Final Focus Challenges for Muon Colliders at 10 and 100 TeV

(1) Perspective

(2) Optics and Spot Size Limitations

(3) High Charge

(4) Multiple Turns, Tolerances & Tuning, Background,...

(5) Single-Pass Option

(6) Conclusions

parameter	symbol	SLC	NLC	CLIC	μ-A	μ-B
cm energy [TeV]	E_{com}	0.1	1	3	10	100
Lorentz factor	γ	10^5	10^6	3×10^6	5×10^4	5×10^5
bunch population [10^{10}]	N_b	4	1	0.4	300	80
hor. emittance [μm]	$\gamma\epsilon_x$	50	4.5	0.6	38	8.7
vert. emittance [μm]	$\gamma\epsilon_y$	8	0.1	0.01	38	8.7
hor. beta [mm]	β_x^*	2.8	12	8	2.1	2.5
vert. beta [mm]	β_y^*	1.5	0.15	0.1	2.1	2.5
hor. spot size [nm]	σ_x^*	1700	235	40	1300	210
vert. spot size [nm]	σ_y^*	900	4	0.6	1300	210
bunch length [mm]	σ_z	1	0.12	0.03	2.2	2.5
rms energy spread [%]	σ_δ	0.1	0.3	0.3	0.042	0.008
pinch enhancement	H_D	2.0	1.45	1.9	1.08	1.11

Non-Issues:

- synchrotron radiation and beamstrahlung: no problem
- small energy spread!

Challenges:

- limited quadrupole strength & high beam energy

  SLC triplet: 100 T/m $\rightarrow$ 500 T/m ?

  but energy increases factor 100 or 1000!
- large geometrical emittances
- high bunch charge: 20–75x SLC, 200–750x CLIC
- multiple passages

Optics

Layout

- 6 models (3 each for 10 & 100 TeV) aided by FFADA code
- chromatic correction (CCX, CCY) and final transformer (20/50 demagnification)
- final doublets (only case '10-TeV e' with triplet)
- vary length, gradients,...

within technological reach: 320 T/m

or $k \approx 2 \times 10^{-3}$ m^{-2} at 100 TeV

$\rightarrow$

large chromaticity, small energy bandwidth

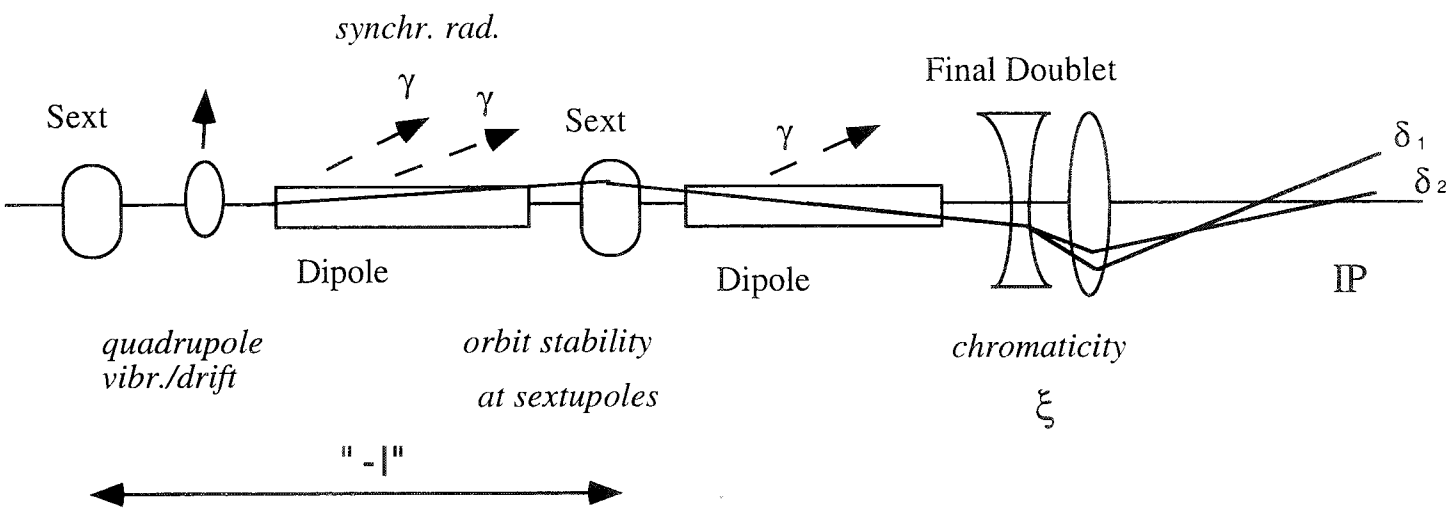
large chromaticity of final quadrupoles

$$\xi = \int K\beta \, ds \qquad \frac{\Delta\sigma_\delta}{\sigma_0} = \xi\delta_{\rm rms}$$

