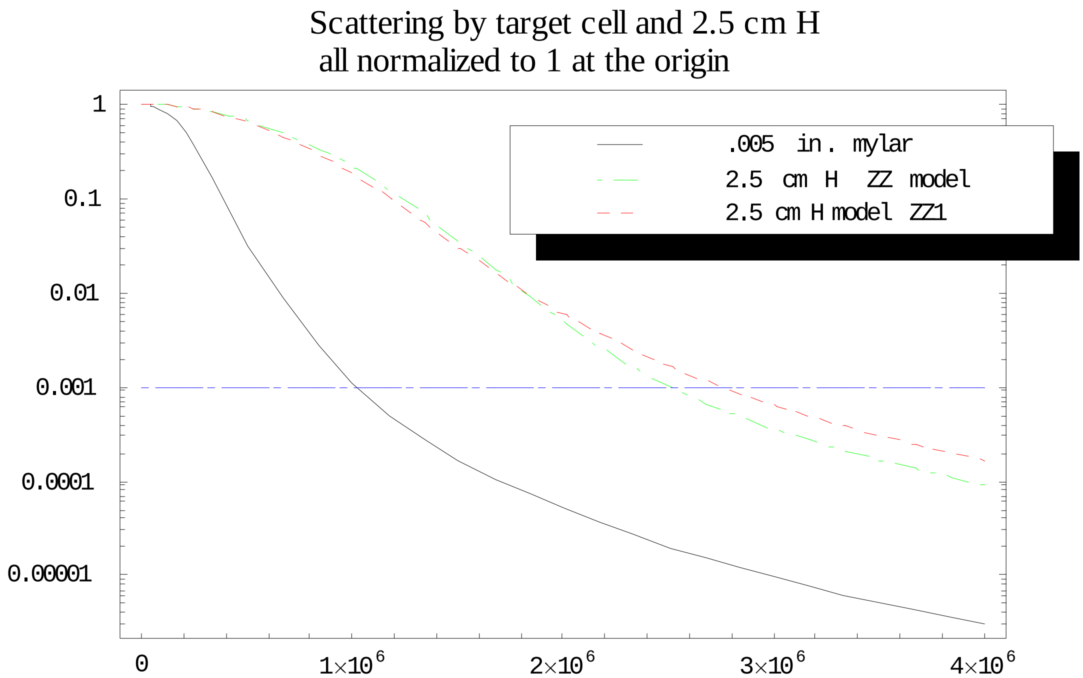


Table 2 gives the integrated probability of scattering a muon between 0 and P_t for the mylar, and the two different models for the 2.5 cm target. The integrals are all normalized to 1.0 and so will all agree at very large P_t.

Table 2.

pt 10^5	mylar	H, ZZ model	H, ZZ1 mode
1	0.132779	0.0147336	0.0159594
2	0.424053	0.0575937	0.0622047
3	0.686566	0.124763	0.134119
4	0.84	0.210517	0.224876
5	0.912032	0.308039	0.326535
6	0.946732	0.410321	0.431199
7	0.96577	0.510986	0.532014
8	0.976734	0.604897	0.623844
9	0.983151	0.688492	0.703536
10	0.987111	0.759834	0.769827
11	0.989742	0.818431	0.822987
12	0.991603	0.864913	0.864328
13	0.992982	0.900643	0.895715
14	0.994036	0.927356	0.919155
15	0.994864	0.946862	0.936515
16	0.995527	0.960841	0.949374
17	0.996067	0.970731	0.958969
18	0.996513	0.977684	0.96622
19	0.996887	0.982577	0.971784
20	0.997202	0.986049	0.97612

Table 3 gives the integrated probability for scattering between a P_t = 2.5 MeV/c and P_t, and can be used to explore the difference in the tails of the models.

Table 3.

pt 10^5	mylar	H, ZZ model	H, ZZ1 model
25	0.0010323	0.00763842	0.0116805
30	0.00158087	0.0100956	0.0162802
35	0.00190765	0.0112812	0.0185883
40	0.00211818	0.0119746	0.0199524
45	0.00226182	0.0124226	0.0208382
50	0.00236421	0.0127311	0.0214499
55	0.00243979	0.0129534	0.0218915
60	0.00249716	0.0131193	0.0222214
65	0.00254175	0.0132466	0.0224748
70	0.00257709	0.0133464	0.0226738
75	0.00260558	0.0134263	0.022833
80	0.00262888	0.0134912	0.0229625
85	0.00264818	0.0135447	0.0230692
90	0.00266434	0.0135894	0.0231582
95	0.00267801	0.013627	0.0232333
100	0.00268968	0.013659	0.0232972

AN OPEN QUESTION

There is still an open question that must be answered. Lebrun's MUCOOL#30 shows effects of the magnetic field on the scattering. It is not understood how to include these effects in the simulation, or if it is important. I don't believe that this is a fundamental problem, but it needs attention.

SUMMARY

The premise of this note is that the simulation program can tell us what we need to measure in a scattering experiment and with what accuracy. Note that in the above experiment we have only been talking about one scattering cell, whereas in the actual channel there may be of the order of 100 such cells. This is the main reason for being concerned about an accurate measurement of the tails.

In addition to the scattering experiment, the hardware must also meet tolerance standards that can be set by the simulation. Hence, when this information is available, actual elements of the hardware should be built and tested. These tests will not only include field measurements, but also will have to include radiation sensitivity tests. The hydrogen cells and rf cavities will need to be studied in a beam to verify that they do perform properly.

An actual target station must be built to verify the target performance as well as to nail down the yields. I note in passing that the CERN antiproton source missed by a factor of

2 in predicting its target yields, and even more to the point, we missed at FNAL also be a factor of two in our source! Fortunately, we do have a target experiment in place, so that is not the subject of this note

A section of the induction linac will have to be building and measured, or if low frequency RF is used, modules of that system will have to be constructed and measured.

Finally, the instrumentation to tune the channel will have to be invented. Again, this relies heavily on the simulation studies first defining what is necessary.

Thus the program looks very different than the original MUCOOL. It will require a beam and a test area. It will involve a large experimental program, which must be integrated with the simulation studies. It may not be less expensive, but it is broken up into smaller pieces. It should be pointed toward making a proposal for a major test that is ultimately integrated with a real machine.