The Potential of Optical Stochastic Cooling After-Burner

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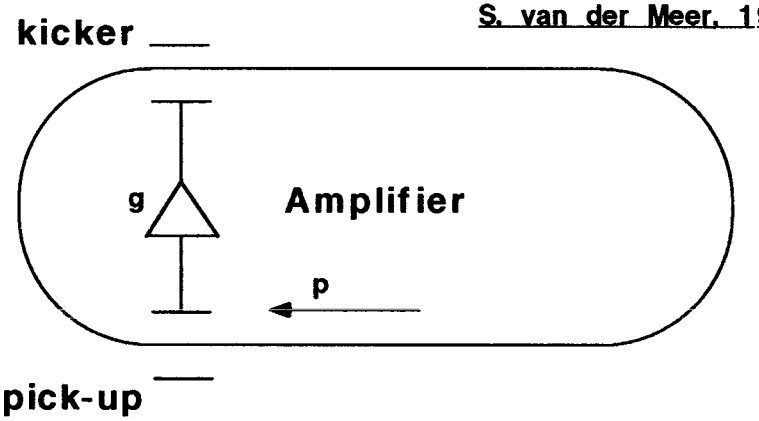
Outline of the talk

1. A brief introduction to the stochastic cooling
2. Specifics of Optical Stochastic Cooling
3. Cooling of muons

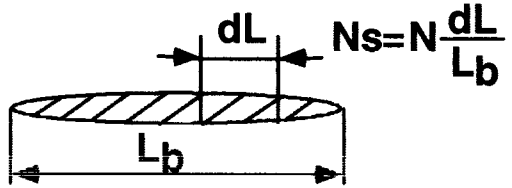
A brief introduction to the stochastic cooling concept

S. van der Meer, 1968



Each particle receives the kick which can be considered as a superposition of two parts:

- 'coherent' kick - in this part the particle affects itself via the cooling loop ----> damping
- 'incoherent' kick - a combination of many random kicks from the other sample particles ---> excitation



A brief introduction cont'd

Why is cooling possible?

Coherent kick (correction effect) accumulates each time the particle passes the cooling scheme, while incoherent kicks (random excitation) accumulate as a square root of the number of passes. (*We assume a perfect mixing of particles within the bunch on the way from the kicker to the pickup*).

Damping time depends on the number of particles in the sample.

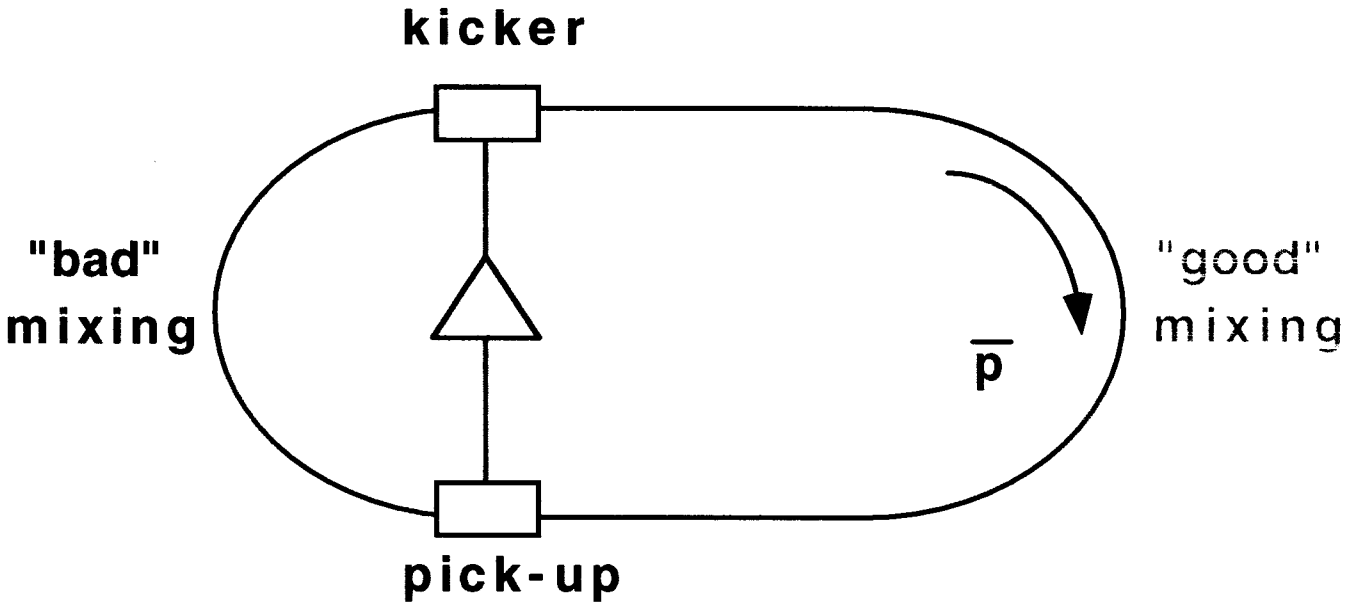
The less particles in the sample the faster the damping.

In the most optimal conditions $\tau = N_s \bullet$ (*revolution time*)

For a given longitudinal density of the particles in the bunch, the only way to reduce a number of sample particles is to reduce a sample length. Thus, a progress in the stochastic cooling technique is mainly associated in a design of a more wider bandwidth system of the pickup-amplifier-kicker.

Mixing

- "good" mixing between kicker and pick-up
- "bad" mixing (no mixing) between pick-up and kicker

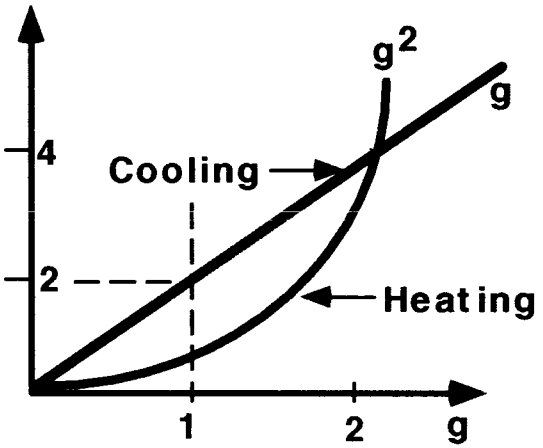


A brief introduction cont'd

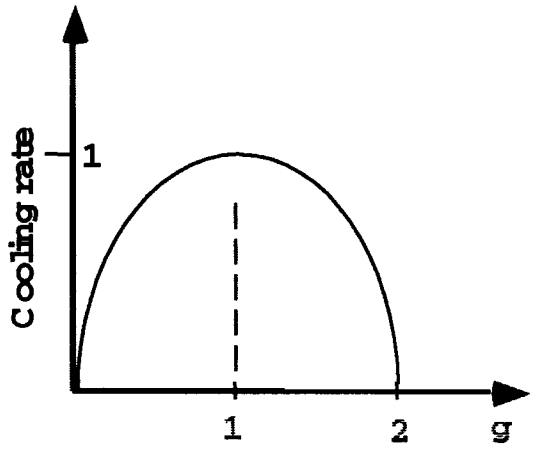
D. Möhl, "Stochastic Cooling for Beginners", CERN