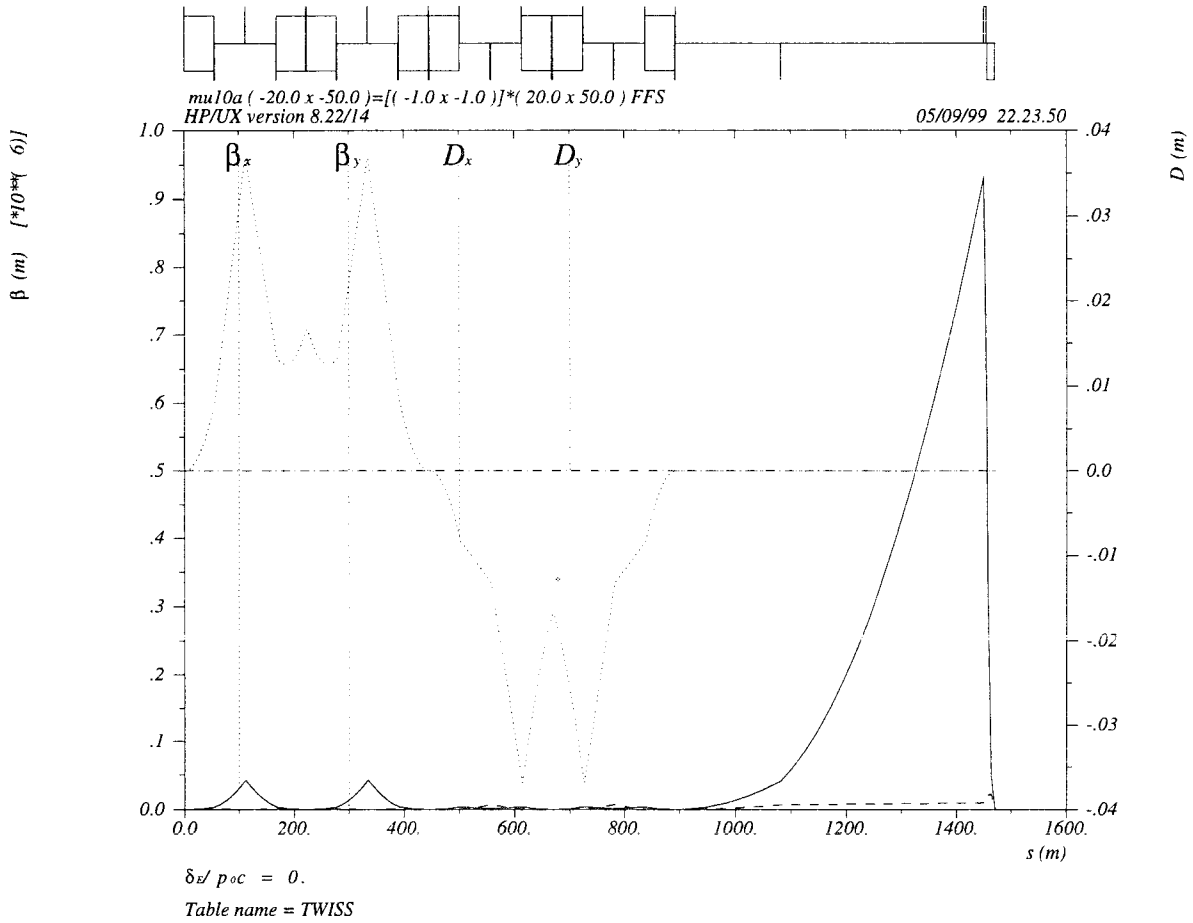
$$\text{total beam size: } \sigma^2 = \sigma_0^2 + \Delta\sigma_\delta^2$$

SLC: $\xi \approx 6000$, $\delta_{\rm rms} \approx 0.1$ %; one requires: $\Delta\sigma_\delta/\sigma_0 < 1/5$ (2% blow up) $\rightarrow$ chromatic correction to within 3×10^{-2} ($\xi_c \leq 180$)

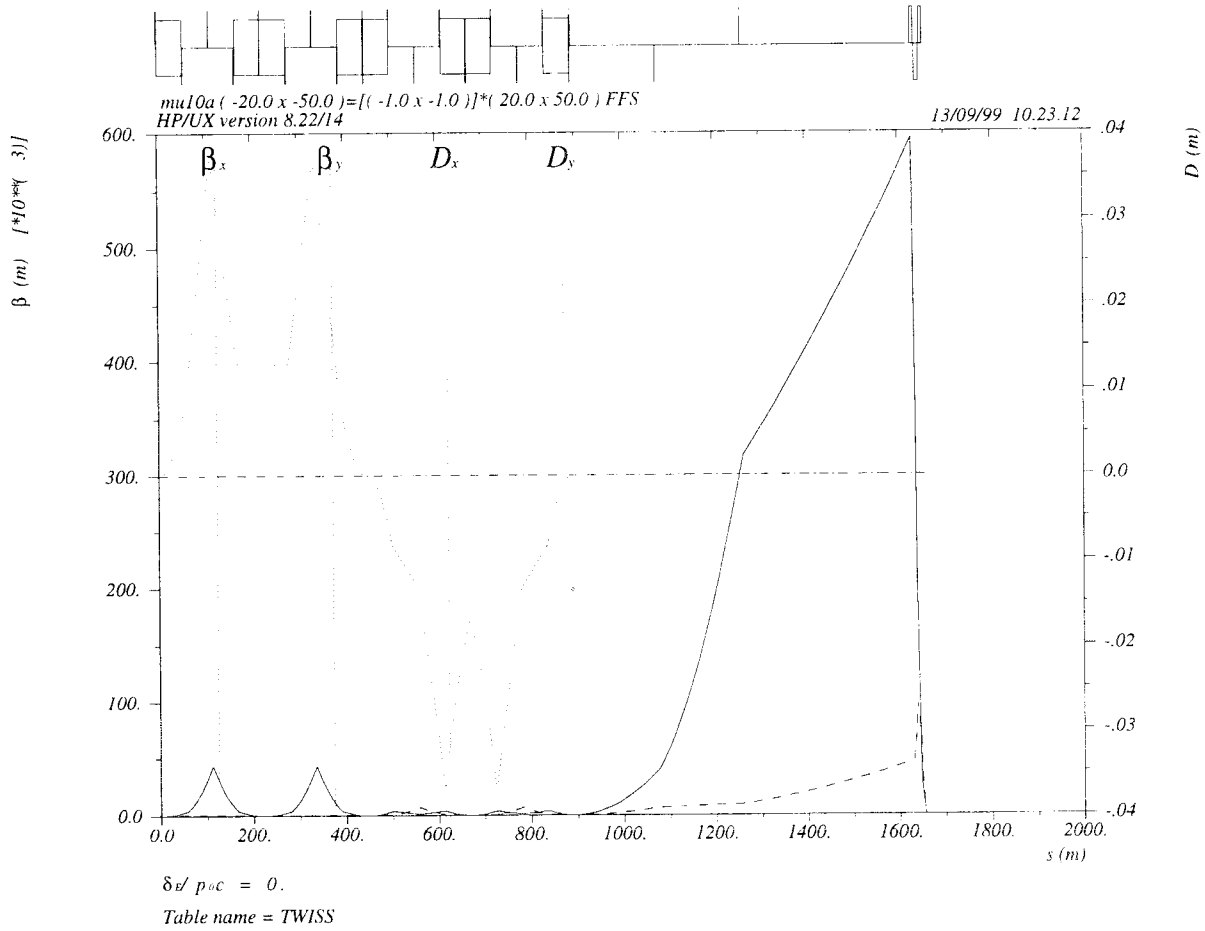
NLC: $\xi \approx 30000$, $\delta_{\rm rms} \approx 0.3$ %; $\rightarrow$ chromatic correction to within 2×10^{-3} ($\xi_c \leq 60$)



