

**Dejan Trbojevic
September 30, 1999
Montauk, Long Island
Muon Collider
Workshop**

FFAG Lattice Presented as a Trigonometrical Problem

- **Introduction**
- **Why the K.R. Symon FFAG proposal finally got the attention after 44 years (1954)??**
- **Basic Parameters – trigonometry**
- **Two Examples**
- **Possible Improvements**

Where are the advantages?

- **The FFAG proposal by K.R. Symon**
 - Mura-KRS-6 "The FFAG Synchrotron MARK I (November 12, 1954) : too LARGE CIRCUMFERENCE FACTOR and too LARGE REQUIRED APERTURE.
- **Advantages:**
 - **FIXED MAGNETIC FIELD**
 - **Large range of particle's energy**
 - **RF simple**
 - **POWER SUPPLY simple**
 - **no transition energy**
- **APPLY FFAG to:**
 - **accelerate particles quickly. Muons are transferred to the Collider or Neutrino storage ring, or in the case of heavy ions or protons in the medical therapy treatment use them at the fixed field for a range of energies.**

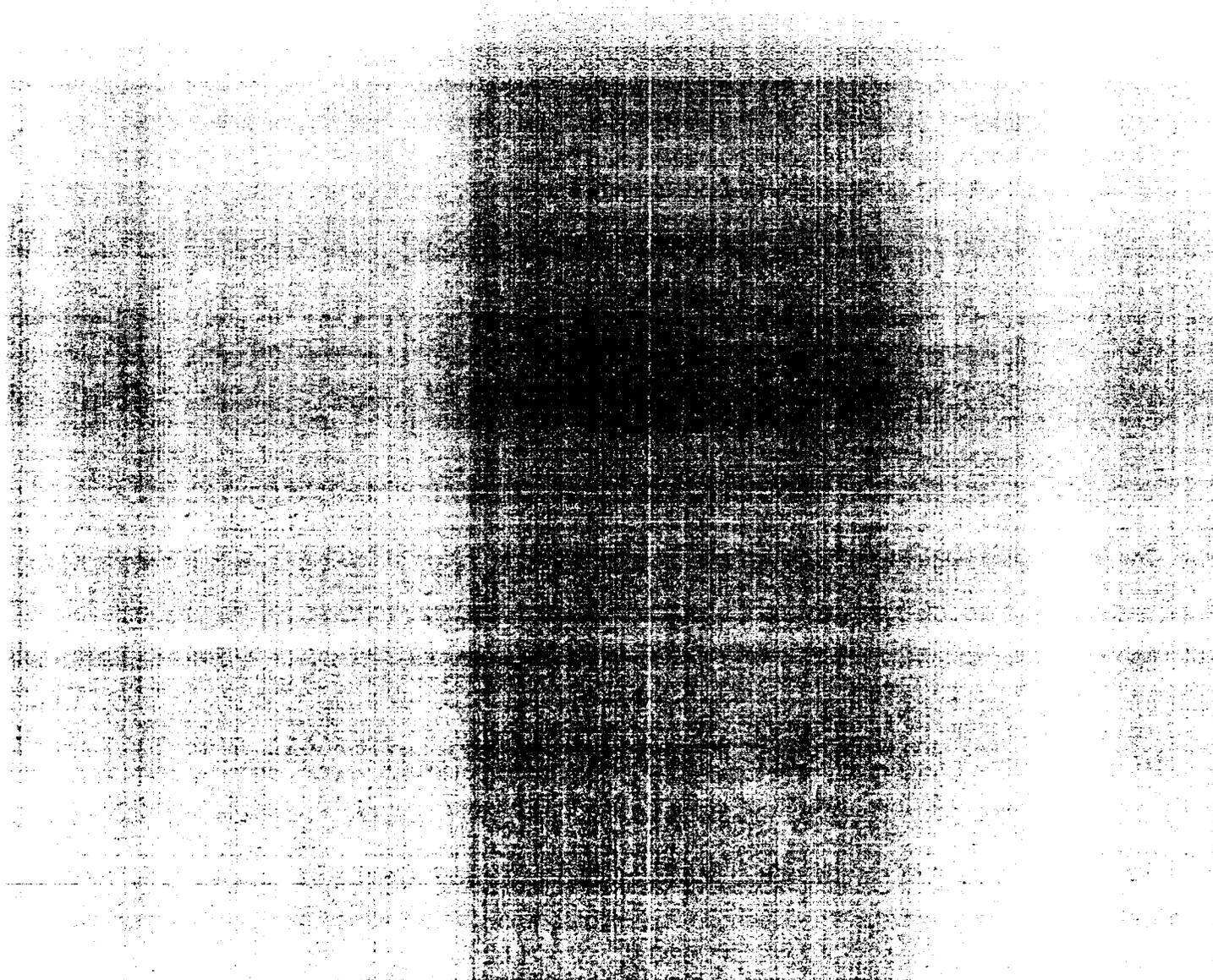
What is the goal?

- **Construct a lattice where the dispersion will oscillate between positive and negative values but not exceeding 7 cm without opposite bending magnets.**
- **Keep tunes away from the integers and half integers during the acceleration – which should happen in the minimum number of turns.**
- **Keep the chromaticities small which in turn will have the tune variations small due to this enormous dp/p range.**
- **The beam size is dominated most of the time (away from p_0) by the momentum offset.**

What are the basic parameters?