$$x_{n+1}^{i} = x_{n}^{i} - \frac{g}{N_s}\sum_{k}^{N_s} x_{n}^{k}$$

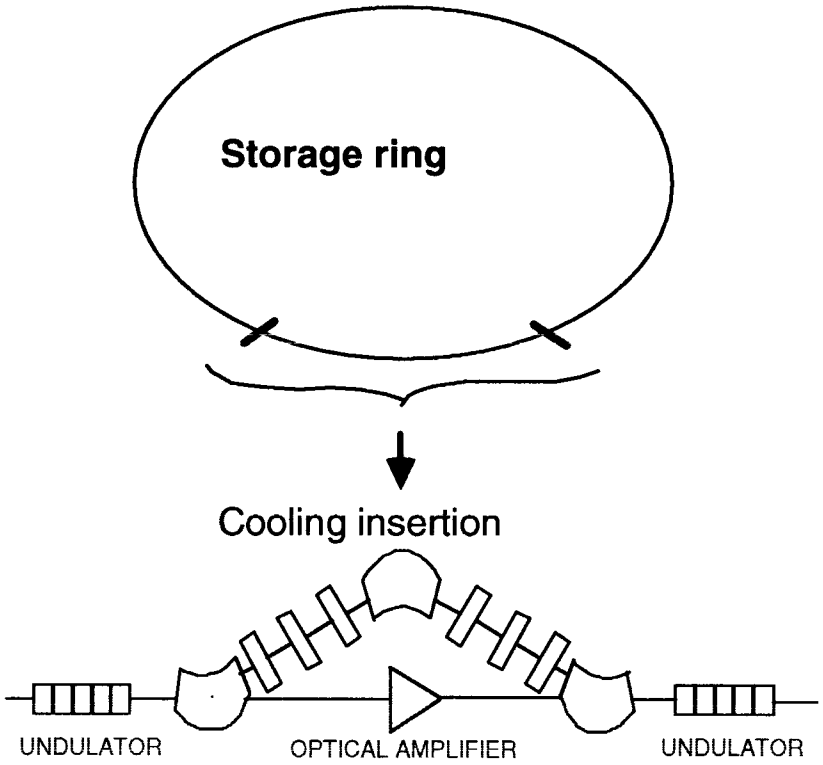
$$\text{decrement} = -\frac{\Delta(x^2)}{x_{rms}^2} = \frac{2g - g^2}{N_s}$$



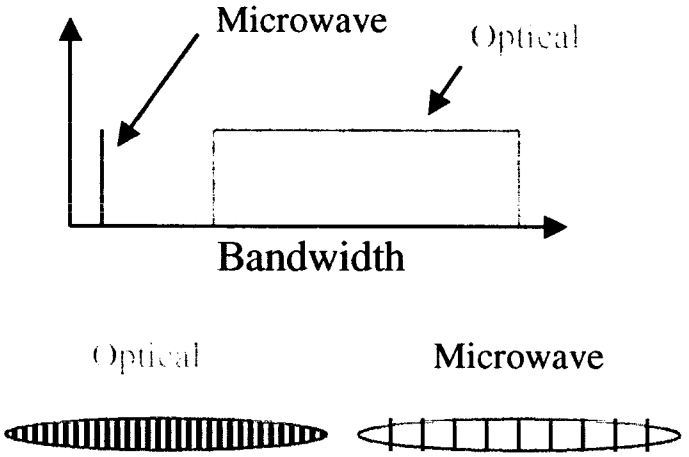
$$\text{maximum decrement} = \frac{1}{N_s} \text{ at } g = 1$$



Optical Stochastic Cooling



A.A. Mikhailichenko, M.S. Zolotorev, "*Optical stochastic cooling*", Phys. Rev. Lett. , V.27, N25, (1993)



OSC uses optical amplifier and undulators as a pick-up and a kicker.

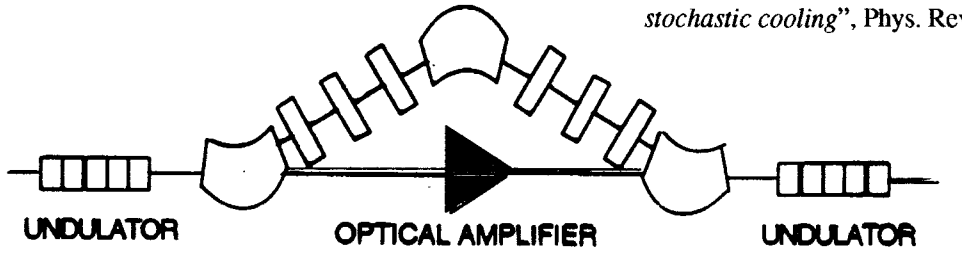
The amplifier bandwidth is ~10^{13} Hz.

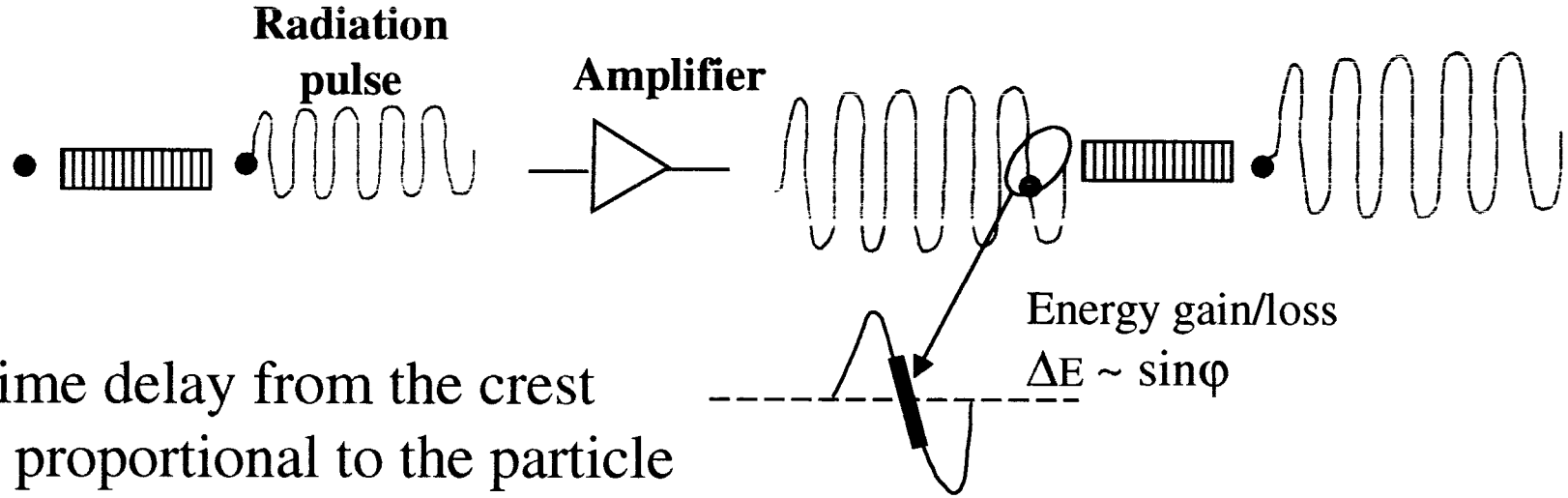
(Compare with ~10^9 Hz for microwave stochastic cooling)

Correspondingly, OSC has a potential for ~10^4 faster damping.

Transit-Time Method of OSC

M.S. Zolotorev, A.A. Zholents, *"Transit-time method of optical stochastic cooling"*, Phys. Rev. E, V.50, N4, (1994)3087.