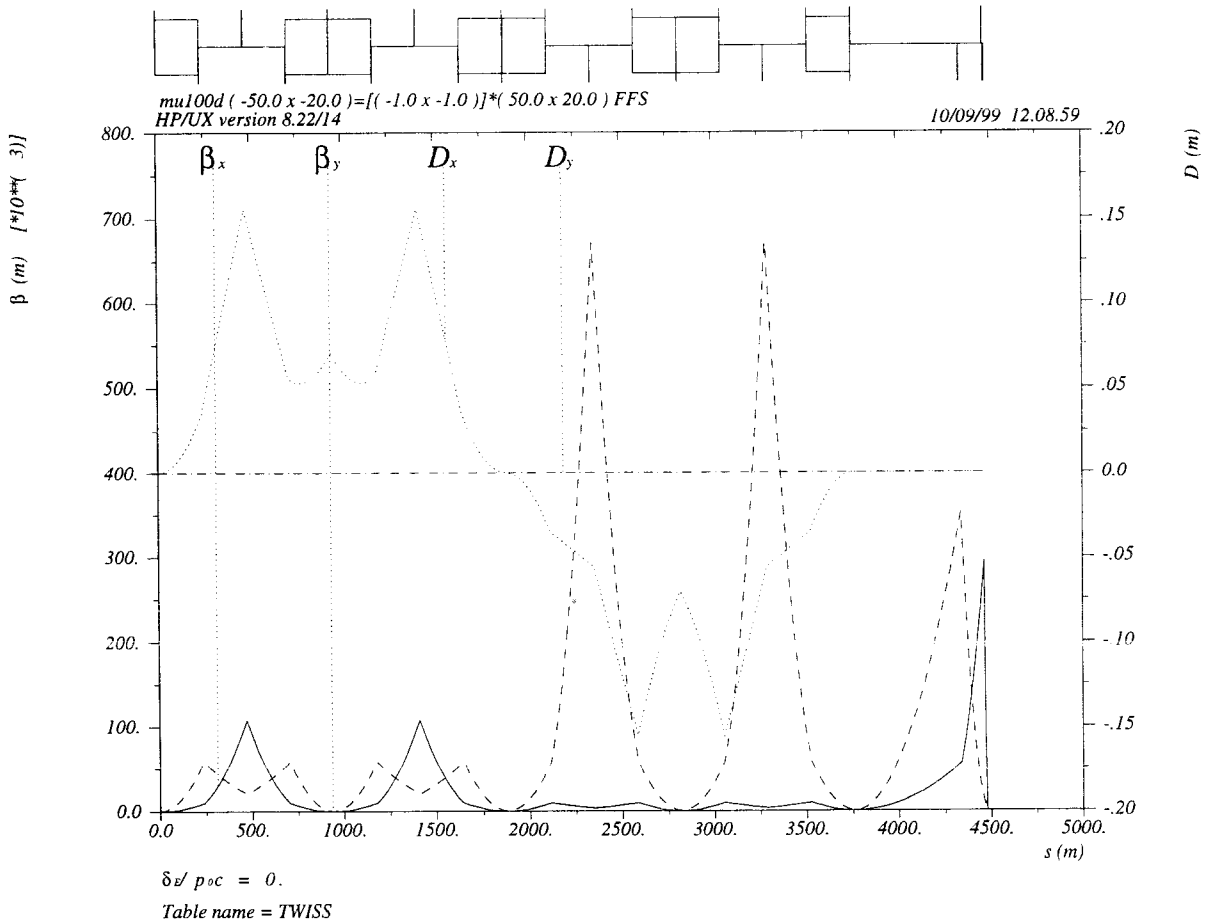
Schematic of a conventional final focus.



Twiss functions for a 10-TeV final focus ('10-TeV a') with a maximum quadrupole gradient of 320 T/m.



Twiss functions for a 10-TeV final focus ('10-TeV e') with a maximum quadrupole gradient of 320 T/m, and a final triplet.



Twiss functions for a 100-TeV final focus ('100-TeV d') with a maximum quadrupole gradient of 7200 T/m.

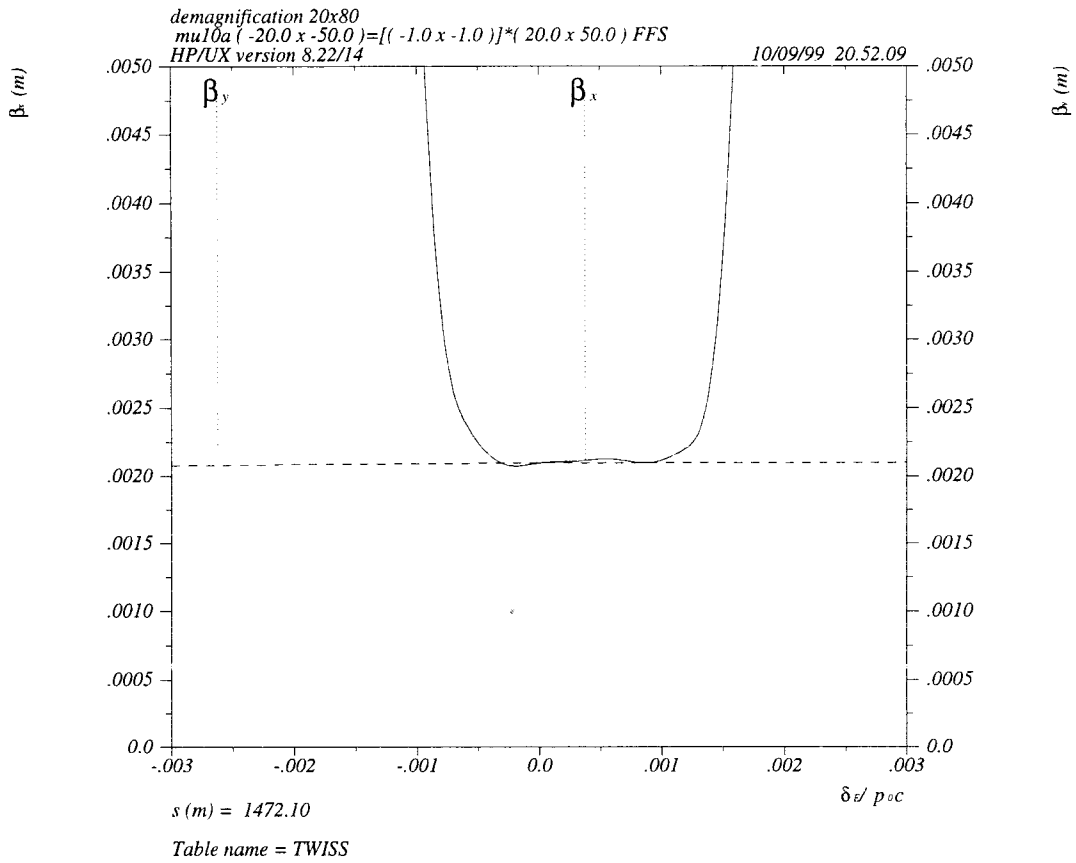
optics	$(\partial B_y/\partial x)_{\max}$	ξ_x	ξ_y	L	$(\Delta p/p)_{\mathrm{FW}}$
10-TeV a	320 T/m	56000	3500	1472 m	0.15%
10-TeV c	3200 T/m	17000	1500	790 m	0.28%
10-TeV e	320 T/m	15000	5400	1660 m	0.22%
100-TeV a	3200 T/m	39500	7000	4500 m	0.14%
100-TeV c	3200 T/m	39500	7000	3000 m	0.12%
100-TeV d	7200 T/m	26000	4700	4500 m	0.21%