- **Required Range of Energies (or dp/p)**
 - the “central” energy or momentum p_0 is in two examples presented later set to 10 GeV. The acceleration would be if possible from 5 GeV up to 15 GeV.
 - Aperture limitation is defined by the maximum value of the DISPERSION function: $\Delta x < D_x * dp/p < +/- 35 \text{ mm}$
 - if the $0.5 < dp/p < 1.5$ then:
 - $D_x < 70 \text{ mm}$
- **Why is the Minimum Emittance Lattice for the electronic Storage Rings Relevant?**
 - The normalized dispersion amplitude corresponds to the $\langle H \rangle^{1/2} !!!$

Basic Parameters - trigonometry



Low Emittance Lattices for Electron Storage Rings Revisited *

D. Trbojevic and E. Courant
Brookhaven National Laboratory
Upton, New York 11973, USA

Abstract

Conditions for the lowest possible emittance of the lattice for electron storage rings are obtained by a simplified analytical approach. Examples of electron storage lattices with minimum emittances are presented. A simple graphical presentation in the normalized dispersion space (Floquet's transformation) is used to illustrate the conditions and results.

1 INTRODUCTION

Electron storage rings are often used to produce high brilliance synchrotron light sources. The lower the transverse electron beam emittance the higher the brightness of the emitted light. The light is emitted in the direction of motion of the relativistic electron beam. The synchrotron light fans out tangentially to the dipole arc. At the point of emission an rms solid angle of the emitted light is inversely proportional to the Lorentz factor ($1/\gamma$) - beam energy. The transverse horizontal emittance is:

$$\epsilon_x = \frac{C_q \gamma^2}{J_x \rho} \langle H \rangle_{\text{dipole}}, \quad (1)$$

$$H = \gamma_x D_x^2 + 2\alpha_x D_x \dot{D}_x + \beta_x \dot{D}_x^2, \quad (2)$$

where $C_q = 3.84 \cdot 10^{-13}$ m, the horizontal partition factor $J_x \approx 1 + \frac{1}{\gamma} \frac{R}{\rho} \approx 1$, $\langle H \rangle = 1/L_d \int_0^{L_d} H ds$, α , β , and γ are the Courant-Snyder parameters, while D_x and \dot{D}_x are the horizontal dispersion function and its slope. The minimum transverse emittance is achieved by minimizing the average integral of the function H , over the length of the dipole. To illustrate this requirement it is useful to use Floquet's coordinate transformation [1] for the dispersion function as:

$$\chi = \frac{D_x}{\sqrt{\beta}} \quad \text{and} \quad \xi = \dot{D}_x \sqrt{\beta_x} + D_x \frac{\alpha_x}{\sqrt{\beta_x}}, \quad (3)$$

then the function H is presented as:

$$H = \chi^2 + \xi^2, \quad (4)$$

$$\chi = \sqrt{H} \sin \phi, \quad \xi = \sqrt{H} \cos \phi, \quad (5)$$

where ϕ is the betatron phase advance. In the normalized dispersion coordinate system the vertical axis is chosen to be χ . The dispersion function satisfies the second order differential equation of motion:

$$\ddot{D}(s) + K_x(s)D(s) = \frac{1}{\rho}, \quad (6)$$

where $K_x(s) = \frac{1}{\rho} - \frac{1}{B\rho} \frac{\partial B_x}{\partial x}$. The presence of a dipole in the lattice is manifested in the normalized χ and ξ dispersion space as a change along the horizontal axis of $\Delta\xi = \Delta\dot{D}_x \sqrt{\beta}$, where $\Delta\dot{D}_x = \theta$ (change of the slope of the dispersion function is equal to the bending angle of the dipole). The amplitude of the function H will depend on the dipole position in the lattice. Outside the dipoles ($\frac{1}{\rho} = 0$) a solution of the homogeneous differential equation 6 represents a circle as equation 4 shows. The Chasman-Green lattice where the dipoles are approximated by the thin lens approximation, is presented in the normalized dispersion diagram in fig.1. Two dipoles lie on the horizontal axis. A part of the lattice with the zero dispersion is located at the origin of the χ and ξ normalized dispersion diagram. The end of the second dipole in fig. 1 is connected to the beginning of the first dipole by the semi-circle shown in equation 4. The minimum possible radius of the semi-circle corresponds to the smallest possible emittance. The size of the radius is determined by the dipole. Fig.1 shows both the idealized case where two vectors represent dipoles in the lowest possible Chasman-Green lattice and a single vector in the middle of the ξ axis representing the lowest possible emittance lattice. The real lattices are presented as well at the same fig.1 where dipoles have curved lines (\sqrt{H}) due to a change of the betatron phase throughout them.

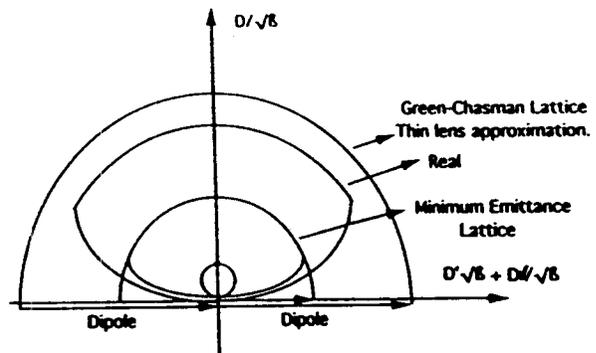


Fig. 1 Thin element approximation of the Chasman-Green minimum emittance lattice and of the lowest emittance lattice - where dipoles are horiz. vectors.

*Work performed under the auspices of the U.S. Department of Energy.

The size of the radius is determined by the dipole. Fig.1 shows both the idealized case where two vectors represent dipoles in the lowest possible Chasman-Green lattice and single vector in the middle of the ξ axis representing the lowest possible emittance lattice. The real lattices are presented as well at the same fig.1 where dipoles have curved lines (\sqrt{H}) due to a change of the betatron phase throughout them.