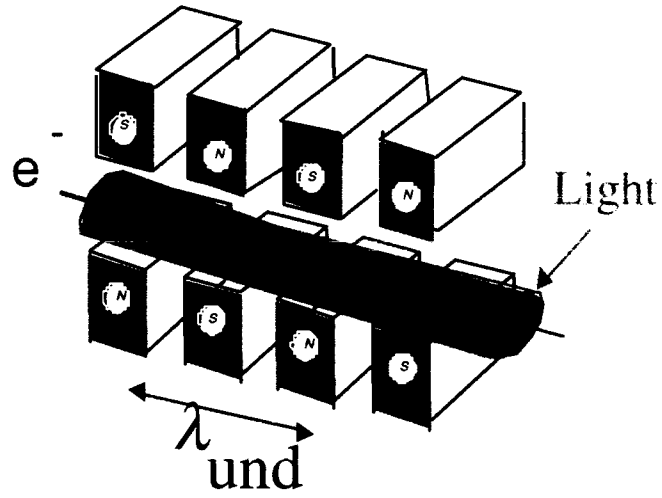


Time delay from the crest is proportional to the particle deviation from the equilibrium in the energy and spatial coordinates.

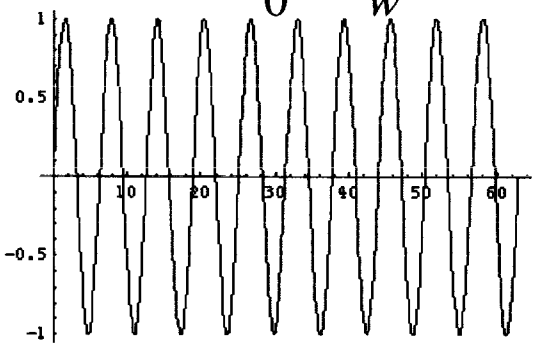
Time-of-flight lattice parameters must be continuously adjusted to compensate for decreasing beam emittance and energy spread.

Energy kick in the undulator

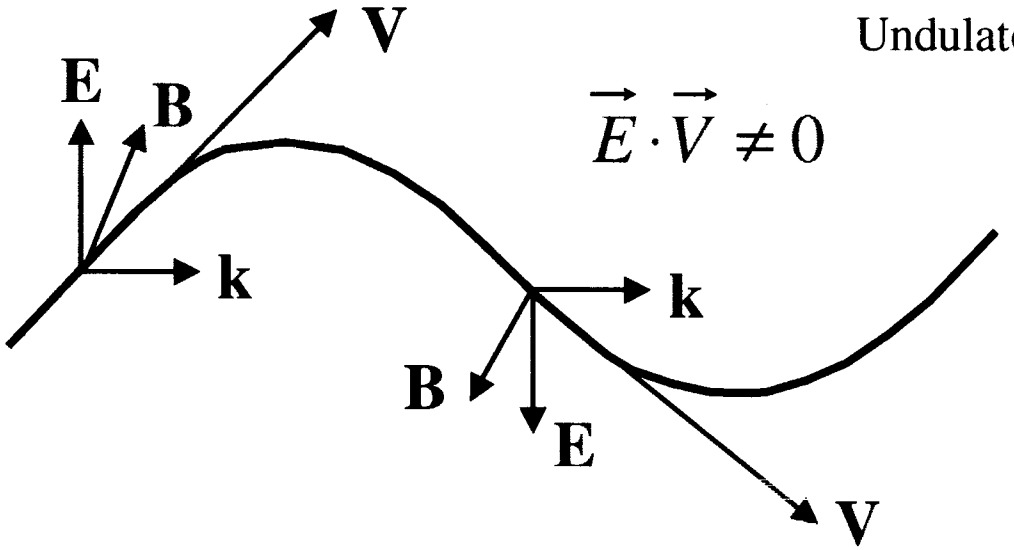
Electron trajectory through undulator



$$B = B_0 \sin(k_w z)$$



Magnetic field in the undulator



$$\vec{E} \cdot \vec{V} \neq 0$$

Undulator period — Light wavelength

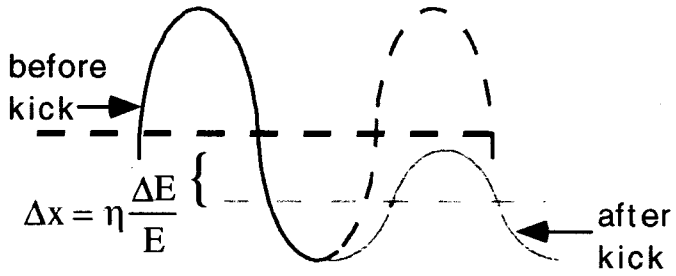
$$\lambda_{und} = 2\gamma^2 \lambda / (1 + K^2/2)$$

Undulator parameter:

$$K = \frac{eB_0 \lambda_w}{mc^2 2\pi}$$

Transit-time method cont'd

In order to have both longitudinal and transverse cooling a pick-up and a kicker should be installed in a position with a non-zero dispersion function (*similar to the Palmer's method of the momentum cooling*).



Betatron oscillations before and after energy kick

$$x = \left(x_\beta + \eta \frac{\delta E}{E} \right)_{\substack{\text{before} \\ \text{kick}}} = \left[x_\beta^{(c)} + \eta \left(\frac{\delta E}{E} + \frac{\Delta E}{E} \right) \right]_{\substack{\text{after} \\ \text{kick}}}$$

Transverse kick

$$\rightarrow \Delta x = x_\beta^{(c)} - x_\beta = -\eta \frac{\Delta E}{E}$$

energy kick

Cooling rates

Inverse damping time expressed in the number of passes through cooling section

Longitudinal damping

$$\frac{1}{\langle n_\varepsilon \rangle} = \frac{1}{e(N_s + n)} f_e(x, r) \qquad 0 < f_e(x, r) < 1$$

noise of the amplifier ~ 50

Transverse damping

$$\frac{1}{\langle n_x \rangle} = \frac{1}{e(N_s + n)} f_x(x, r) \qquad 0 < f_x(x, r) < 1$$

$$x = \frac{\text{delay due to the betatron oscillations}}{\text{delay due to the energy off-set}}$$

$$r = \frac{\text{synchrotron beam size in the undulator}}{\text{betatron beam size in the undulator}}$$

Noise of the optical amplifier

The amplifier is not affected by a thermal noise. Practically, there are no thermal photons with the energy of signal photons because: $e^{-\frac{\hbar\omega}{kT}} = e^{-60}$

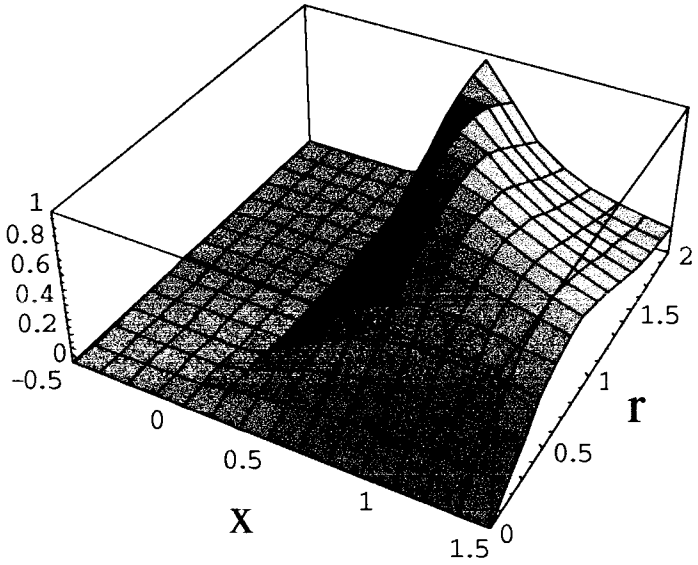
Noise of the amplifier is due to the spontaneous emission.