rms beam energy spread: 0.042% at 10 TeV

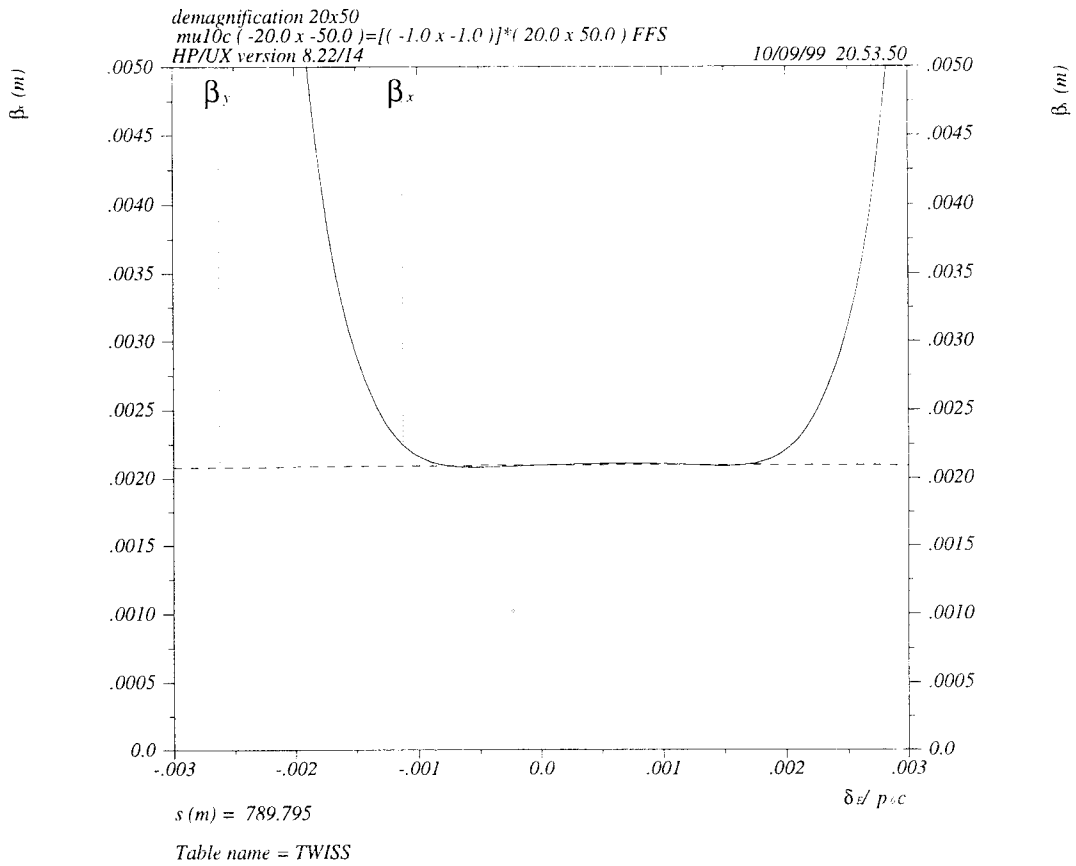
0.008% at 100 TeV

'10-TeV e' with triplet, all others based on doublets

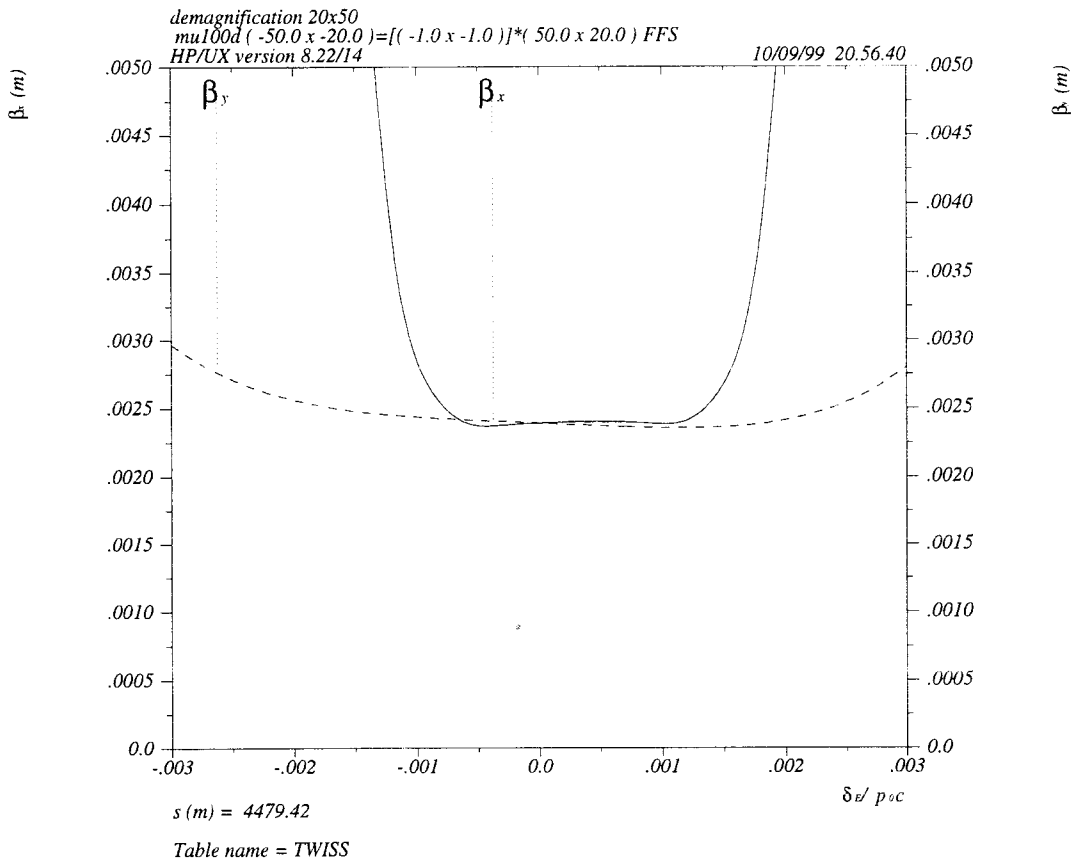
increasing quadrupole gradient & length improves the bandwidth



Momentum bandpass for optics '10-TeV a' (320 T/m).