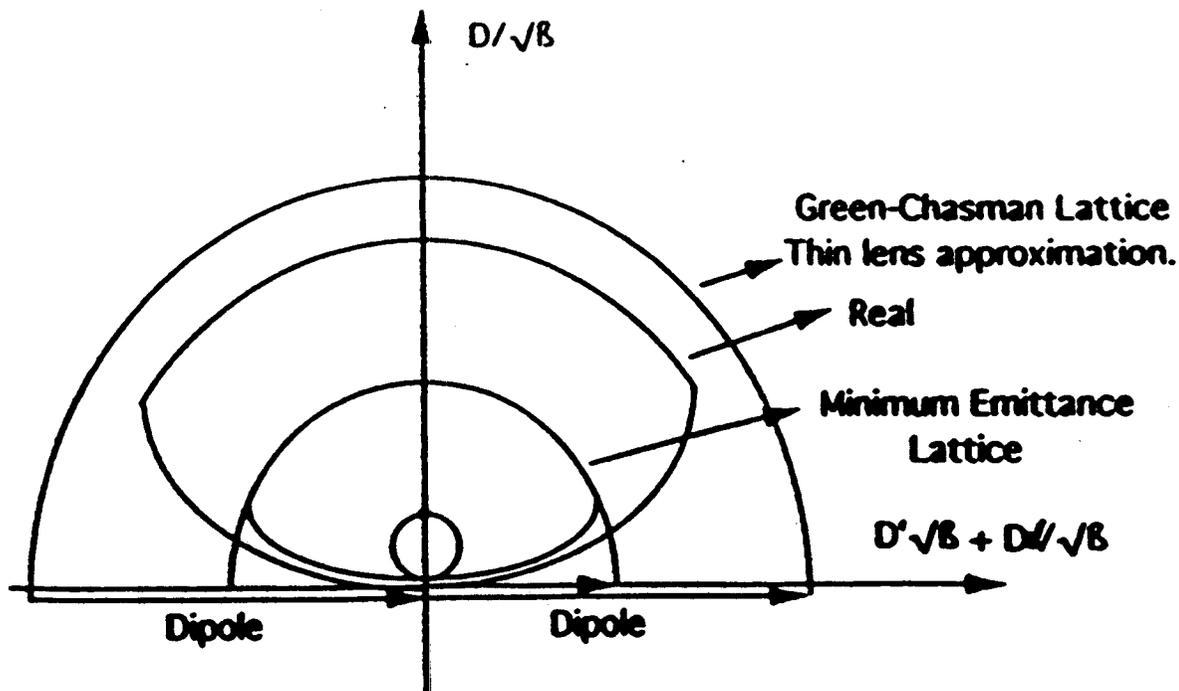
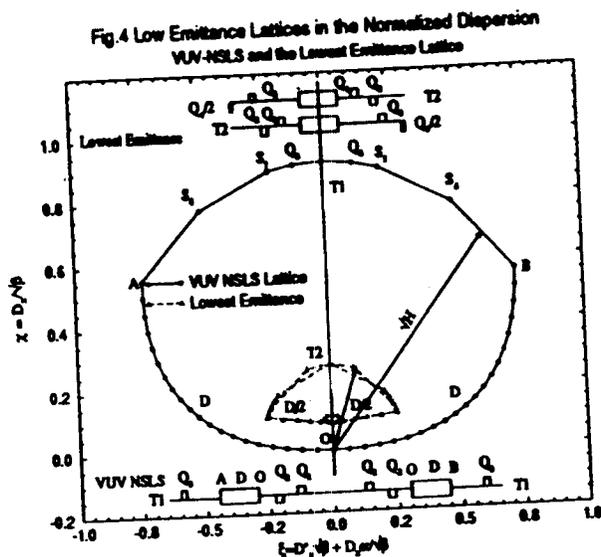


Fig. 1 Thin element approximation of the Chasman-Green minimum emittance lattice and of the lowest emittance lattice - where dipoles are horiz. vectors.

5 EXAMPLES OF THE MINIMUM EMITTANCE LATTICES

An example of a minimum emittance lattice is shown in fig.4. There are two basic modules constructed by using the same initial conditions.



The electron energy was chosen to correspond to the energy of the UV ring of the National Synchrotron Light Source (NSLS) at Brookhaven National Laboratory. To be able to compare this example with the other similar lattices the dipole length and number was chosen to be the same as in the UV NSLS ring ($L_d=1.5$ m and $N_d=8$). The basic module consists of the dipole surrounded by two triplets where the central quadrupole is the defocusing quadrupole. In the second module which starts at the middle of the dipole there are two defocussing quadrupole separated by a drift. The chromaticity of this example is higher than the one of the existing UV ring in NSLS. But the sextupole correctors have a very reasonable strength of $KSF=4.38$ m^{-2} and $KSD=-3.77$ m^{-2} . The quadrupole gradients were chosen not to exceed 15 T/m. The transverse emittance of the present UV ring at NSLS was reduced to the minimum possible value of $8.68 \cdot 10^{-7}$ mrad, compared to the existing emittance of $1.3 \cdot 10^{-6}$ mrad. By the same procedure few other examples have been designed with the parameters of the X-ray ring at NSLS and of the X-ray ring of Argonne National Laboratory (ANL). The emittance of the X-ray ring at the NSLS was reduced to the minimum possible emittance by a factor of 8.8, while an example of the X-ray ring at the ANL was reduced by a factor of 11.27. The examples to be compared with the X-ray NSLS and to the ANL X-ray had the same lattice elements, most importantly the same dipoles. In the X-ray ring example to be compared to the NSLS ring an additional quadrupole was introduced.