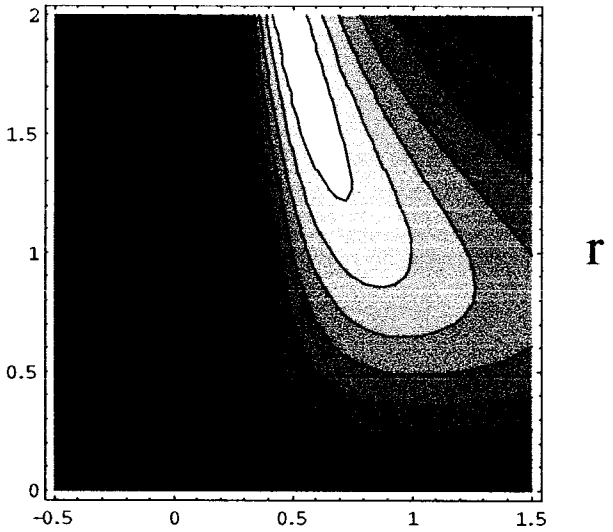
The equivalent noise of the spontaneous emission at the amplifier front end is one photon per mode. It is like "having extra ~50 muons" in a sample.

For undulator radiation: $(\text{number of photons}) \approx \frac{4}{137}\left(\frac{K^2/2}{1+K^2/2}\right)(\text{number of muons})$

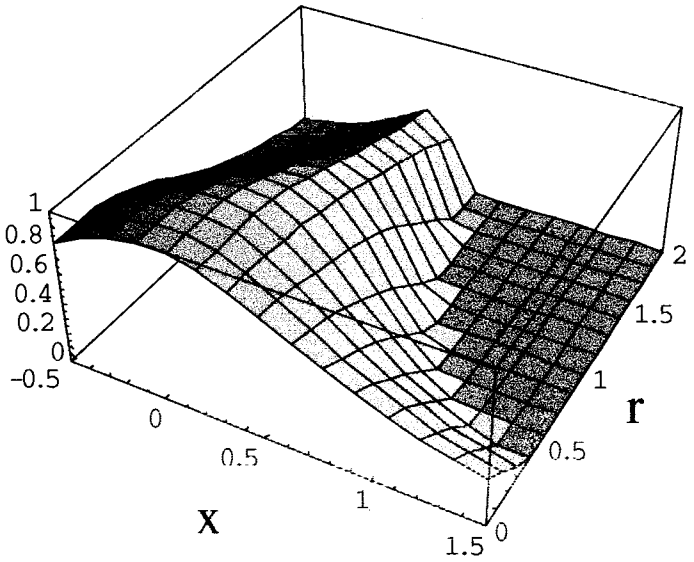
Cooling rates cont'd

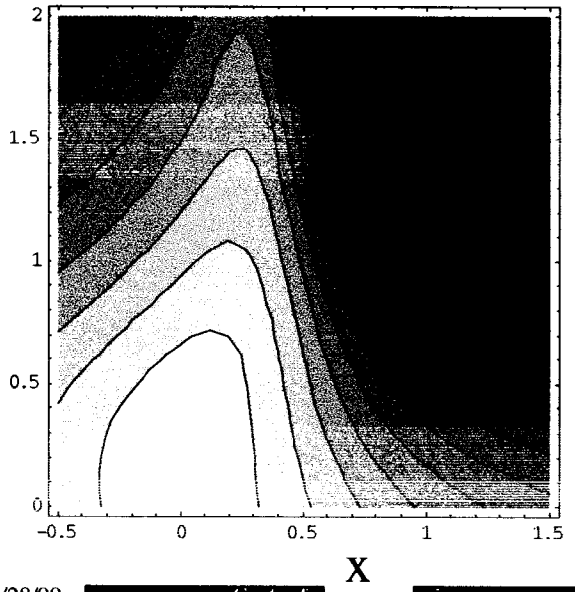
$f_e(x,r)$





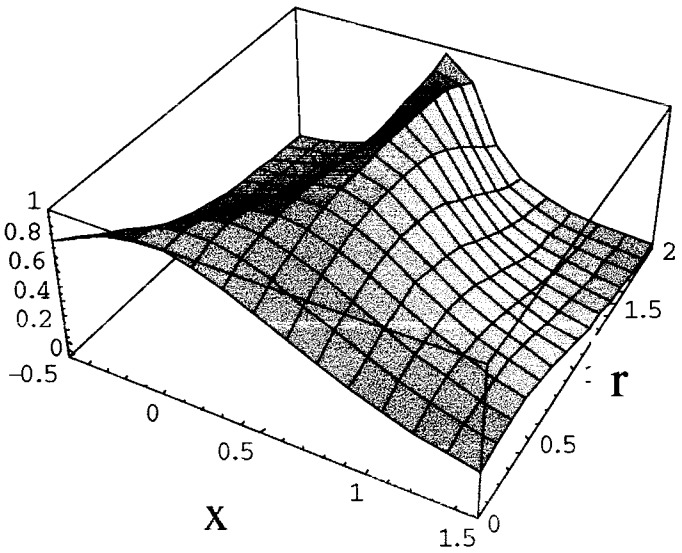
$f_x(x,r)$

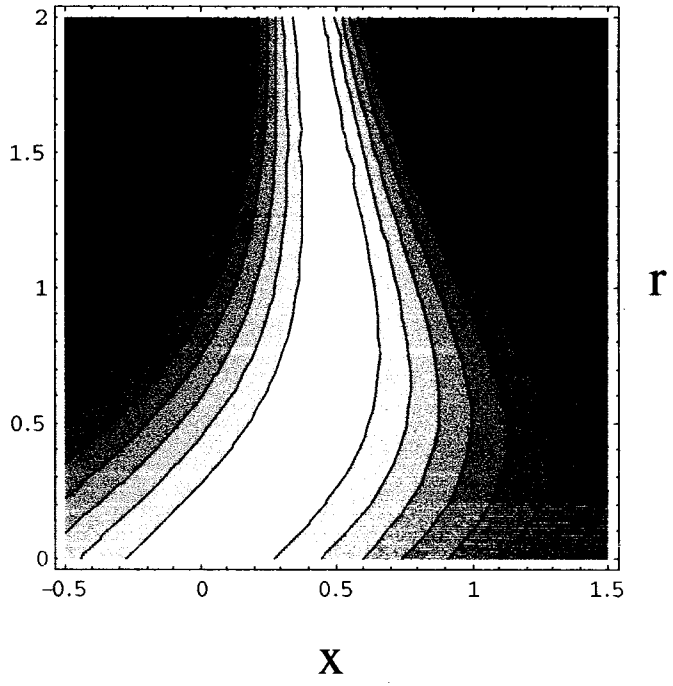




Cooling rates cont'd

$$f_e(x,r) + f_x(x,r)$$





Amplification factor

Amplitude gain of the amplifier:

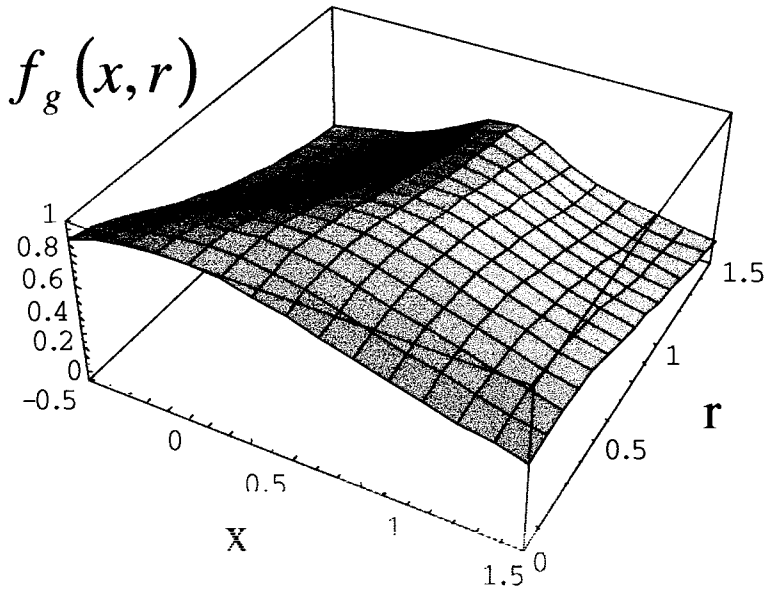
central wavelength of the amplifier ~ 800nm