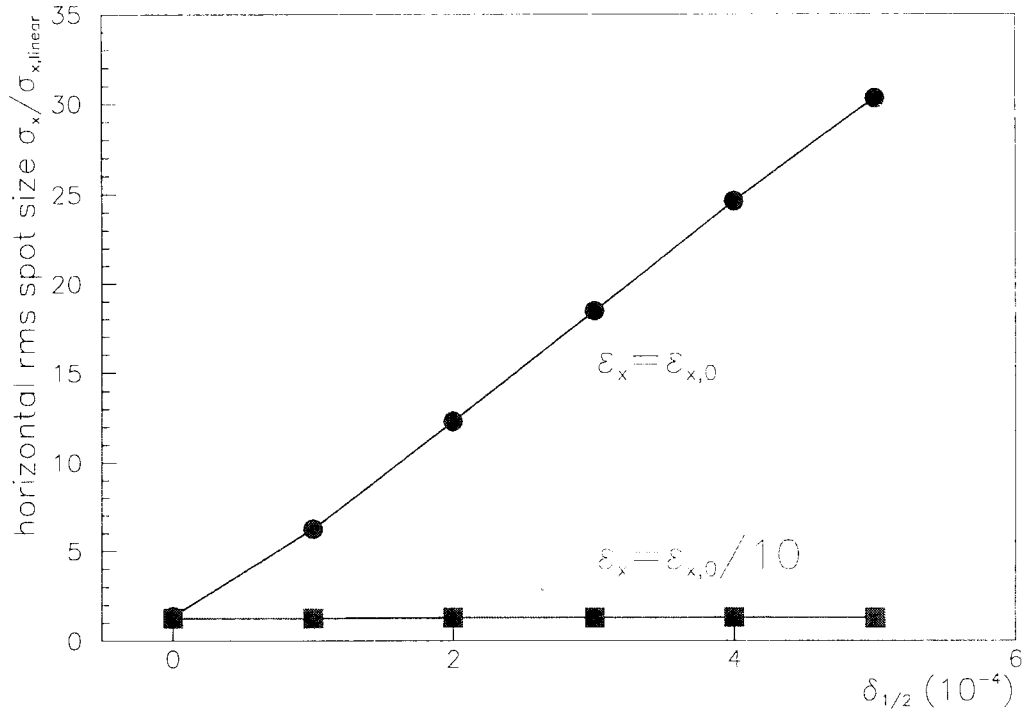


Momentum bandpass for optics '10-TeV c' (3200 T/m).



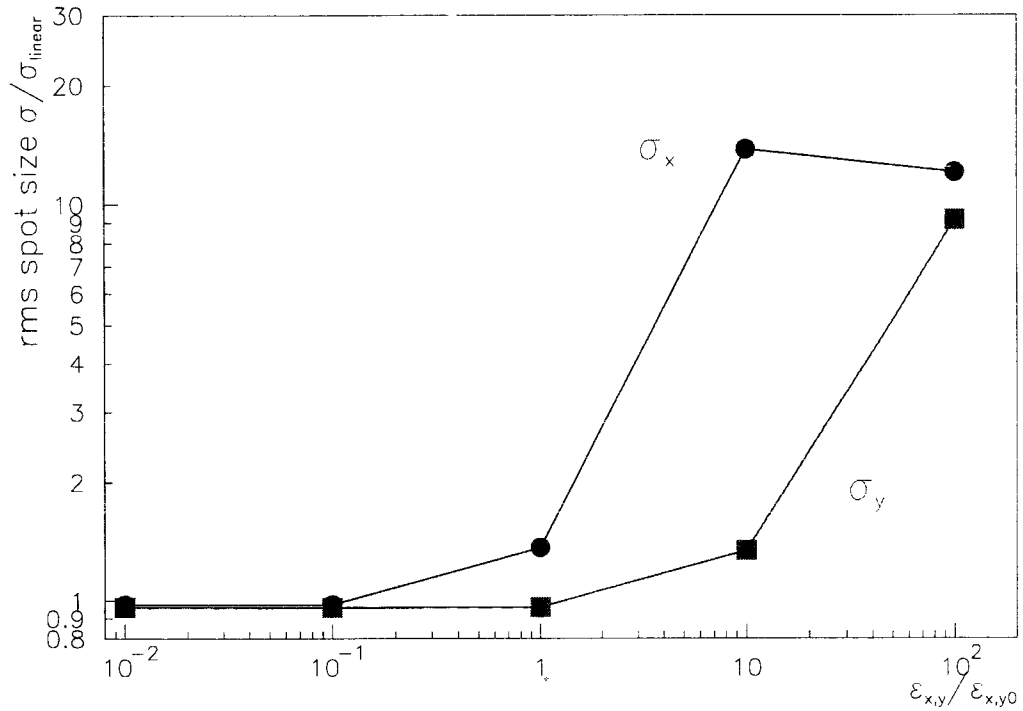
Momentum bandpass for optics '100-TeV d' (7200 T/m).

tracking



Horizontal rms IP spot sizes vs. half width of flat energy distribution. With $\pm 0.042\%$ flat energy spread: $\sigma_x = 35\ \mu$m! ($\sigma_y \approx 1.3\ \mu$m still). The blow-up is due to higher-order chromaticity.

correct spot size for cold beam!



Rms IP spot sizes vs. transverse emittance, for optics '10-TeV c', and zero energy spread. The vertical axis shows the simulated spot size divided by the linear spot size; the horizontal axis the emittance normalized to the design emittance.

correct spot size for cold beam !