6 CONCLUSION

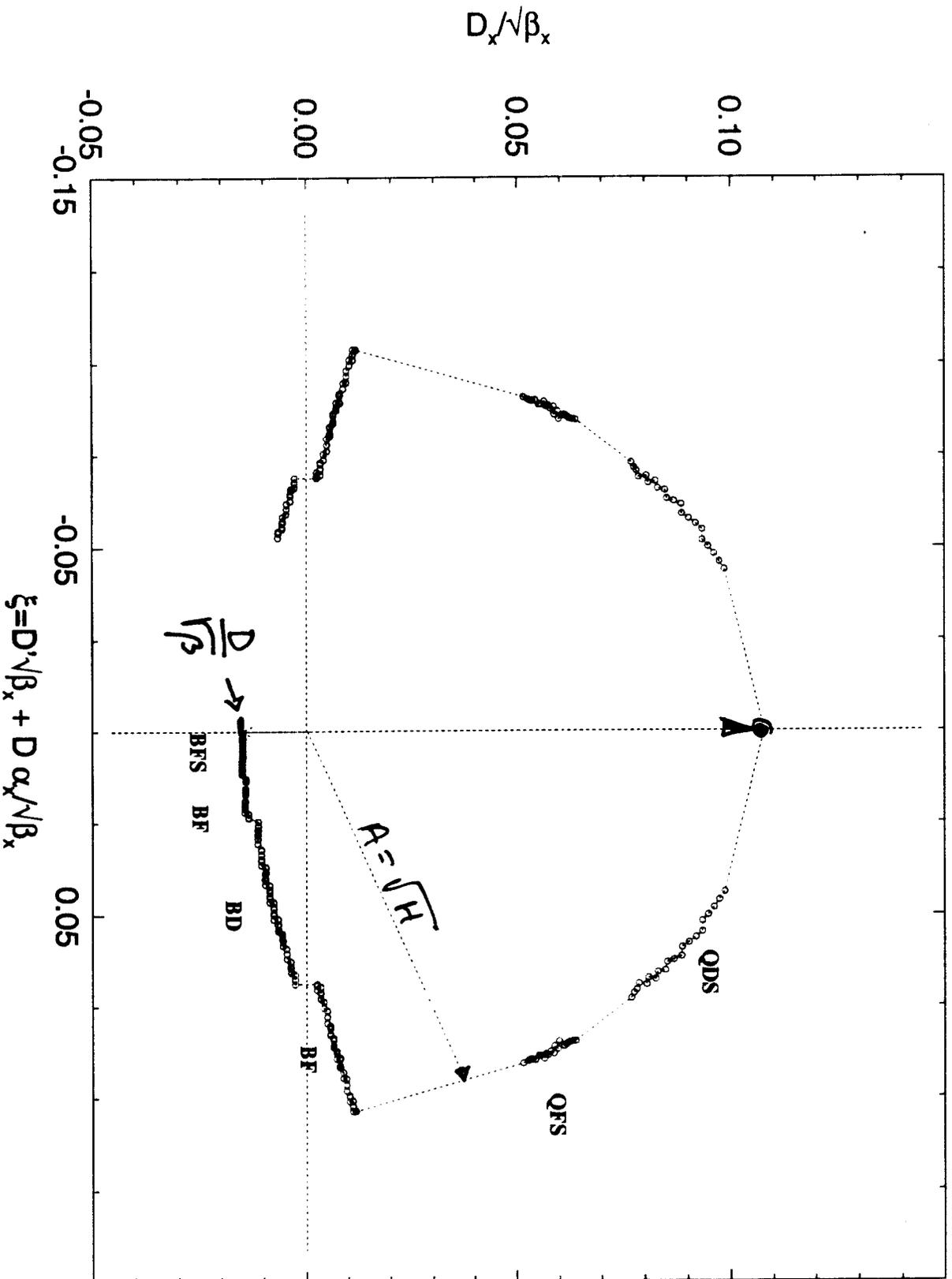
We repeated the analysis [4] for obtaining the conditions for the minimum emittance lattice of the electron storage rings. Conditions for the minimum emittance show that dispersion function in the middle of the dipoles should have a small value but not zero $D_0 = \frac{\beta L_d}{24}$ and that the horizontal betatron function should have a minimum at the middle of the dipole with a value of $\beta_0 = \frac{L_d}{2\sqrt{15}}$. These initial conditions were applied and few examples of the minimum possible emittance were designed and presented in detail. We used Floquet's coordinate transformation to illustrate the design procedure. It is possible to reduce the sextupole second order tune shift with amplitude by constructing lattice modules where betatron phase differences between sextupoles is multiple of 90° . We show that the transverse emittance of the existing electron storage rings can be easily reduced to the minimum possible value by either different settings of the existing quadrupoles or, in some examples, by introducing an additional quadrupole or by repositioning the existing quadrupoles. We hope that this work will help in the future electron storage ring design as well as existing rings.

7 REFERENCES

- [1] E. D. Courant and H.S. Snyder, *Ann. Phys.* 3, 1 (1958).
- [2] L.C. Teng, TM-1269, Fermilab Internal Report, June 1984, p.1-9.
- [3] L.C. Teng, TM-1269-A, Fermilab Internal Report, December 1984 1984, p.1-6.
- [4] S.Y. Lee and L.C. Teng, Conference Record of the 1991 IEEE Particle Accelerator Conference, May 6-9, 1991, San Francisco, California, p. 2679-2681.
- [5] D. Trbojevic, D. Finley, R. Gerig, and S. Holmes, in Proceedings of Second European Particle Accelerator Conference, Nice, 1990, edited by P. Martin and P. Mandrillon, pp. 1536-1538.
- [6] Tom L. Collins, "Distortion Functions", Fermi National Laboratory, Fermilab-84/114, Internal Report, October 23, 1984, 1-45 pp.
- [7] S.Y. Lee, K.Y. Ng and D. Trbojevic, Fermi National Accelerator Laboratory, Fermilab-FN-595, October 1992, 1-65 pp.
- [8] S.Y. Lee, K.Y. Ng, and D. Trbojevic, *Physical Review E*, Volume 48, No 4, October 1993, pp. 3040-3048.