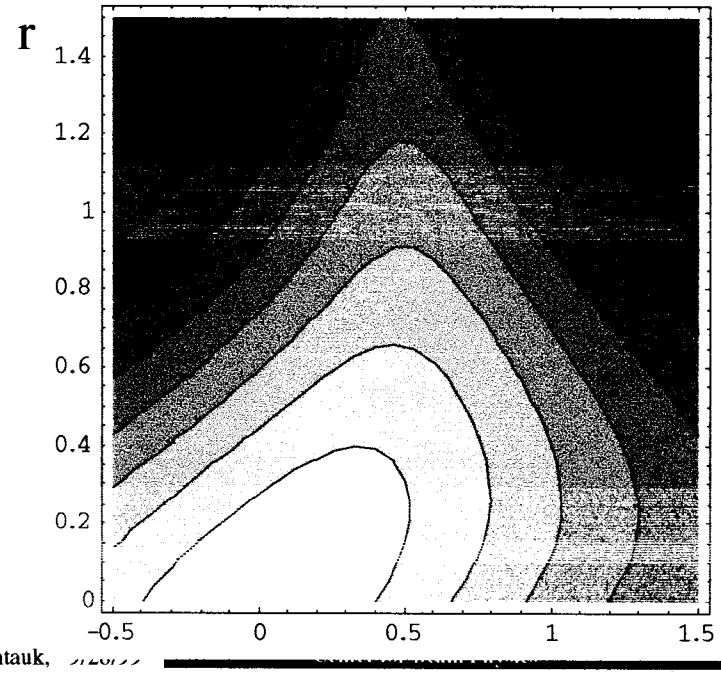
energy spread of muons (relative)

$$g \approx \frac{1.}{8\pi\sqrt{e}\,(N_s + n)}\left(\frac{1+K^2/2}{K^2/2}\right)\frac{\gamma\lambda\sigma_e}{r_\mu} f_g(x,r) \qquad 0 < f_g(x,r) < 1$$

muon classical radius

undulator parameter ≈ 2





Undulator

Matching conditions for the undulator:

undulator period

$$\lambda \approx \frac{\lambda_{\text{und}}}{2\gamma^2}\left(1 + K^2/2\right)$$

Undulator parameter:

peak magnetic field

$$K = \frac{eB\lambda_{\text{und}}}{mc^2 2\pi}$$

At E=100 GeV and K=2: $B \approx 10\text{T}$, $\lambda_{\text{und}} \approx 0.5\text{m}$

Amplification factor cont'd

Amount of the energy in the amplified light consumed in one damping time:

beam energy

normalized longitudinal emittance

$$\delta E \approx \frac{1}{\pi} E_b \frac{\varepsilon_{\parallel} \sigma_e}{M \; r_\mu}$$

relative bandwidth of the Ti:Sapphire crystal

Gain narrowed bandwidth of the amplifier:

$$\frac{1}{M} = \frac{\Delta f}{f} \approx \frac{0.4}{0.5\,\mathrm{Ln}(g)}$$

Sample length defined by the bandwidth: $M\,\lambda/3$

Longitudinal density of muons: $3N_s / M\,\lambda$

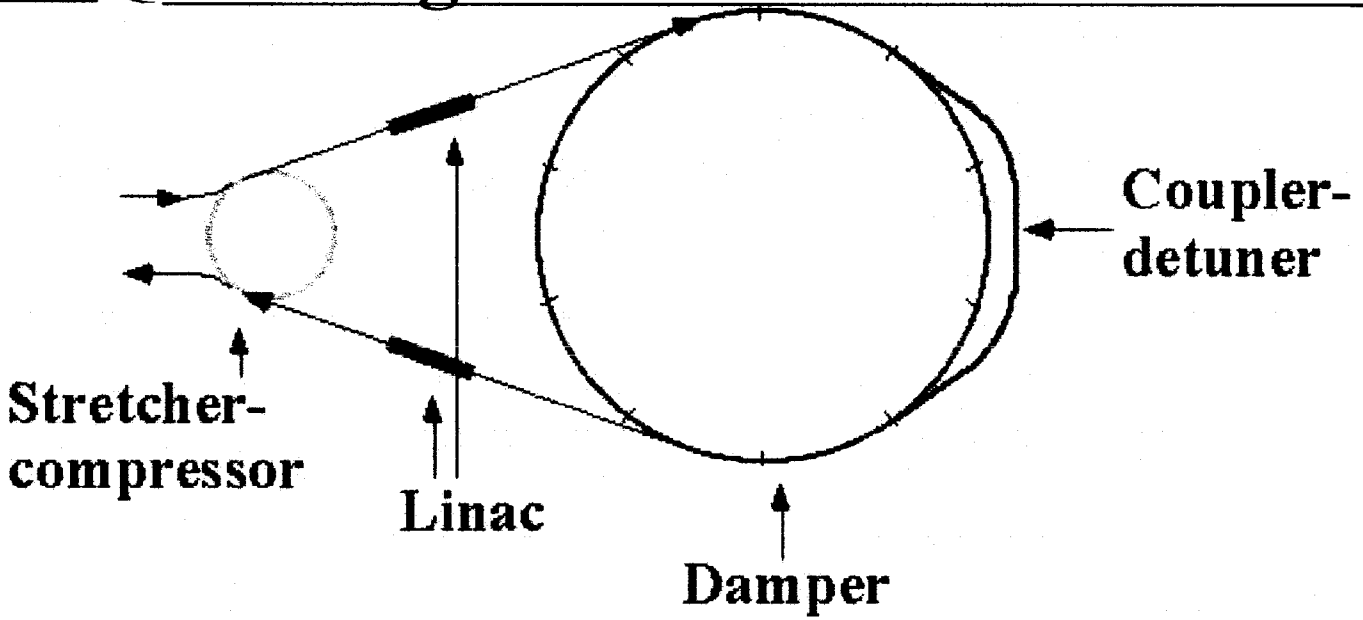
A schematic of the amplifier

A. A. Zholents, M. Zolotorev, "*An amplifier for optical stochastic cooling*", Proc. Part. Acc. Conf. 1997.

Amplifier

Bandwidth, FWHM	4 0 THz	1-Pumping laser
Total gain	~90 dB	2-Ti:sapphire crystal
		3-Lenses

A scheme for optical stochastic cooling of muons



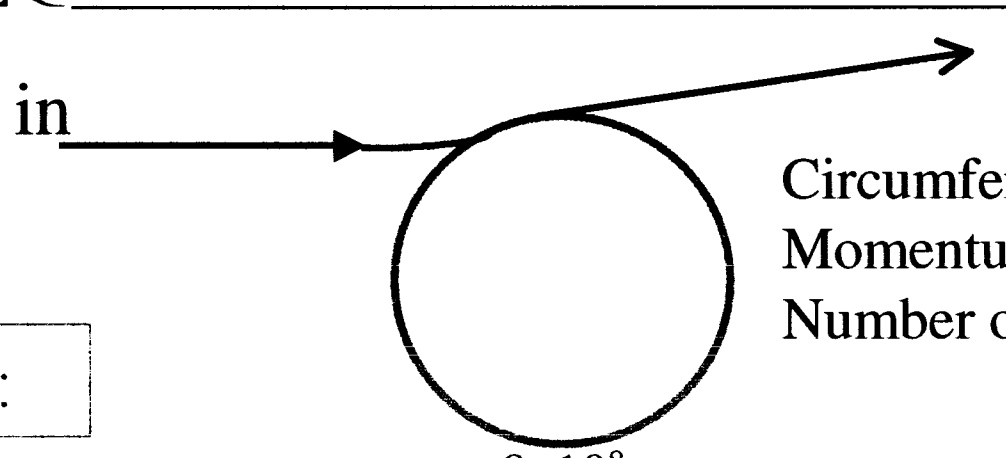
•Stretcher-compressor: a 290 m ring with a momentum compaction >0.5

•Linac: an induction linac, E=50 MeV.