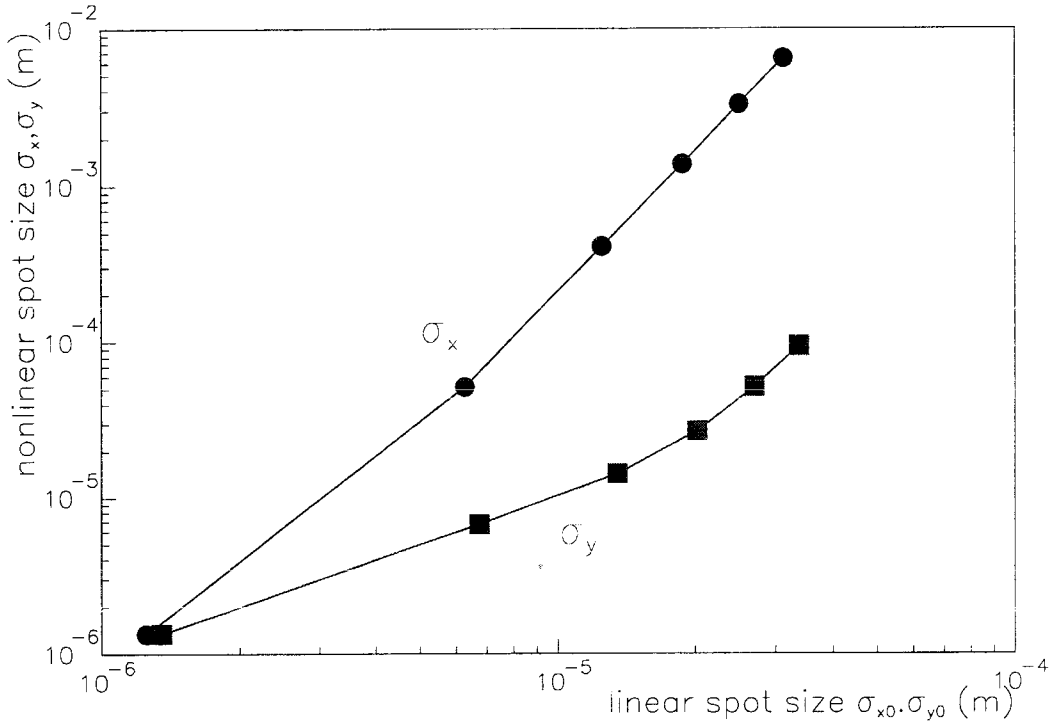
Kinematic Terms

Limit due to nonlinear kinematic terms (W. Wan). General trajectory equations:

$$
\begin{aligned}
x'' &= \frac{e}{p}\sqrt{1+z'^2+x'^2}\left[x'B_s-(1+z'^2)B_x+x'z'B_z\right] \\
z'' &= -\frac{e}{p}\sqrt{1+z'^2+x'^2}\left[z'B_s-(1+z'^2)B_z+x'z'B_x\right]
\end{aligned}
$$

with $p=\gamma mv$ the particle momentum.

To study the effect, we track a group of particles backwards and forwards through the final focus, with and without the higher-order kinematic terms, respectively.



Spot size increase due to the kinematic terms as a function of the linear spot size, for the optics '10-TeV a' with a constant $\beta^*_{x,y}$ and varying emittance. At 100 TeV, the effect is 10 times smaller.

Synchrotron Radiation (SR)

$m_\mu \approx m_e/200 \rightarrow$ SR much weaker than for e^+e^-.

rms energy spread induced by SR in a bending magnet of length l_B with a total bend angle θ_B:

$$\delta_{\rm rms} = \left(\frac{55}{24\sqrt{3}} r_\mu \lambda\!\!\!^{-}_\mu \gamma^5 \frac{\theta_B^3}{l_B^2} \right)^{1/2}$$

For example, $\theta_B = 2.5$ mrad, $l_B = 1000$ m, $E_{beam} = 50$ TeV, gives $\delta_{\rm rms} = 4 \times 10^{-9}$.

Oide effect (spot-size increse due to SR radiation in the last quadrupole magnets) is no issue either.

Beamstrahlung

Beamstrahlung is synchrotron radiation emitted in the field of the opposing beam. It can be characterized by two parameters: Υ and n_γ. Apart from a factor 2/3, the Υ parameter is equal to the critical energy of the beamstrahlung in units of the incident beam energy.

Average value of Υ:

$$\Upsilon = \frac{5}{6} \frac{r_\mu^2 \gamma N_b}{\alpha \sigma_z \sigma_x (1 + \sigma_y/\sigma_x)}$$

For 100 TeV: $\Upsilon \approx 3 \times 10^{-5}$.

Number of beamstrahlung photons emitted per muon:

$$n_\gamma \approx \frac{2\alpha r_\mu N_b}{\sigma_x (1 + \sigma_y/\sigma_x)}$$

which evaluates to $n_\gamma \approx 0.4$. This is not completely insignificant.

Average energy loss: $\delta_B \approx \Upsilon n_\gamma/2$, or $\delta_B \approx 7 \times 10^{-6}$ at 100 TeV.

High Charge

energy spread induced between sextupoles and final quadrupoles must be small to avoid spot-size increase

$$(\Delta\delta)_{\rm rms} \ll 1/\xi_{x,y}$$

high bunch charge

$\rightarrow$ wake fields

space charge

Longitudinal Wake Fields I

Resistive-wall wake

$$\frac{d\sigma_E}{ds} = 0.205\,\frac{Ne^2}{b^2}\left(\frac{s_0}{\sigma_z}\right)^{3/2} \qquad \text{(cgs units)}$$

where $s_0 = \left(\frac{cb^2}{2\pi\sigma_c}\right)^{1/3}$ with b chamber radius, c speed of light, σ_c conductivity. For Al chamber ($\sigma_c \approx 3.2 \times 10^{17}$ s^{-1}) with $b = 1$ cm radius: $s_0 = 25$ μm, and

$$(\Delta\delta)_{\rm rms} \approx 3 \times 10^{-8} \quad \text{at 10 TeV for } \Delta s \approx 1.5 \text{ km}$$

$$(\Delta\delta)_{\rm rms} \approx 2 \times 10^{-9} \quad \text{at 100 TeV over } \Delta s \approx 4.5 \text{ km.}$$

Longitudinal Wake Fields II

Geometric wake: untapered collimator of radius a, pipe radius $b \rightarrow$

$$(\Delta\delta)_{\rm rms} \approx 0.444 \times \frac{N r_\mu}{\gamma \sigma_z} \ \ln \frac{b}{a}$$

e.g., one collimator at 1.5 cm in a 2-cm beam pipe.

$$(\Delta\delta)_{\rm rms} \approx 7 \times 10^{-8} \qquad \text{at 10 TeV}$$

$$(\Delta\delta)_{\rm rms} \approx 1.6 \times 10^{-9} \qquad \text{at 100 TeV}$$

Tapered collimator:

$$(\Delta\delta)_{\rm rms} \approx 0.44 \times \frac{N r_\mu}{\gamma \sigma_z} (1 - \tilde{\eta}_1) \ \ln \frac{b}{a}$$

where $\tilde{\eta} = \min(1.0, \eta_1)$ with $\eta_1 = \frac{g\sigma_z}{(b-a)^2}$ and g the length of the taper. <u>Note:</u> $(\Delta\delta)_{\rm rms} = 0$ for $\tilde{\eta}_1 = 1$, For a bunch length of 2.2 mm, and $(b-a)$ equal 5 mm, $\tilde{\eta}_1 = 1$ for taper lengths above 1 cm.