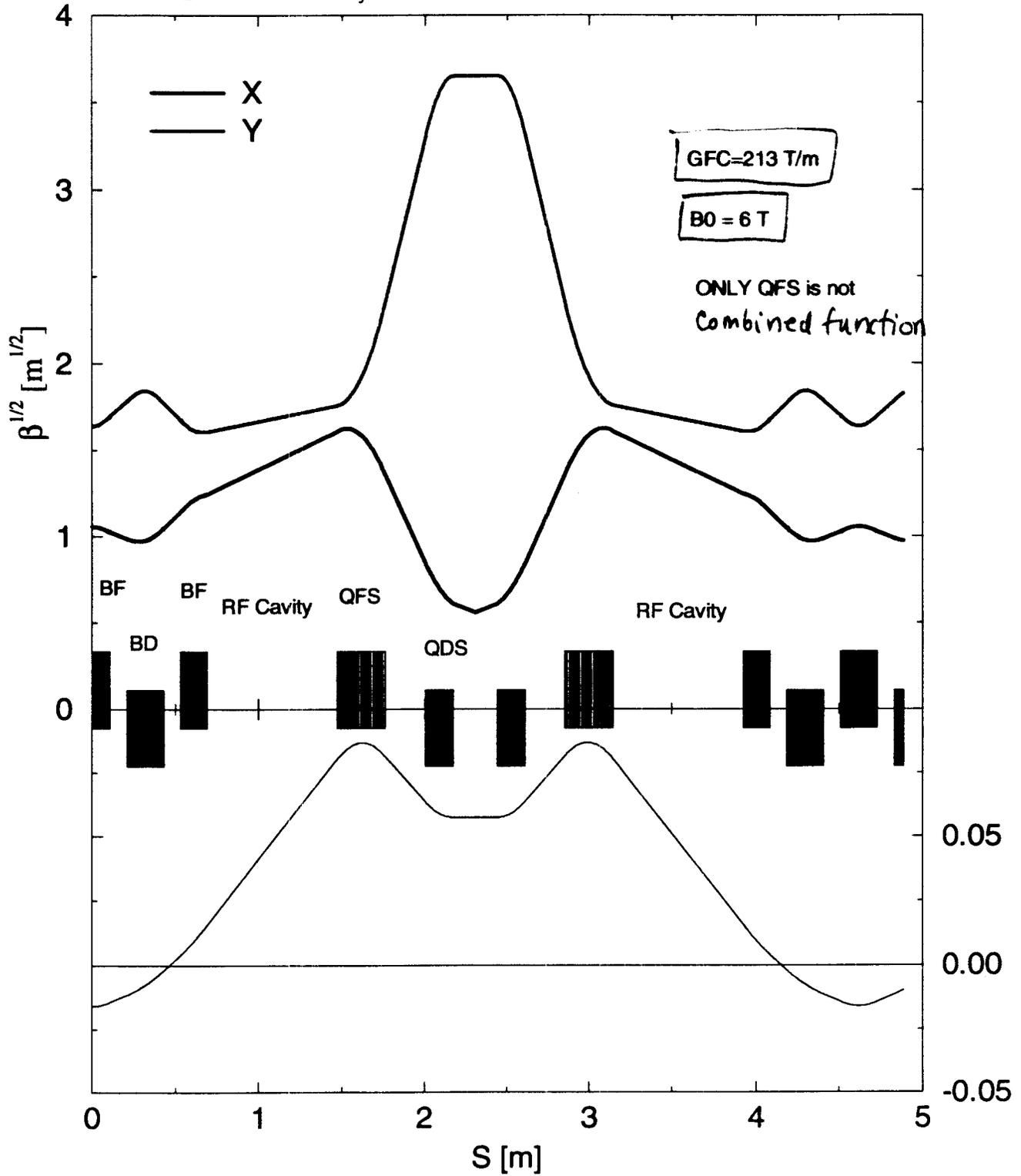
Normalized Dispersion in the FFAG Lattice

No Reverse Bending Dipoles



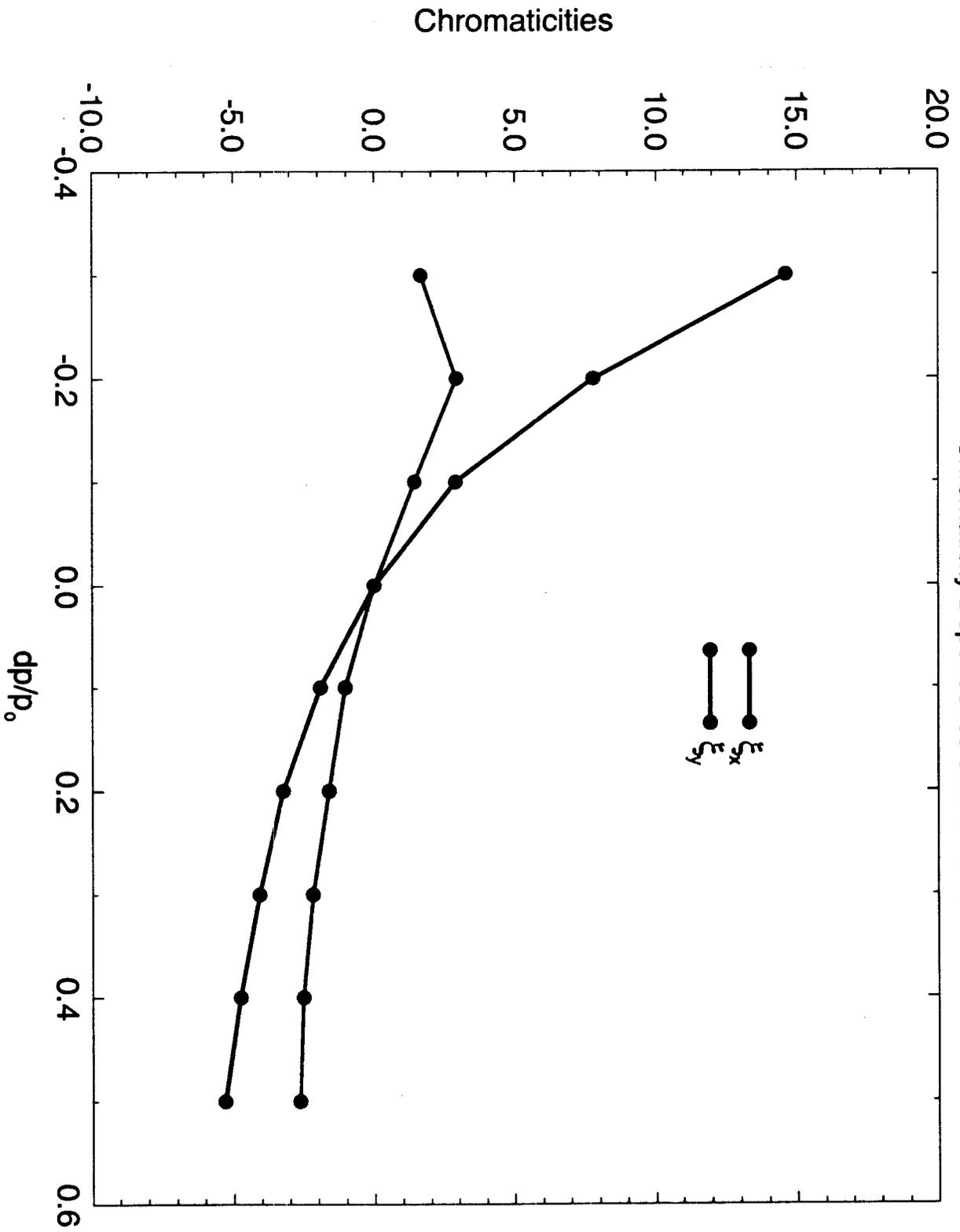
FFAG Without Opposite Bends - Length=157 m