•Damper: 3 rings for 3 cooling stages with 10 cooling systems in each ring; optical amplifiers are shared between rings.

•Coupler-detuner: a bypass in each ring for x-y emittance exchange and a tune shift from interger resonance.

Stretcher-compressor



Circumference = 290m
Momentum compaction = 0.52
Number of turns =950

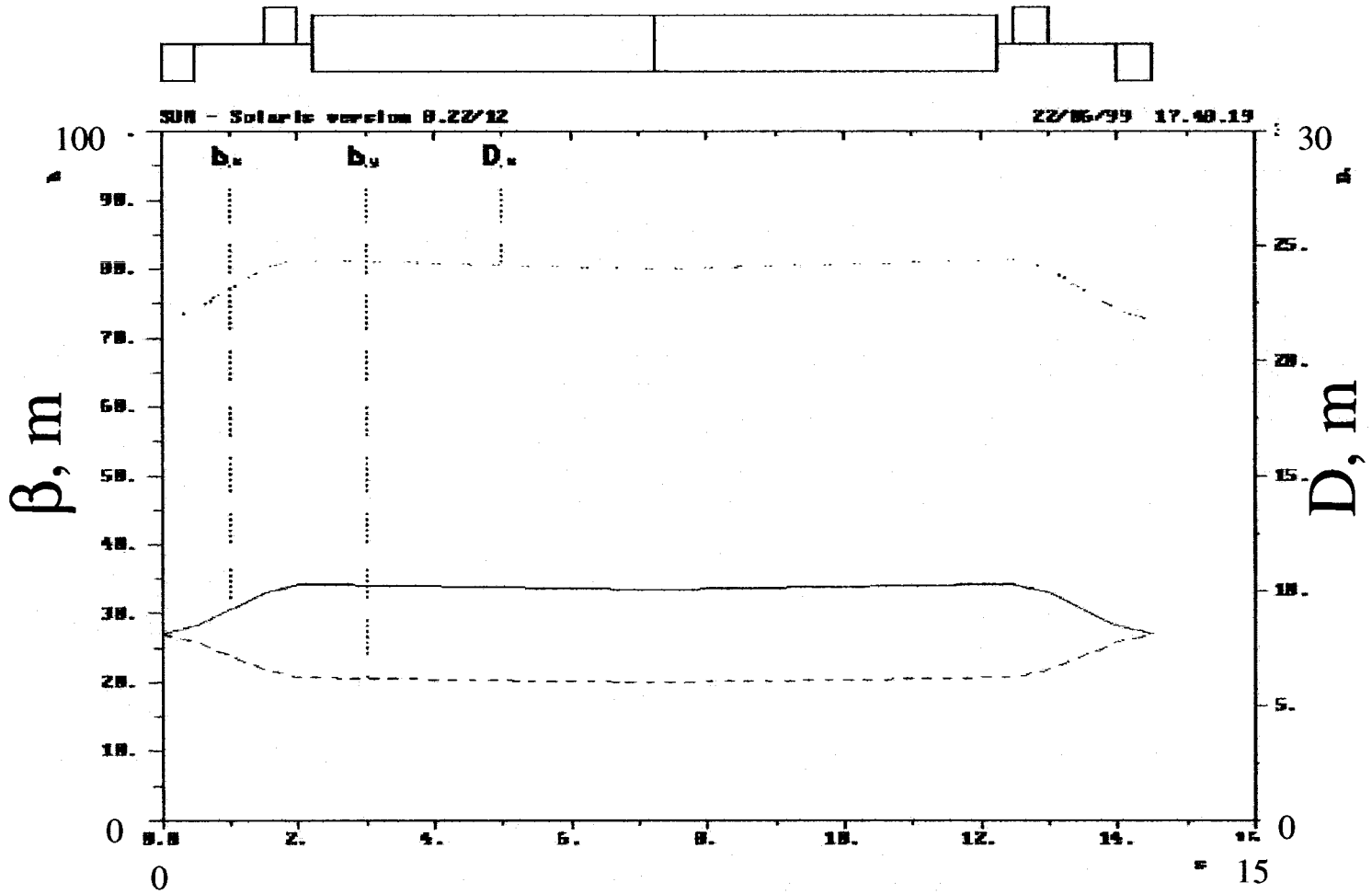
in:

Number of muons = 8×10^8
Normalized transverse emittance = 5×10^{-3}cm
Normalized longitudinal emittance = 4 cm
Uncorrelated energy spread, σ = 21MeV
Bunchlength, σ = 20 cm

out:

Number of muons = 5×10^8
Normalized transverse emittance = 5×10^{-3}cm
Normalized longitudinal emittance = 4 cm
Uncorrelated energy spread, σ = 0.14MeV
Bunchlength, σ = 3070 cm
Correlated head-tail energy spread = 21 MeV

Stretcher-compressor cont'd



Standard cell: 1/20 part of the ring

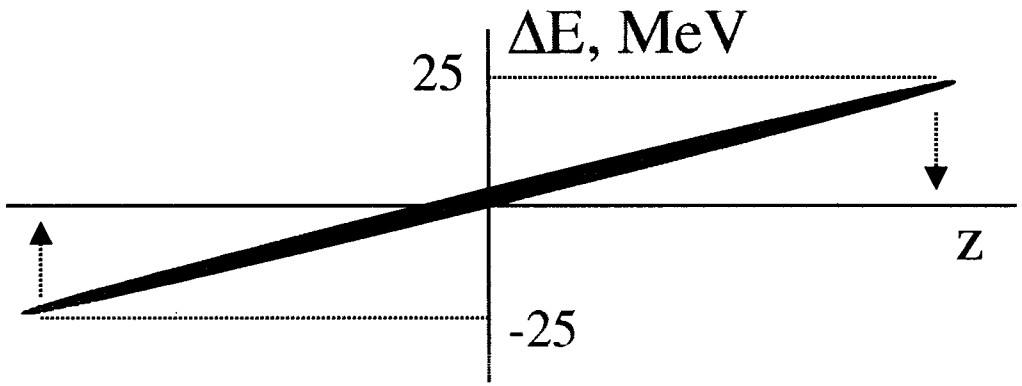
Linac

Removes the correlated energy spread

Induction linac:

pulse duration, $2.5\sigma = 250$ ns

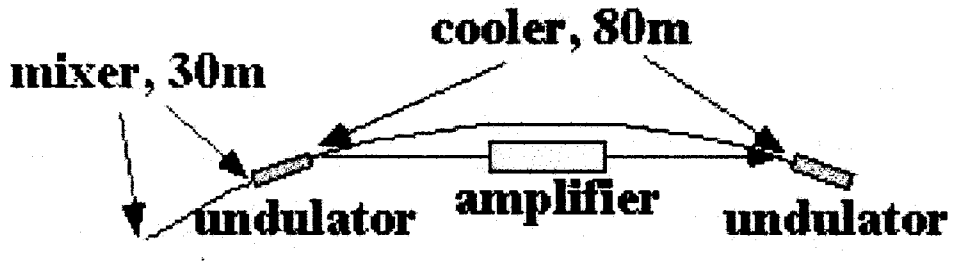
energy gain, $2.5\sigma = \pm 25$ MeV



Damper

Damping is performed in 3 stages with 3 rings of 1100m in circumference. Each ring has 10 cooling sections and a coupler-detuner bypass. Muons stay in each ring for about 8 damping times (4 for x-E cooling and 4 for y-E cooling). Damping time is 33 turn. Every ~ 33 turns muons are turned into the bypass for x-y rotation and detuning from integer resonance and returned to the ring. Three ring share the amplifiers.