Longitudinal Space Charge

difference in energy spread between two locations '1' and '2' may be estimated as

$$(\Delta\delta)_{\rm rms} \approx 0.28 \times \frac{2Nr_\mu}{\sqrt{2\pi}\sigma_z\gamma} \ln\left(\frac{(\sigma_x+\sigma_y)_2 b_1}{(\sigma_x+\sigma_y)_1 b_2}\right)$$

where b_1 and b_2 the beam pipe radii. Ratio of the sum of beta functions can be a factor 20. Then $(\Delta\delta)_{\rm rms} \approx 2.6 \times 10^{-7}$, which is still acceptable.

More detailed tracking simulations could be performed using the real space-charge force.

Transverse Wake Fields I

Resistive-wall wake:

$$\Delta y' = 2\frac{N r_\mu}{\gamma \sigma_z}\frac{L}{b^3} \; < f_R > \; \sqrt{\lambda \sigma_z}\,(\Delta y)$$

with (Δy) centroid offset, $< f_R >= 0.82$ average over the longitudinal wake, and $\lambda = \rho/(120\pi)$. ($\lambda = 0.045,\ 0.15,\ 1.2$ nm for *Cu, W, Ti*). *E.g.*, *W* with b=2 cm, and $L = 100$ m, the jitter enhancement at 10 TeV is

$$\Delta y/y \approx \underset{5\times 10^{-6}}{\cancel{0.14}}\ \beta[\mathrm{m}]$$

This can be significant!

Transverse Wake Fields II

Geometric wake: for untapered collimator

$$\Delta y' = \frac{2Nr_\mu}{\gamma}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) y$$

where a and b are the radius of the collimator and the beam pipe, respectively, and y the centroid offset. For 10 TeV, $a = 1$ cm, $b = 1.5$ cm, the jitter enhancement is

$$\Delta y/y \approx 10^{-5}\beta[\mathrm{m}]\ \cancel{y[\mathrm{m}]}$$

Tapered collimator:

$$\Delta y' = \frac{Nr_\mu}{\gamma\sigma_z}\frac{2(b-a)^2}{abg} < f_G > \ y$$

with $< f_G >= 0.282$.

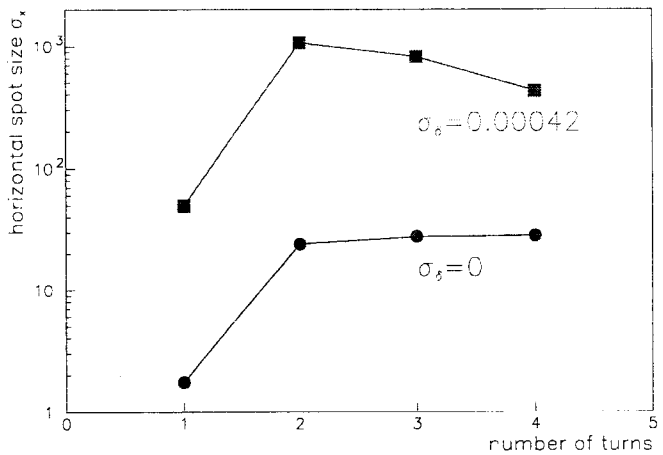
Multiple Turns

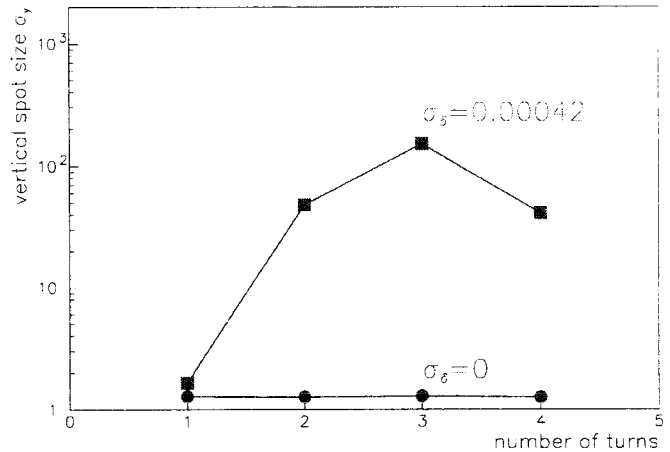
Collider parameters at 10 and 100 TeV, as provided by the workshop organizers.

energy	10 TeV	100 TeV
circumference	15 km	100 km
repetition rate	27 Hz	7.9 Hz

At 10 TeV the muons are stored for 740 turns, and at 100 TeV for 380 turns.

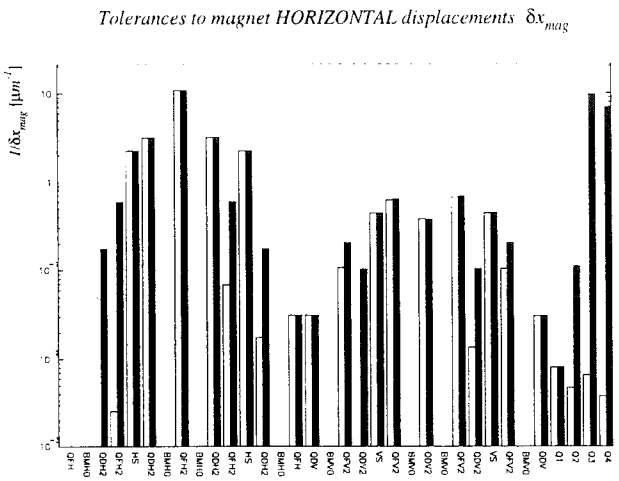
At the end of the 10-TeV store the luminosity has decreased by 50%, and the two beam currents by about 30%. Much smaller decay at 100-TeV. 'Top-off' possible? (needs fast cooling)

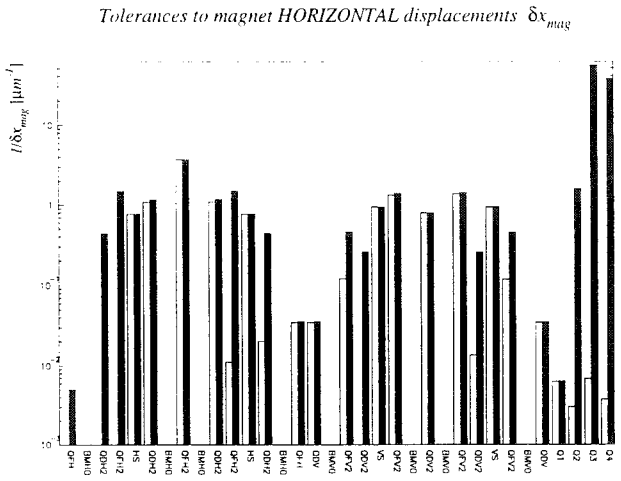




Spot size on first few turns, for optics '1-TeV c'. Blow-up for $\delta_{\rm rms} = 0$ possibly caused by imperfections of the $-I$ or of the phase advance between the sextupoles and the final doublet, and/or by a tiny mismatch between turns? Huge spot-size increase with $\delta_{\rm rms} \neq 0$, due to higher-order chromaticity?