$v_x = 22.8332$ $v_y = 6.98121$ FILE = c11_plot.mad

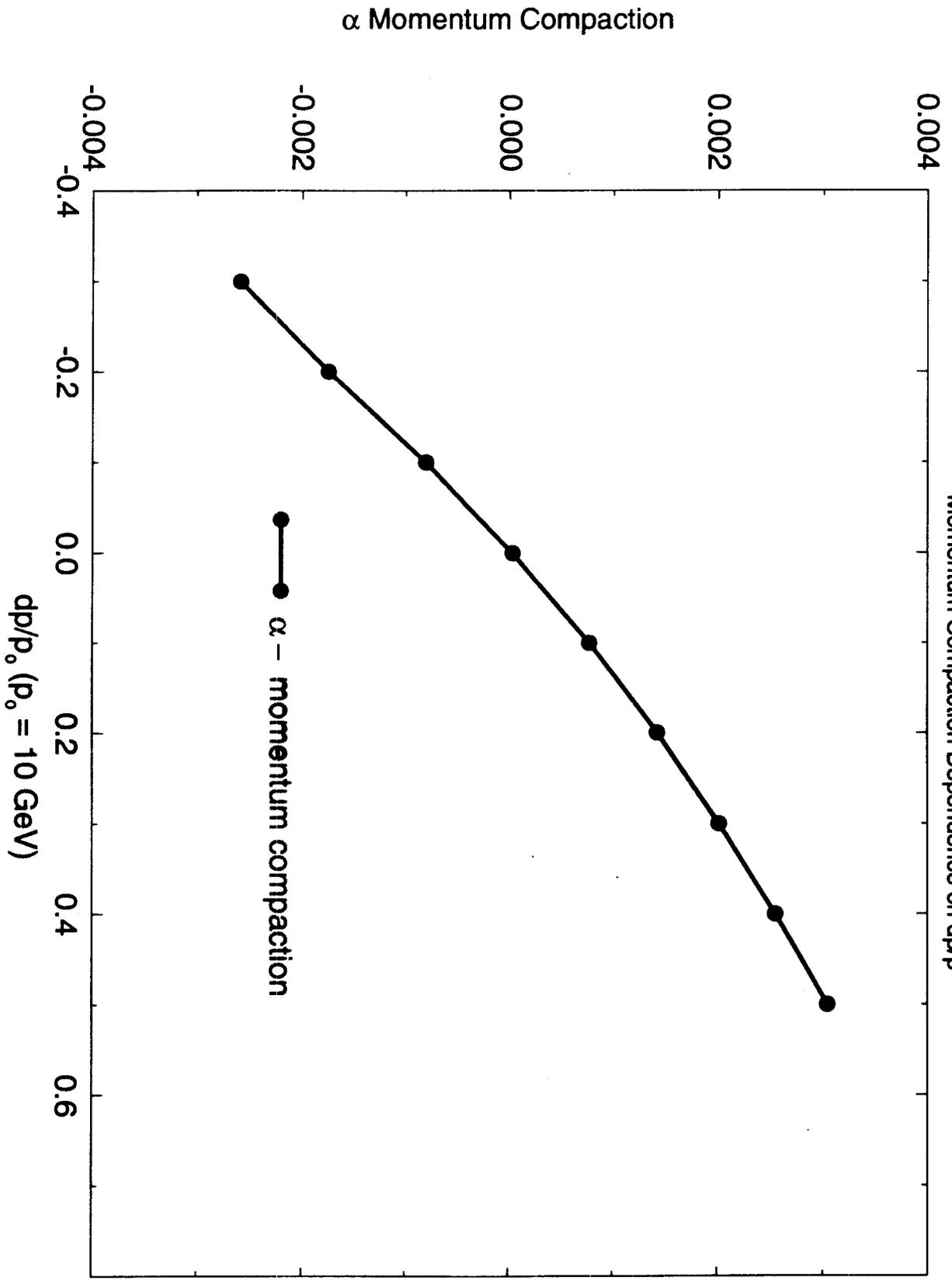


FFAG no opposite bends

Chromaticity Dependence on Momentum