Cooling section:



At the end the 6D emittance is reduced in ~2×10^{11} times - limited by intrabeam scattering.

Damper cont'd

Requirements for the amplifiers at the beginning of cooling:

amplitude gain $= 2.5 \times 10^4$
bandwidth $= 13$ THz
light pulse energy per single pass $= 5$ mJ

Total light energy provided by each amplifier over the entire cooling time is 300 mJ;

Average amplifier power at 50 Hz is 15 W.

Damper cont'd

Fast damping is achieved by choosing N_s=25.

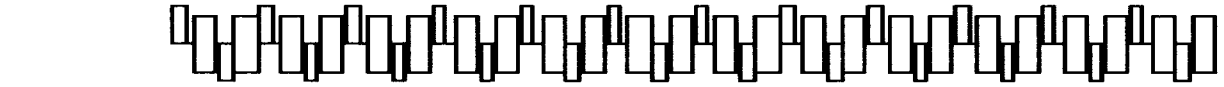
This defines longitudinal density = 6×10^5 muon/cm and a total number of muons* = 5×10^8

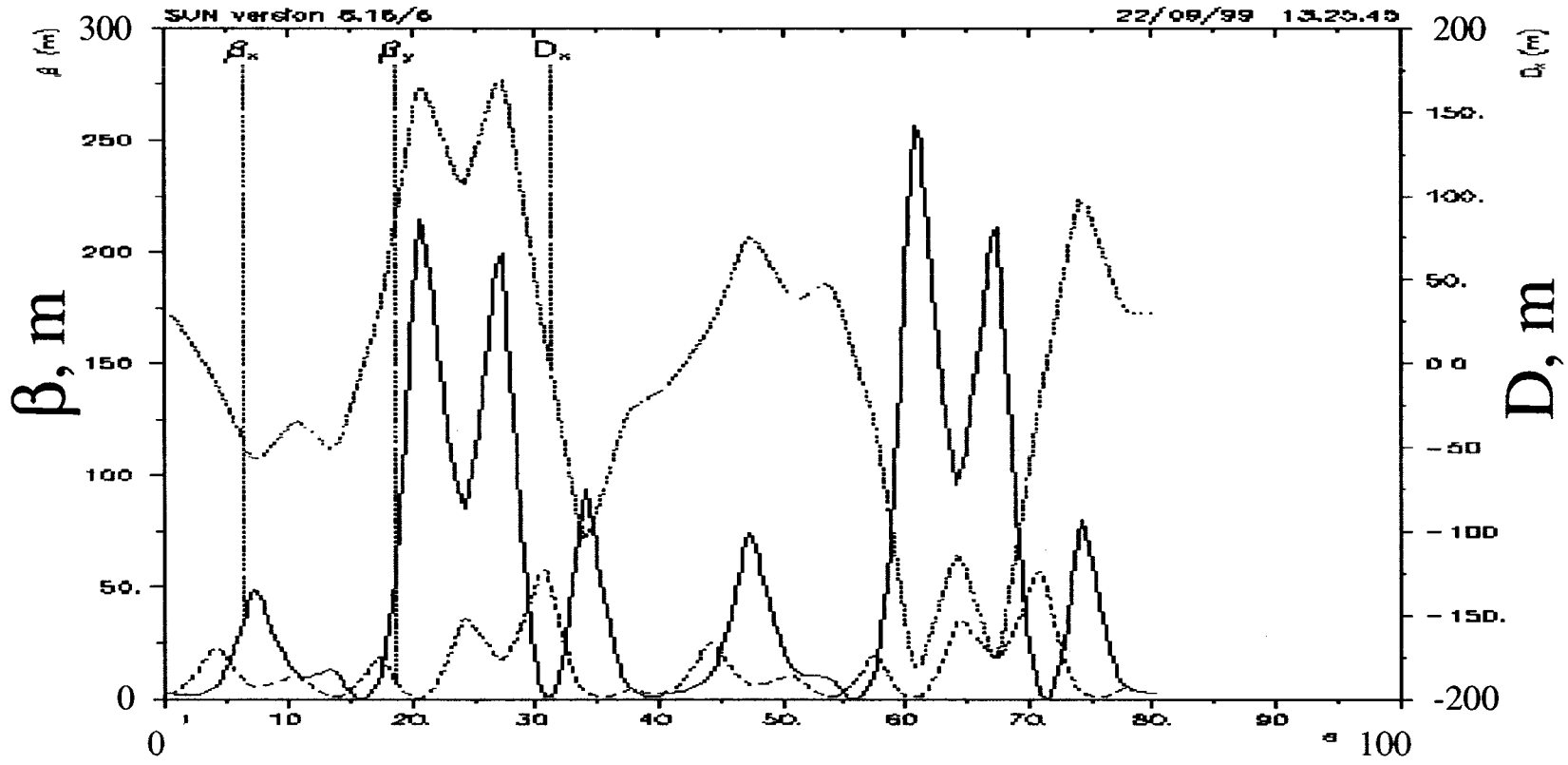
*)beam energy spread is taken 1.3×10^{-6}

300m dispersion function in the undulator provides equal synchrotron and betatron sizes at a 1.3×10^{-6} energy spread. (Beta-function = 2.5 m).