Tolerances



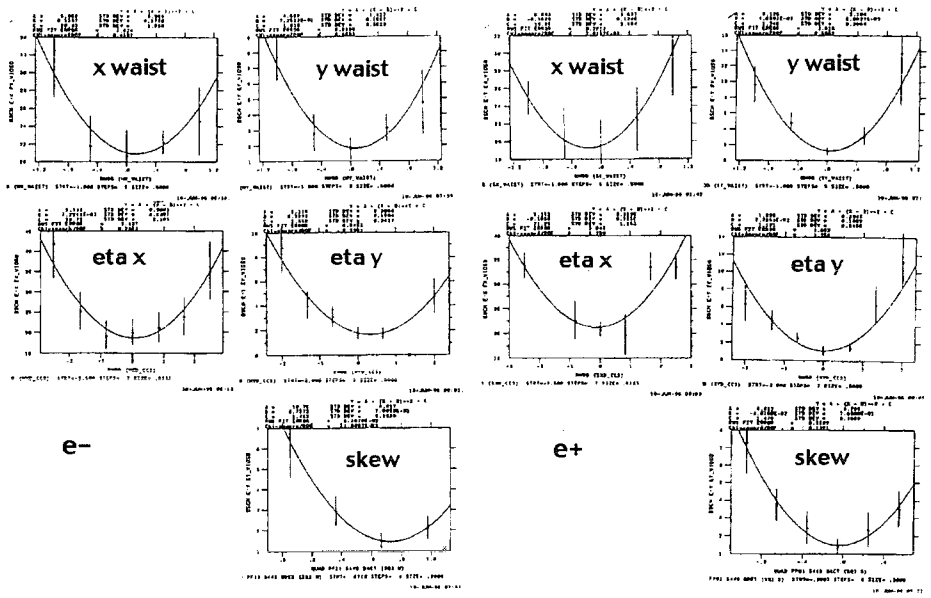


Horizontal stability tolerances for 2% luminosity loss on a single passage, calculated with FFADA. Left: optics '10-TeV a'; right: optics '100-TeV c'. At 100 TeV, the final quadrupoles must be stabilized on a level of 20 nm.

Multi-turn tolerances?

Multi-turn feedback for orbit and lower-order aberrations?

Spot-Size Tuning at SLC



Typical tuning scans at the SLC interaction point. Dispersion, waist and coupling were corrected every few hours, by varying orthogonal multiknobs, and for each knob value determing the rms beam sizes via beam-beam deflections. The knobs were set to the minimum of the fitted parabola. Later: beamstrahlung and luminosity signalxs.

Spot-Size Tuning at Muon Collider

- both beams are affected by tuning, unless restricted to 1st turn.
- independent control via electrostatic magnets?
- turn-to-turn matching (pulsed magnets?)
- match of outgoing dispersion, waist, etc.
- effect on betatron tunes
- possibility of complete tuning scan in one cycle?
- no (or little) beamstrahlung signal

Background, Heat Load, and Collimation Issues

Two sources of background and heat load:

- muon decay

  at 10 TeV 10^8 muons are lost per km; and 2×10^6 at 100 TeV

- beam halo

Only the second source exists in electron-positron colliders.

Double-approach to background reduction at 2 TeV:

- sweeping dipoles (40% of length, and 8.5 T strength; a dipole even between IP and last quadrupole)

- tungsten liner, several cm thick;
  $\rightarrow$ lower quadrupole gradient!

Should/can this approach be extended to 100 TeV?

Single-Pass Collider Option

The design of a muon ring collider at multi-TeV energies faces severe, perhaps insurmountable problems:

- neutrino radiation could limit the ring collider to energies below a few TeV;
- beam has to survive hundreds of passes through a final-focus system more challenging than that of the SLC, retaining the same constant emittance. Non-trivial!

Similarily, several difficulties lie in the way of electron-positron linear colliders at multi-TeV energies: beamstrahlung, coherent pair creation at high Υ, and associated degradation of the luminosity spectrum and large background.

Injecting a muon beam with similar properties as contemplated for the standard ring muon collider, with

bunches/train	$n_b = 150$
repetition rate	$f_{rep} = 75$ Hz
bunch spacing	$t_{sep} = 0.67$ ns
muons/bunch	$N_b = 4 \times 10^9$

into a high-gradient compact linac such as CLIC solves all the above problems: the beamstrahlung at 10 TeV is still negligible, there is no need to preserve the emittances after the collision, and the beam can be dispersed downwards onto a dump, thereby reducing the density of neutrino radiation by orders of magnitude.

parameter	symbol	e^+e^- CLIC	μ CLIC A	μ CLIC B
com energy	E_{cm}	3 TeV	10 TeV	100 TeV
luminosity [10^{35} cm^{-2}s^{-1}]	L	1.1	14/10^4	780/10^4
beam energy	E_b	1.5 TeV	5 TeV	50 TeV
normalized tr. emittances	$\gamma\epsilon_{x,y}$	0.6, 0.01 μm	50 μm	8.7 μm
6-d. n. emittance [10^{-15} m^3]	$\gamma^3\epsilon_{6d}$	5	50000/4	1500/4
rms energy spread	$\delta_{\rm rms}$	1%	1%	0.1%
rms bunch length	σ_z	30 μm	43 μm/4	43 μm/4
IP beta functions	$\beta^*_{x,y}$	8, 0.1 mm	10 μm	10 μm
IP spot sizes	$\sigma_{x,y}$	40 , 0.6 nm	100 nm	14 nm
beamstrahlung energy loss	δ_B	32%	4 $\times 10^{-9}$	4 $\times 5 \times 10^{-7}$
Upsilon parameter	Υ	9	4 $\times 4 \times 10^{-7}$	4 $\times 3 \times 10^{-5}$
photons/lepton	n_γ	2.2	0.004	0.003
luminosity enhancement factor	H_D	1.92	1.00	1.00

Conclusions

- 1-turn final-focus optics not trivial due to limited quadrupole strength and large emittances; possible cures: plasma lenses, dynamic focusing, advances in high-field magnets...? cooling
- multiple passages imply many additional challenges concerning spot-size stability, tolerances, tuning,...
- single-pass collider à la CLIC is one option which avoids these last (and other) problems and could achieve a high luminosity