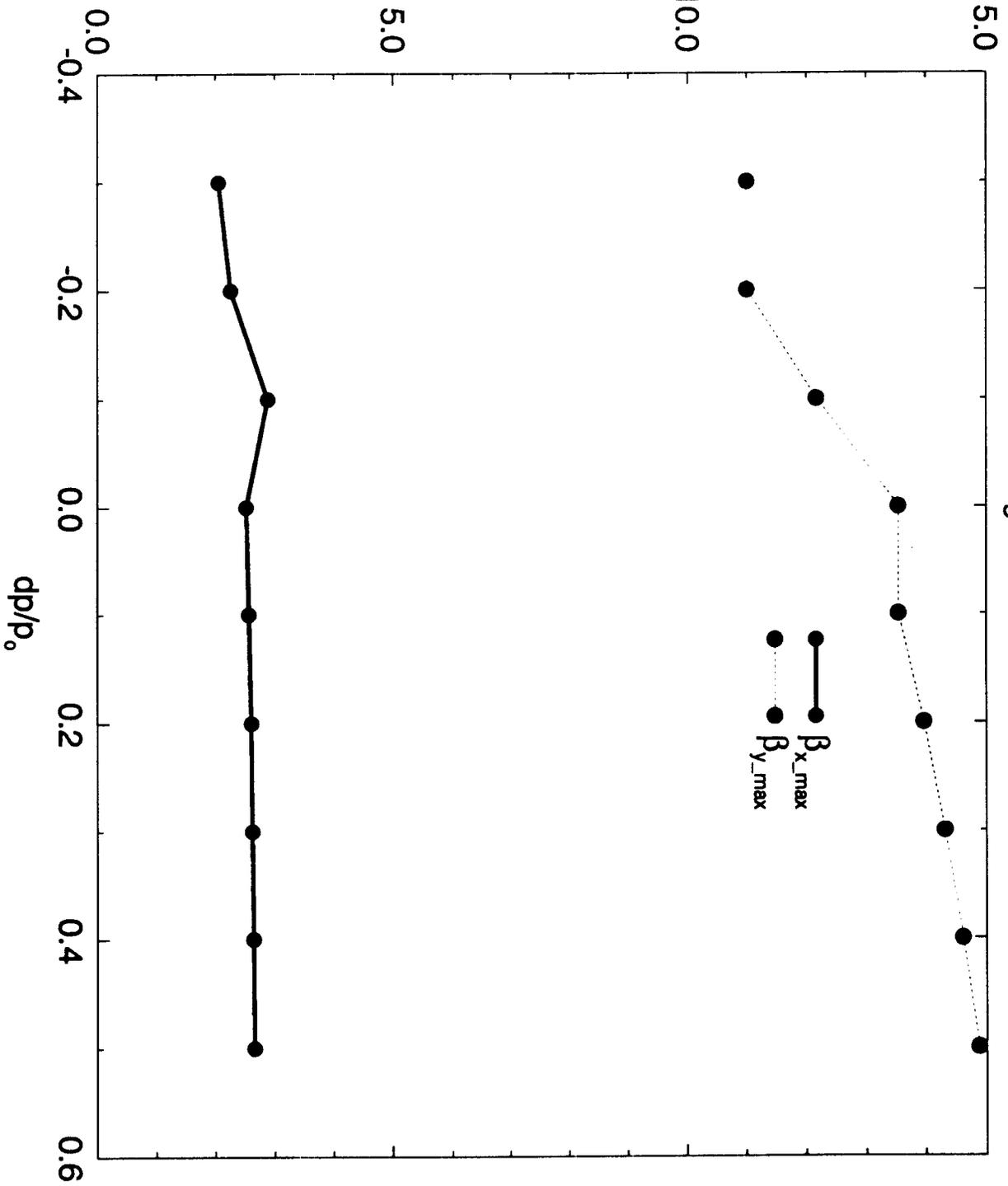
FFAG with no Oposite Bends - 156 m, Muon Energy = 10 GeV
Momentum Compaction Dependence on dp/p