Damper cont'd

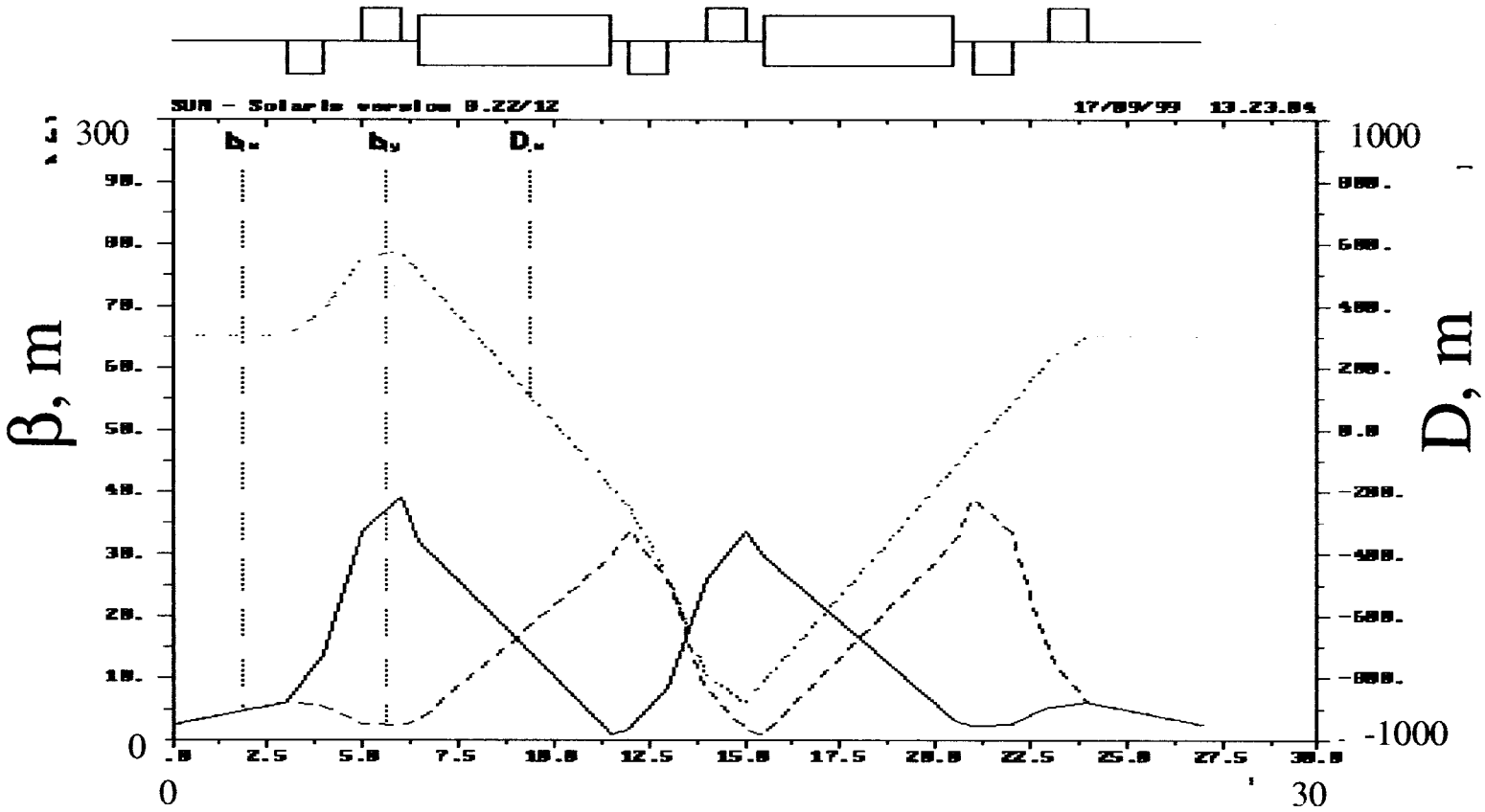
Cooler (I)





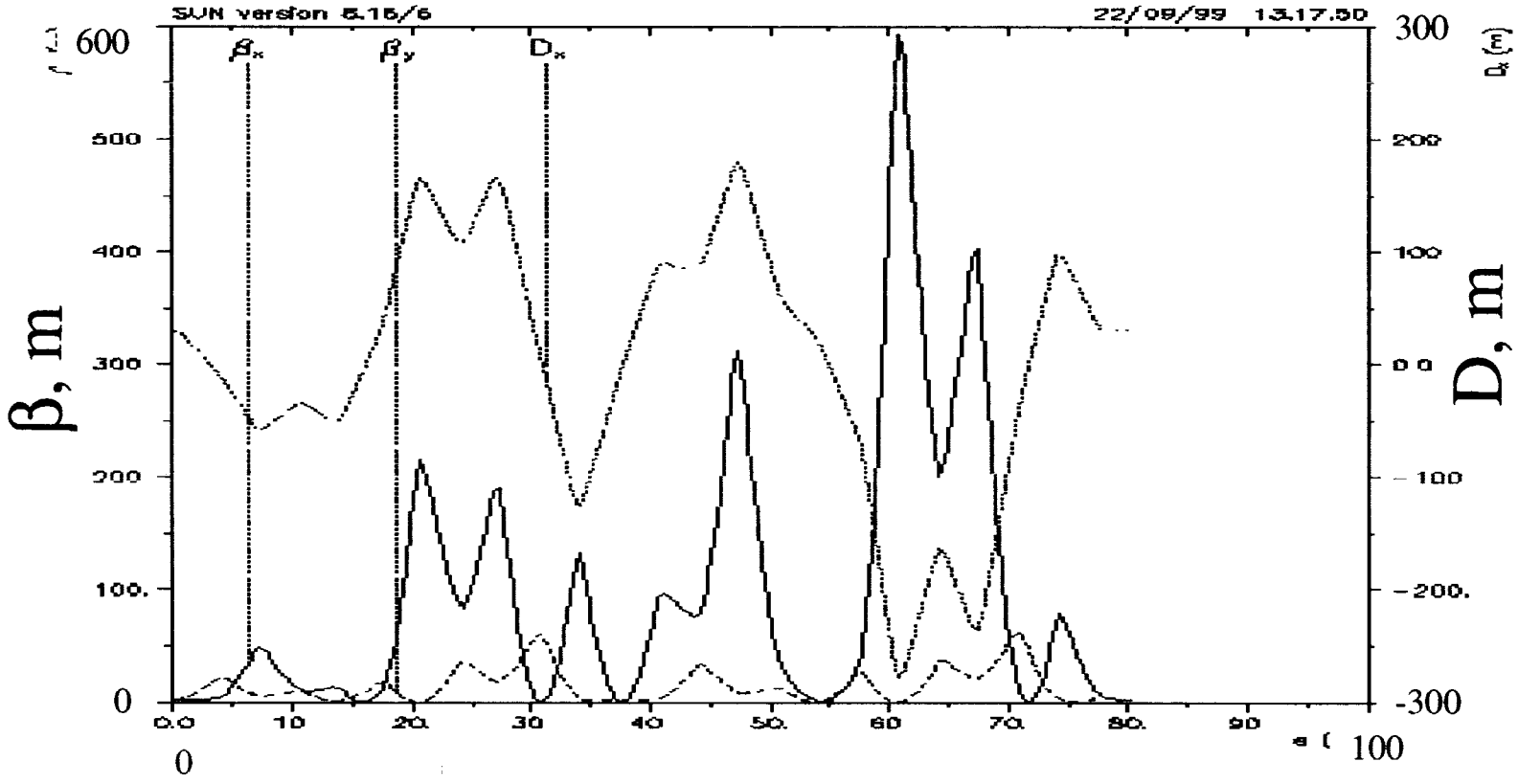
Damper cont'd

Mixer



Damper cont'd

Cooler (II)



Variation of the quadrupoles < 15%

Damper cont'd

The output beam parameters:

Number of muons	$= 1.5 \times 10^{8}$
Normalized transverse emittance	$= 3 \times 10^{-8}$ cm
Normalized longitudinal emittance	= 0.4 cm
Uncorrelated energy spread, σ	$= 1.3 \times 10^{-7}$
Bunchlength, σ	= 3070 cm

Luminosity etc.

	This talk	CDR
Energy	= 2 TeV	2 TeV
Beta-function at the IP	= 0.03 cm	0.3cm
Transverse beam size at the IP	= 2×10^{-7} cm	8×10^{-3}cm
Beam divergence at the IP	= 7×10^{-6}	3×10^{-4}
Number of particles (10 bunches combined)	= 10^{9}	2×10^{12}
Energy spread	= 8×10^{-3}	7×10^{-4}
Beam-beam parameter, ξ	= 5.4	0.05
Luminosity	= 10^{35} cm^{-2}s^{-1}	10^{35}cm^{-2}s^{-1}
Particle density at the IP	= 6×10^{22} cm^{-3}	5×10^{18}cm^{-3}

Emittance reached in 1 muon decay time.

$B = 10\ T$; $\sqrt{} = 25 \cdot E\ (TeV)$

$E\ (TeV)$	Normalized emittance, cm
2	$7.5 \cdot 10^{-8}$
10	$1.9 \cdot 10^{-6}$
30	$1.7 \cdot 10^{-5}$
60	$6.7 \cdot 10^{-5}$
100	$1.9 \cdot 10^{-4}$

$\epsilon_1 \approx 1.8 \cdot 10^{-8}\ E^2\ (TeV)$

A. Zholents isolated transparency

Thursday, Session II