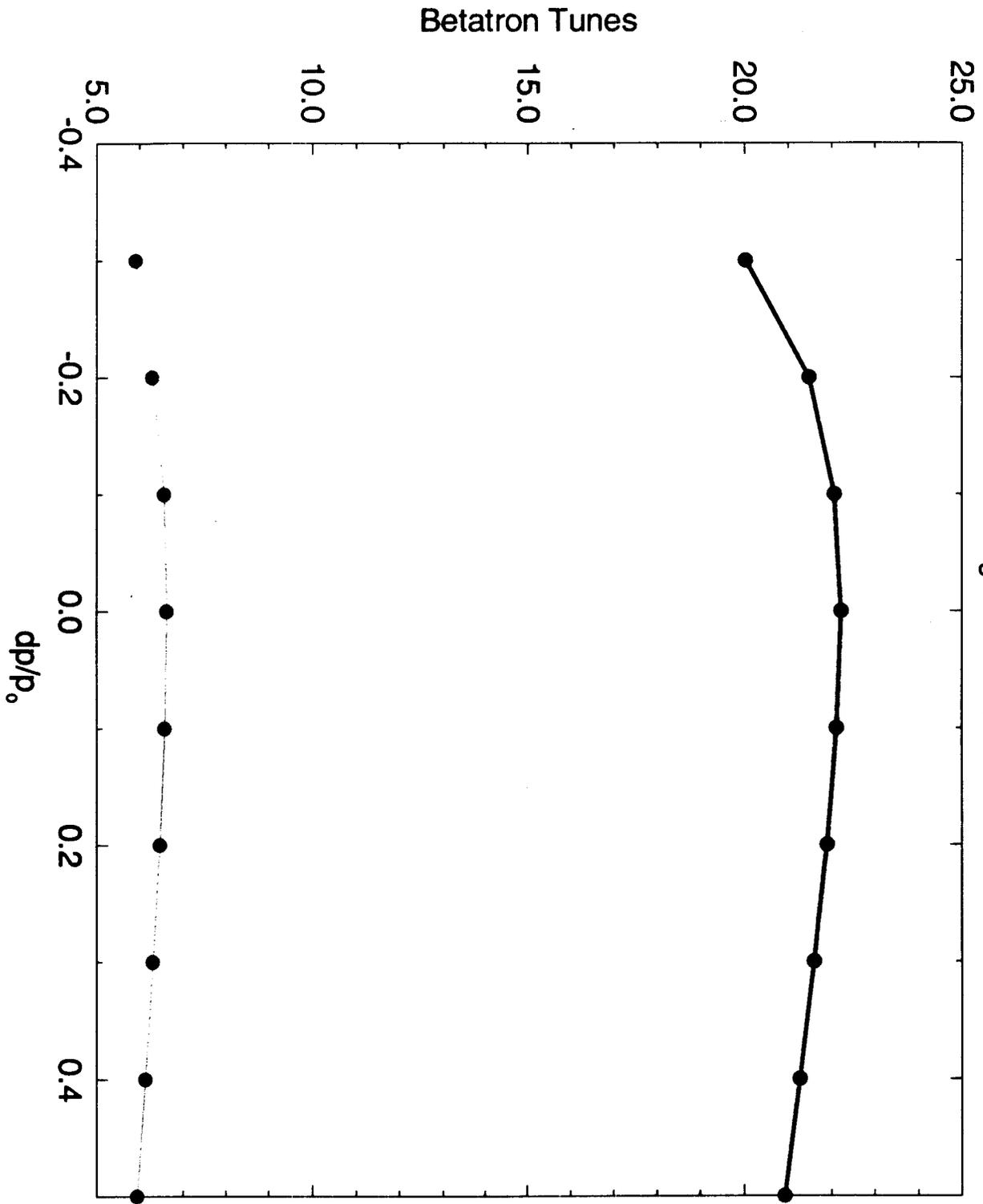
FFAG no opposite bends

Length of the orbit in mm

Maximae of the β_x and β_y (m)

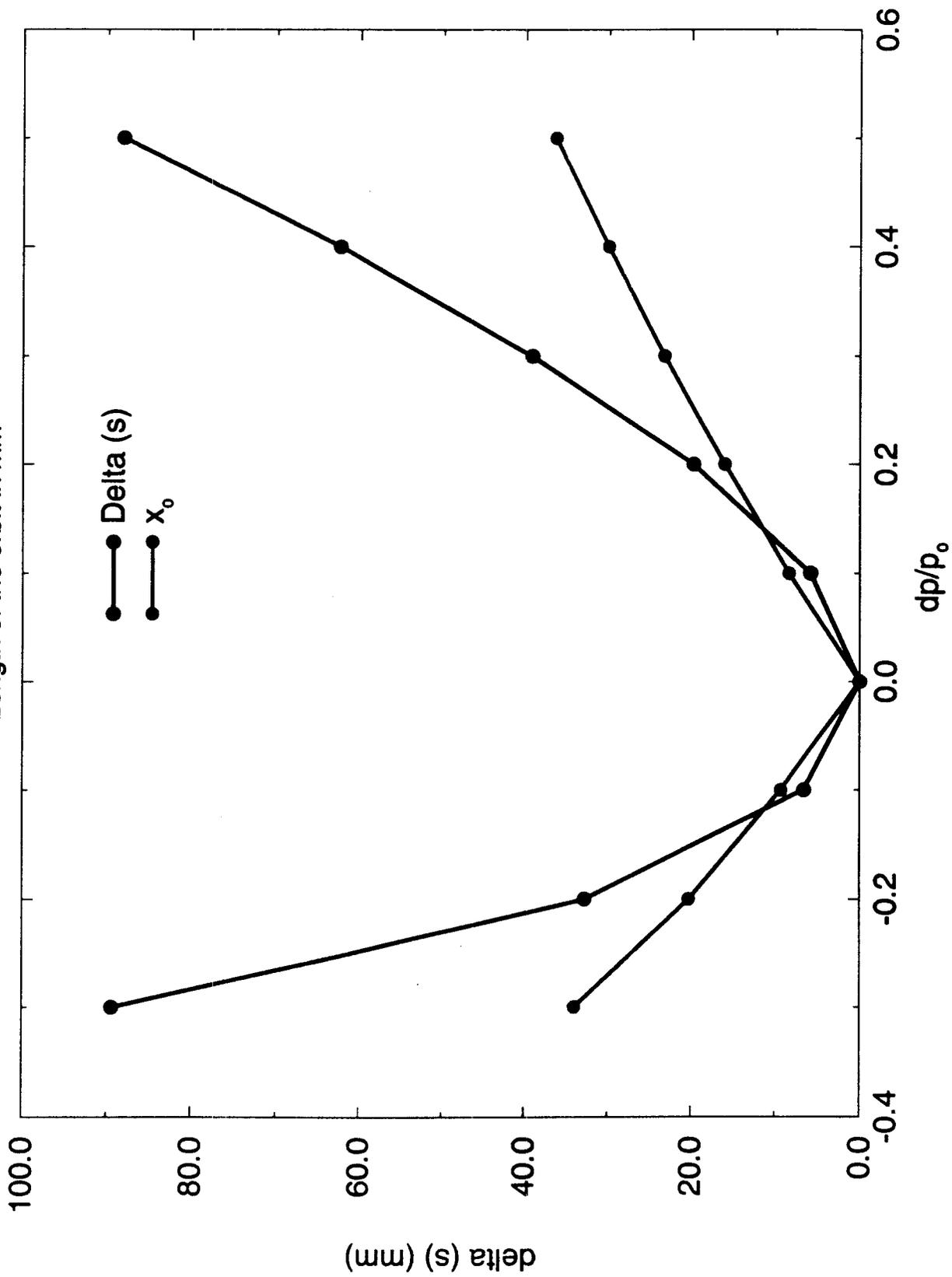


FFAG no opposite bends
Length of the orbit in mm



FFAG no opposite bends

Length of the orbit in mm



FFAG with no opposite BENDS, 156 m (c11.mad)

